VECTORS

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CONTENT

8.1 VECTORS

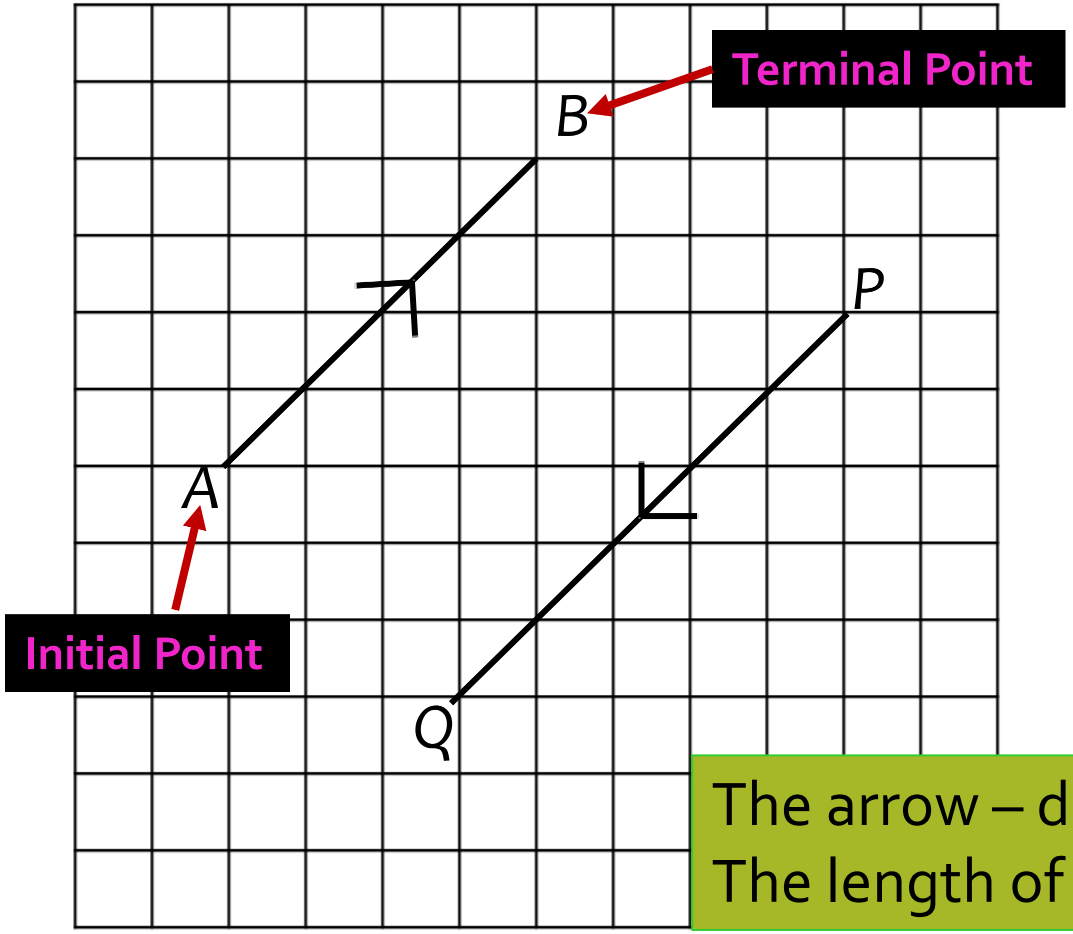
8.1.1 COMPARING DIFFERENCES : VECTOR AND SCALAR

Scalar quantity	Vector Quantity
Has magnitude only	Has magnitude and direction
Example : i) Distance ii) Speed iii)Mass	Example : i) Displacement ii) Velocity iii)Weight

8.1 VECTORS

8.1.2 VECTOR REPRESENTATIONS

1) Directed Line Segment



The arrow – direction
The length of line - magnitude

2) Vector Notations

$\overrightarrow{AB}$	**AB**	$\underset{\sim}{a}$	**a**
$\overrightarrow{PQ}$	**PQ**	$\underset{\sim}{b}$	**b**

8.1 VECTORS

8.1.2 VECTOR REPRESENTATIONS

Zero vectors

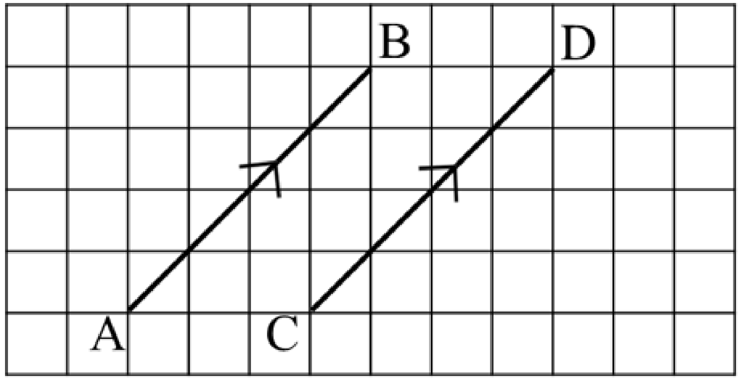
Vectors that consist of zero magnitude and the direction cannot be determined.

Can be represented as $\underset{\sim}{0}$

Equal vectors

Two vectors are the same/equal if both vectors consist of same magnitude and same direction,

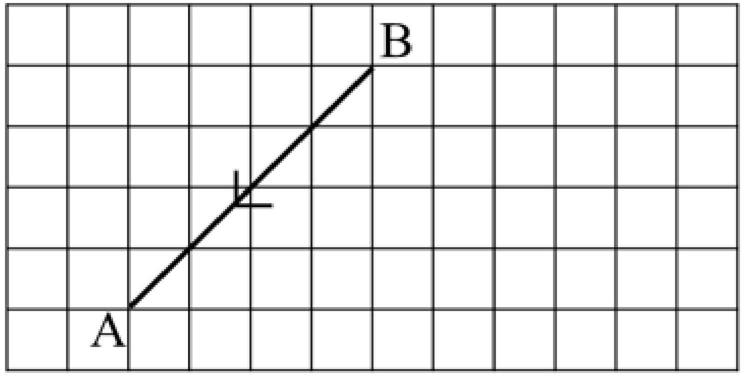
$$\overrightarrow{AB} = \overrightarrow{CD}$$



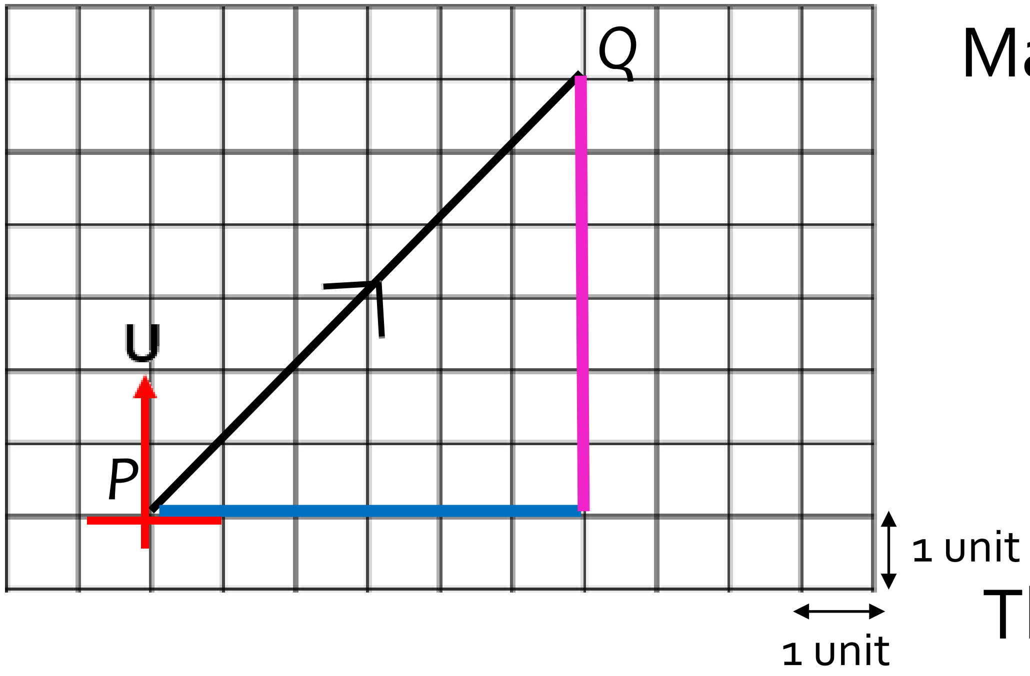
Negative vectors

A vector is negative if the vector consist of the same magnitude but is moving in the opposite direction.

$$\overrightarrow{BA} = -\overrightarrow{AB}$$



Magnitude and Direction



Magnitude of vector $\overrightarrow{PQ}$ $= \sqrt{6^2 + 6^2}$

$= \sqrt{72}$

$= \sqrt{36 \times 2}$

$= 6\sqrt{2}$ unit

The direction of $\overrightarrow{PQ}$ is to the Northeast

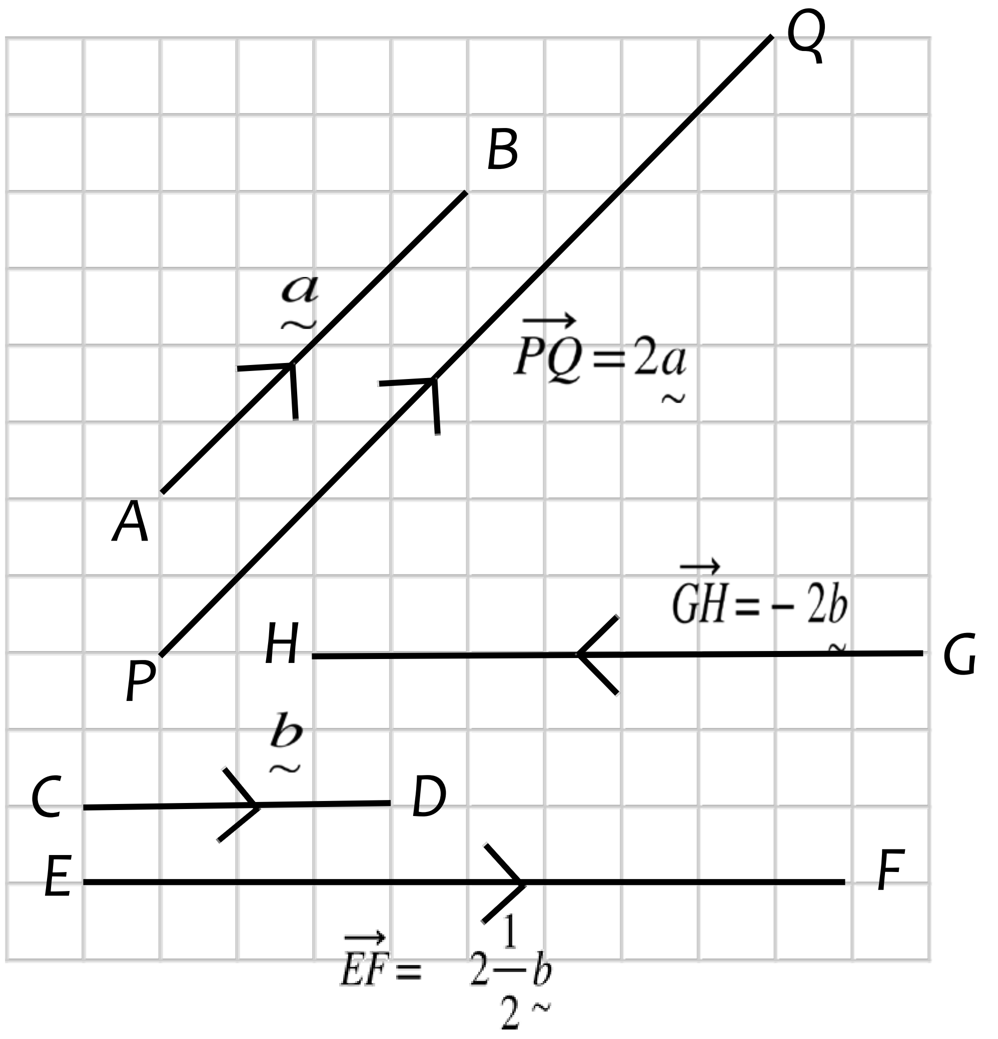
8.1.3 PROPERTIES OF SCALAR MULTIPLICATION ON VECTORS

Conclusion :

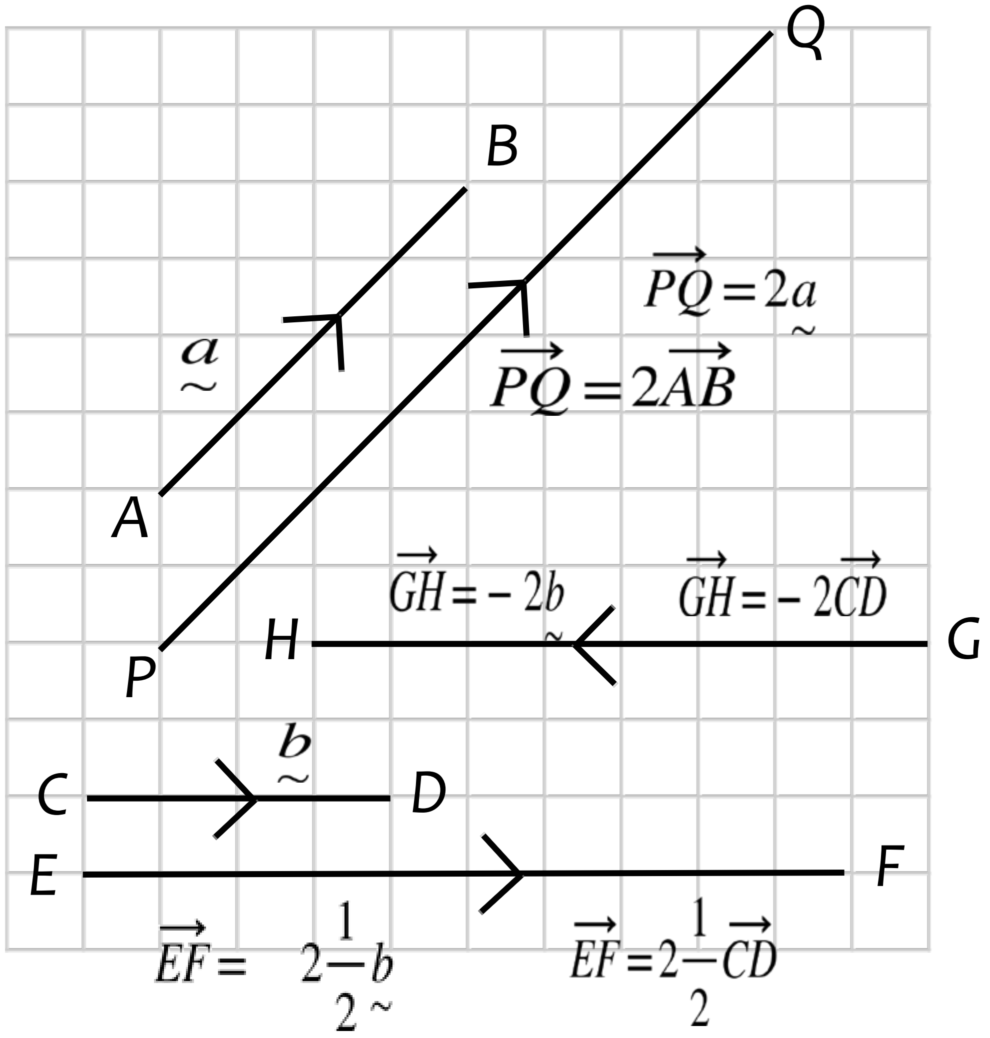
(a) $|k\underset{\sim}{a}| = k|\underset{\sim}{a}|$

(b) $k\underset{\sim}{a}$ is in the same direction with $\underset{\sim}{a}$ if $k > 0$

(c) $k\underset{\sim}{a}$ is in the opposite direction with $\underset{\sim}{a}$ if $k < 0$



8.1.4 PARALLEL VECTORS



Two vectors are parallel if and only if one vector is a scalar multiple of the other vector

If $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are two non-zero vectors and are not parallel, then $h\underset{\sim}{a} = k\underset{\sim}{b}$, thus $h = k = 0$

8.1.4 PARALLEL VECTORS

Conclusion :

(a) Two vectors $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are parallel if

$$\overrightarrow{AB} = \lambda\overrightarrow{CD}$$

where $\lambda \neq 0$

(b) Two vector $\overrightarrow{CD}$ and $\overrightarrow{EF}$ are non-parallel if

$$\overrightarrow{CD} = \lambda\overrightarrow{EF}$$

where $\lambda = 0$

LET'S START

8.1.4 PARALLEL VECTORS

EXAMPLE 1

Given that $\overrightarrow{AB} = 2\boldsymbol{x} + \boldsymbol{y}$ and $\overrightarrow{PQ} = 6\boldsymbol{x} + 3\boldsymbol{y}$, show that $\overrightarrow{AB}$ is parallel to $\overrightarrow{PQ}$.

Solution 1:

$$\overrightarrow{AB} = \lambda\overrightarrow{PQ}$$

$$2\boldsymbol{x} + \boldsymbol{y} = \lambda(6\boldsymbol{x} + 3\boldsymbol{y})$$

$$2\boldsymbol{x} + \boldsymbol{y} = 6\lambda\boldsymbol{x} + 3\lambda\boldsymbol{y}$$

By comparing: $2 = 6\lambda$ or $1 = 3\lambda$

$$\lambda = \frac{1}{3}$$

Don't forget me

$\overrightarrow{AB} = \lambda\overrightarrow{PQ}$

$$\overrightarrow{AB} = \frac{1}{3}\overrightarrow{PQ}$$

Conclusion : vector $\overrightarrow{AB}$ is parallel to vector $\overrightarrow{PQ}$

8.1.4 PARALLEL VECTORS

EXAMPLE 2

Given that $\boldsymbol{a}$ and $\boldsymbol{b}$ are two non-zero vectors and they are not parallel to each other and $(2h-1)\boldsymbol{a} + 3k\boldsymbol{b} = 3\boldsymbol{a} + 9\boldsymbol{b}$, find the values of h and k.

Solution 2 :

Method 1

Compare the coefficient of $\boldsymbol{a}$ and $\boldsymbol{b}$.

$$2h - 1 = 3 \qquad 3k = 9$$

$$h = 2, k = 3$$

Method 2

$h = k = 0$

$$(2h-1)\boldsymbol{a} - 3\boldsymbol{a} = 9\boldsymbol{b} - 3k\boldsymbol{b}$$

$$(2h-4)\boldsymbol{a} = (9-3k)\boldsymbol{b}$$

$$2h - 4 = 0 \qquad 3k = 9$$

$$h = 2, k = 3$$

8.1 VECTORS

Given that $PQ : QR = 2:3$ $\frac{PQ}{QR} = \frac{2}{3}$

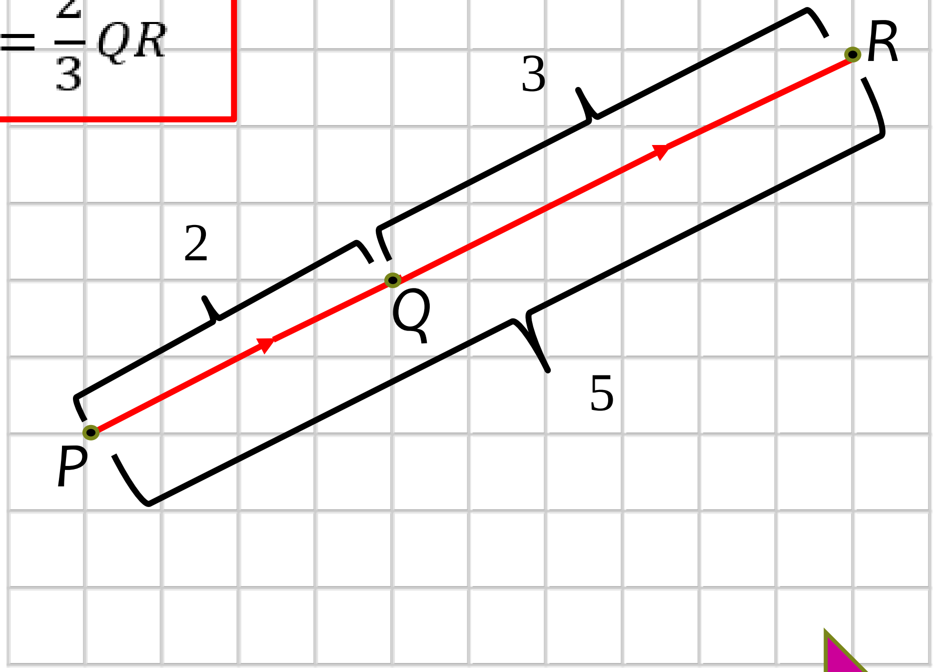
Condition for *P*, *Q* and *R* are collinear

i) the first vector is a scalar multiple of second vector

$PR = \frac{5}{2}PQ$

$PR = \frac{5}{3}QR$

$PQ = \frac{2}{3}QR$



ii) Both vectors are parallel

$\overrightarrow{PR} // \overrightarrow{PQ}$ $\overrightarrow{PQ} // \overrightarrow{QR}$

iii) **has a common point**

P, Q and R are collinear

LET'S START

8.1.4 PARALLEL VECTORS

EXAMPLE 3

Given $\overrightarrow{PQ} = 3\boldsymbol{x} + 3\boldsymbol{y}$ and $\overrightarrow{PR} = 5\boldsymbol{x} + 5\boldsymbol{y}$, show that P, Q and R are collinear.

Solution 3:

$$\overrightarrow{PR} = \lambda\overrightarrow{PQ}$$

$$5\boldsymbol{x} + 5\boldsymbol{y} = \lambda(3x + 3y)$$

$$5\boldsymbol{x} + 5\boldsymbol{y} = 3\lambda\boldsymbol{x} + 3\lambda\boldsymbol{y}$$

By comparing: $5 = 3\lambda \rightarrow \lambda = \frac{5}{3}$

Thus, $\overrightarrow{PR} = \frac{5}{3}\overrightarrow{PQ}$

Therefore P, Q, R are collinear

Don't forget me

$\overrightarrow{PR} = \lambda\overrightarrow{PQ}$ // $\overrightarrow{PQ} = \lambda\overrightarrow{PR}$

Alternative:

$$\overrightarrow{PQ} = \lambda\overrightarrow{PR}$$

$$3\boldsymbol{x} + 3\boldsymbol{y} = \lambda(5\boldsymbol{x} + 5\boldsymbol{y})$$

$$3\boldsymbol{x} + 3\boldsymbol{y} = 5\lambda\boldsymbol{x} + 5\lambda\boldsymbol{y}$$

By comparing: $3 = 5\lambda \longrightarrow \lambda = \frac{3}{5}$

Thus, $\overrightarrow{PQ} = \frac{3}{5}\overrightarrow{PR}$

LAS7

8.1.4 PARALLEL VECTORS

EXAMPLE 4

Given that $\overrightarrow{PQ} = (k+2)\boldsymbol{x} + 4\boldsymbol{y}$. If PQ is extended to point R with $\overrightarrow{QR} = h\boldsymbol{x} + \boldsymbol{y}$, express k in terms of h.

Solution 4 :

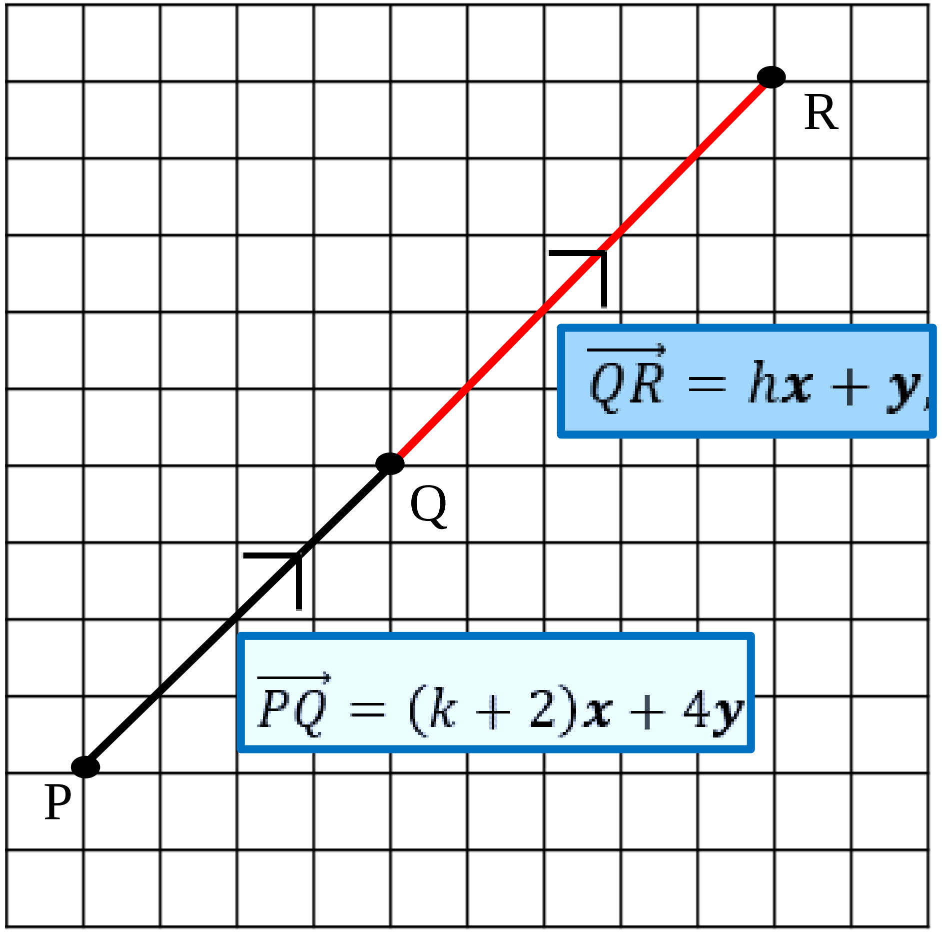
$$\overrightarrow{PQ} = \lambda\overrightarrow{QR}$$

$$(k+2)\boldsymbol{x} + 4\boldsymbol{y} = \lambda(h\boldsymbol{x} + \boldsymbol{y})$$

$$(k+2)\boldsymbol{x} + 4\boldsymbol{y} = h\lambda\boldsymbol{x} + \lambda\boldsymbol{y}$$

$$k+2 = h\lambda \qquad 4 = \lambda$$

$$k = 4h - 2$$

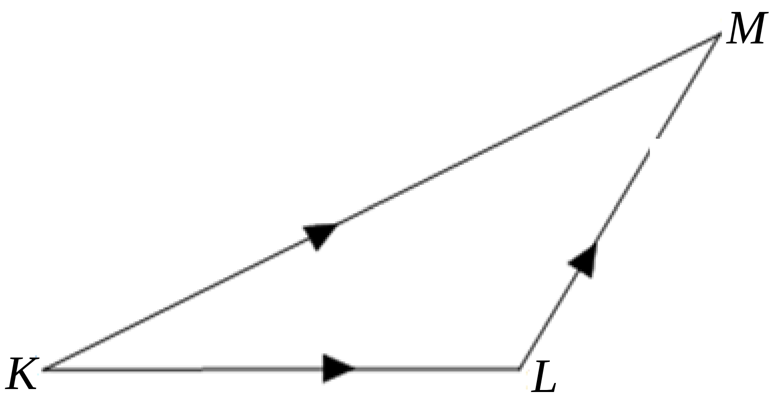


CONTENT

• 8.2 ADDITIONAL AND SUBTRACTION OF VECTORS

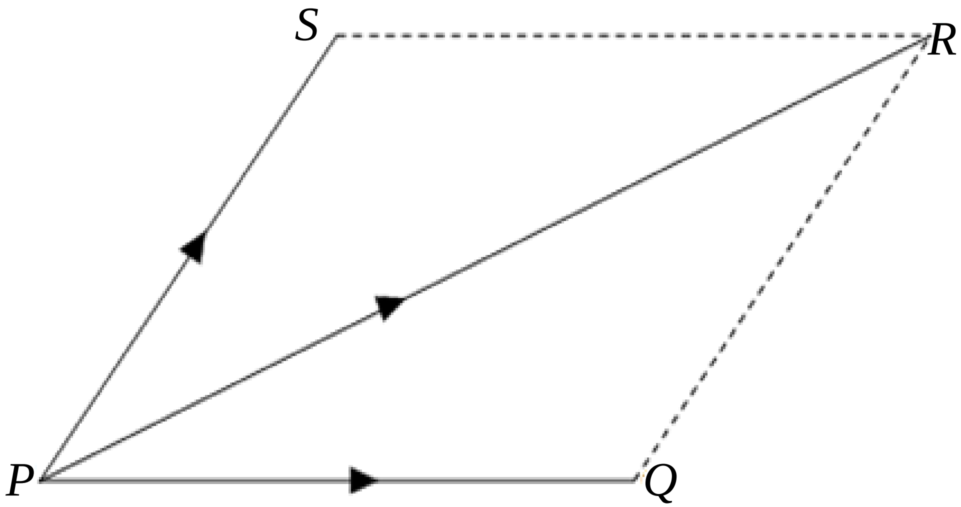
8.2.1 RESULTANT VECTORS OF TWO NON-PARALLEL VECTORS

TRIANGLE LAW



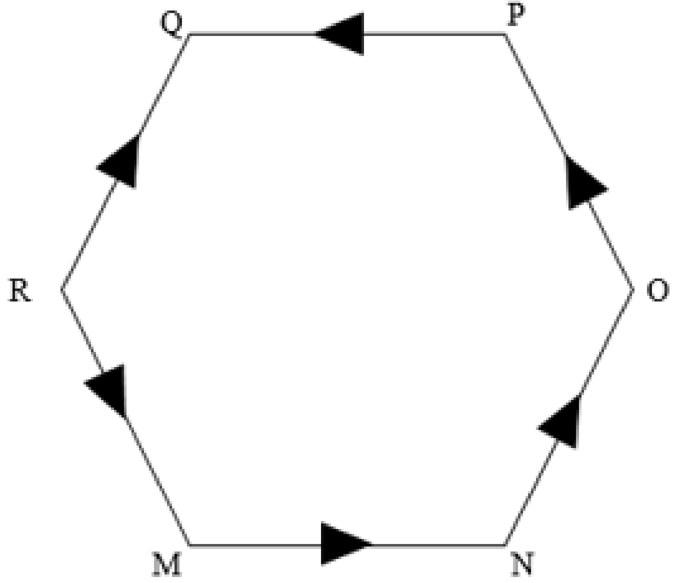
$$\overrightarrow{KM} = \overrightarrow{KL} + \overrightarrow{LM}$$

PARALLELOGRAM LAW



$$\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QR} = \overrightarrow{PS} + \overrightarrow{SR}$$

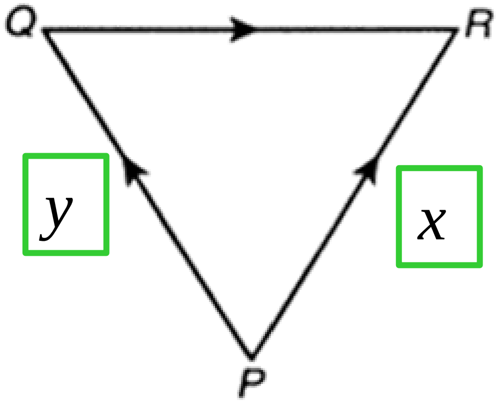
POLYGON LAW



$$\overrightarrow{RQ} = \overrightarrow{RM} + \overrightarrow{MN} + \overrightarrow{NO} + \overrightarrow{OP} + \overrightarrow{PQ}$$

8.2.1 RESULTANT VECTORS OF TWO NON-PARALLEL VECTORS

EXAMPLE 5



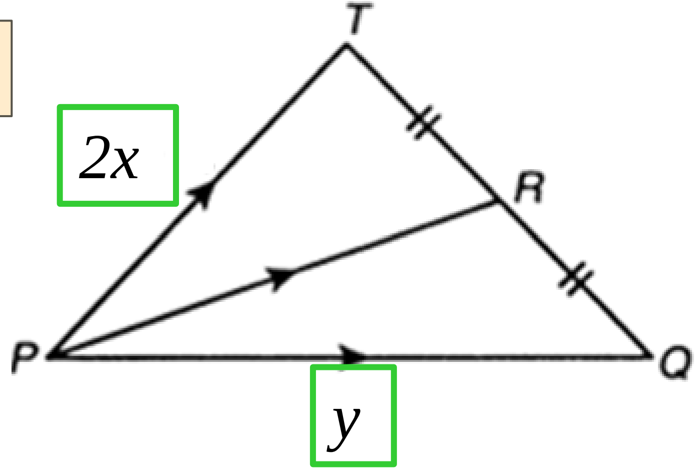
Find vector $\overrightarrow{QR}$

Solution 5:

$$\overrightarrow{QR} = \overrightarrow{QP} + \overrightarrow{PR}$$

$$\overrightarrow{QR} = -y + x$$

EXAMPLE 6



Find vector $\overrightarrow{TR}$

Solution 6:

$$\overrightarrow{TQ} = \overrightarrow{TP} + \overrightarrow{PQ}$$

$$\overrightarrow{TQ} = -2\boldsymbol{x} + \boldsymbol{y}$$

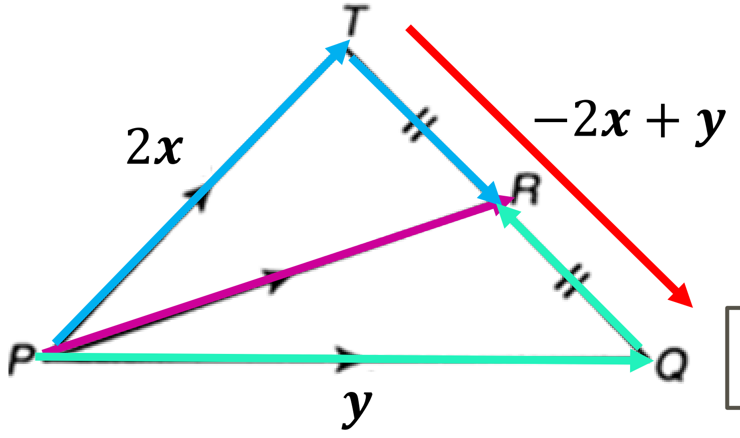
$$\overrightarrow{TR} = \frac{1}{2}\overrightarrow{TQ}$$

$$\overrightarrow{TR} = \frac{1}{2}(-2x + y)$$

$$= -x + \frac{1}{2}y$$

8.2.1 RESULTANT VECTORS OF TWO NON-PARALLEL VECTORS

EXAMPLE 7



Find vector $\overrightarrow{PR}$

Solution 7:

Method 1

$$\overrightarrow{PR} = \overrightarrow{PT} + \overrightarrow{TR}$$

$$\overrightarrow{PR} = \overrightarrow{PT} + \frac{1}{2}\overrightarrow{TQ}$$

$$\overrightarrow{PR} = 2\boldsymbol{x} + \frac{1}{2}(-2\boldsymbol{x} + \boldsymbol{y})$$

$$\overrightarrow{PR} = 2x - x + \frac{1}{2}y$$

$$\overrightarrow{PR} = \boldsymbol{x} + \frac{1}{2}\boldsymbol{y}$$

Method 2

$$\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QR}$$

$$\overrightarrow{PR} = \overrightarrow{PQ} + \frac{1}{2}\overrightarrow{QT}$$

$$\overrightarrow{PR} = \overrightarrow{PQ} - \frac{1}{2}\overrightarrow{TQ}$$

$$\overrightarrow{PR} = \boldsymbol{y} - \frac{1}{2}(-2\boldsymbol{x} + \boldsymbol{y})$$

$$\overrightarrow{PR} = \frac{1}{2}\boldsymbol{y} + \boldsymbol{x}$$

8.2.1 RESULTANT VECTORS OF TWO NON-PARALLEL VECTORS

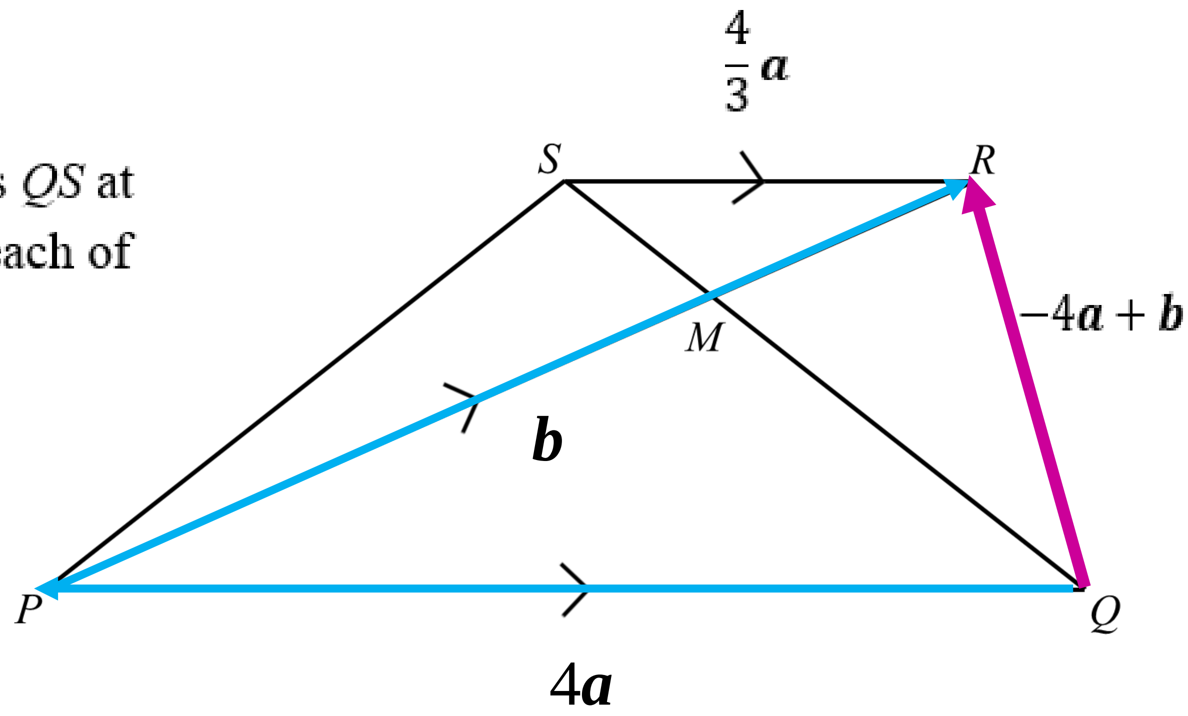
EXAMPLE 8

In the diagram shows, PQ and SR are parallel. PR intersects QS at M. Given that $\overrightarrow{PQ} = 4\boldsymbol{a}$, $\overrightarrow{PR} = \boldsymbol{b}$ and $PQ = 3SR$. Express each of the following vectors in term of $\boldsymbol{a}$ and $\boldsymbol{b}$:

(a) $\overrightarrow{QR}$

(b) $\overrightarrow{PS}$

(c) $\overrightarrow{SQ}$



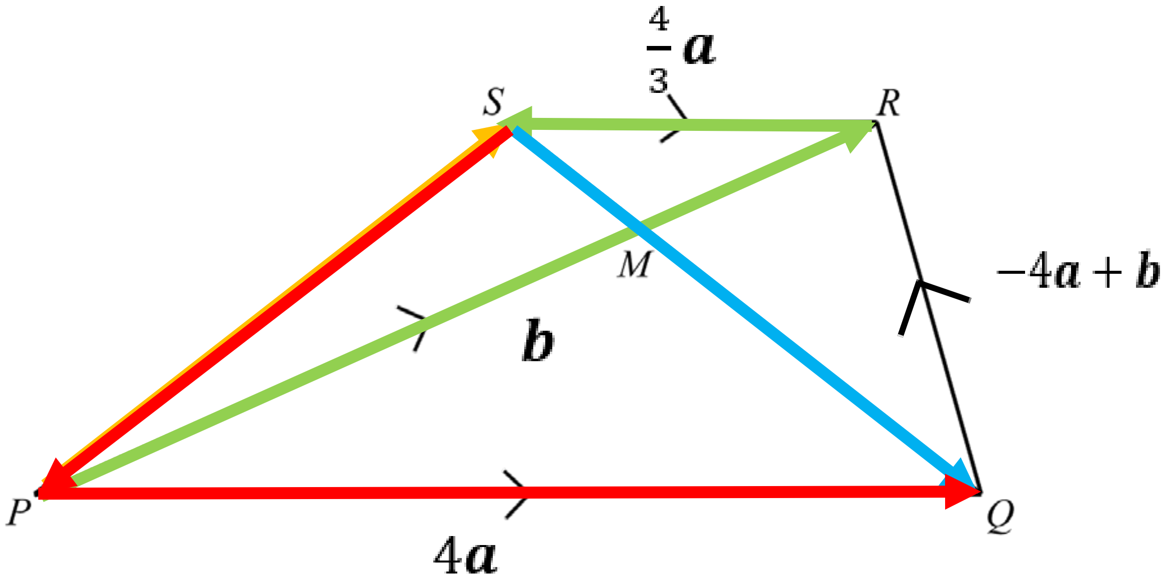
Solution 8a:

$$\overrightarrow{QR} = \overrightarrow{QP} + \overrightarrow{PR}$$

$$\overrightarrow{QR} = -4\boldsymbol{a} + \boldsymbol{b}$$

8.2.1 RESULTANT VECTORS OF TWO NON-PARALLEL VECTORS

EXAMPLE 8



(a) $\overrightarrow{QR}$

(b) $\overrightarrow{PS}$

(c) $\overrightarrow{SQ}$

Solution 8b:

$$\overrightarrow{PS} = \overrightarrow{PR} + \overrightarrow{RS}$$

$$\overrightarrow{PS} = \boldsymbol{b} + \left(-\frac{4}{3}\boldsymbol{a}\right)$$

$$\overrightarrow{PS} = -\frac{4}{3}\boldsymbol{a} + \boldsymbol{b}$$

Solution 8c:

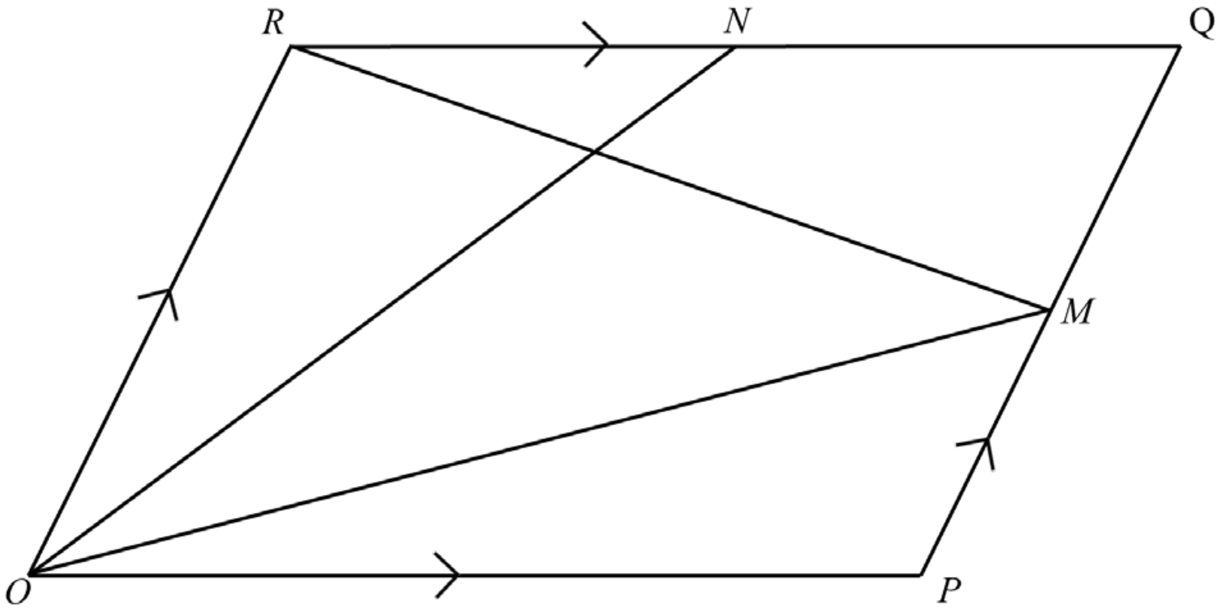
$$\overrightarrow{SQ} = \overrightarrow{SP} + \overrightarrow{PQ}$$

$$\overrightarrow{SQ} = -\left(-\frac{4}{3}\boldsymbol{a} + \boldsymbol{b}\right) + 4\boldsymbol{a}$$

$$\overrightarrow{SQ} = \frac{16}{3}\boldsymbol{a} - \boldsymbol{b}$$

8.2.1 RESULTANT VECTORS OF TWO NON-PARALLEL VECTORS

EXAMPLE 9



The diagram shows a parallelogram $OPQR$. M and N are the midpoints Of PQ and QR respectively.

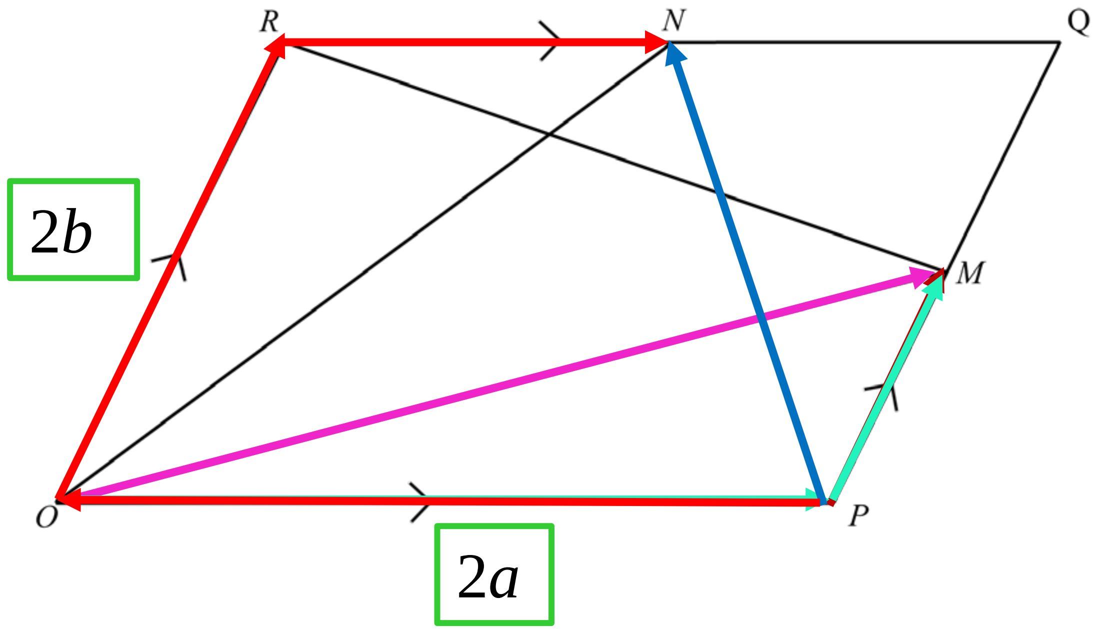
Given that $\overrightarrow{OP} = 2\boldsymbol{a}$ and $\overrightarrow{OR} = 2\boldsymbol{b}$, express in terms of $\boldsymbol{a}$ and/or $\boldsymbol{b}$.

(a) $\overrightarrow{PM}$

(b) $\overrightarrow{OM}$

(c) $\overrightarrow{PN}$

EXAMPLE 9



The diagram shows a parallelogram $OPQR$. M and N are the midpoints Of PQ and QR respectively.

Given that $\overrightarrow{OP} = 2\boldsymbol{a}$ and $\overrightarrow{OR} = 2\boldsymbol{b}$, express in terms of $\boldsymbol{a}$ and/or $\boldsymbol{b}$.

(a) $\overrightarrow{PM}$

(b) $\overrightarrow{OM}$

(c) $\overrightarrow{PN}$

Solution 9a:

$$\overrightarrow{PM} = \frac{1}{2}\overrightarrow{PQ}$$

$$\overrightarrow{PM} = \frac{1}{2}(2\boldsymbol{b}) = \boldsymbol{b}$$

Solution 9b:

$$\overrightarrow{OM} = \overrightarrow{OP} + \overrightarrow{PM}$$

$$\overrightarrow{OM} = 2\boldsymbol{a} + \boldsymbol{b}$$

Solution 9c :

$$\overrightarrow{PN} = \overrightarrow{PO} + \overrightarrow{OR} + \overrightarrow{RN} \quad \text{or} \quad \overrightarrow{PN} = \overrightarrow{PQ} + \overrightarrow{QN}$$

$$\overrightarrow{PN} = -2a + 2b + \boldsymbol{a} = -\boldsymbol{a} + \boldsymbol{2b}$$

LAST

8.2.1 RESULTANT VECTORS OF TWO NON-PARALLEL VECTORS

EXAMPLE 10

In the diagram, $PQRS$ is a parallelogram.

Given that $\overrightarrow{PS} = 6\boldsymbol{a}$, $\overrightarrow{PQ} = 8\boldsymbol{b}$ and $SR : ST = 4:3$. Express in the terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

(a) $\overrightarrow{PT}$
(b) $\overrightarrow{QT}$

$$\frac{SR}{ST} = \frac{4}{3}$$

$$SR = 4, \quad ST = 3, \quad TR = 1$$

$$ST = \frac{3}{4}(8\boldsymbol{b}) = 6\boldsymbol{b}, \quad TR = \frac{1}{4}(8\boldsymbol{b}) = 2\boldsymbol{b}$$

Solution 10a:

$$\overrightarrow{PT} = \overrightarrow{PS} + \overrightarrow{ST}$$

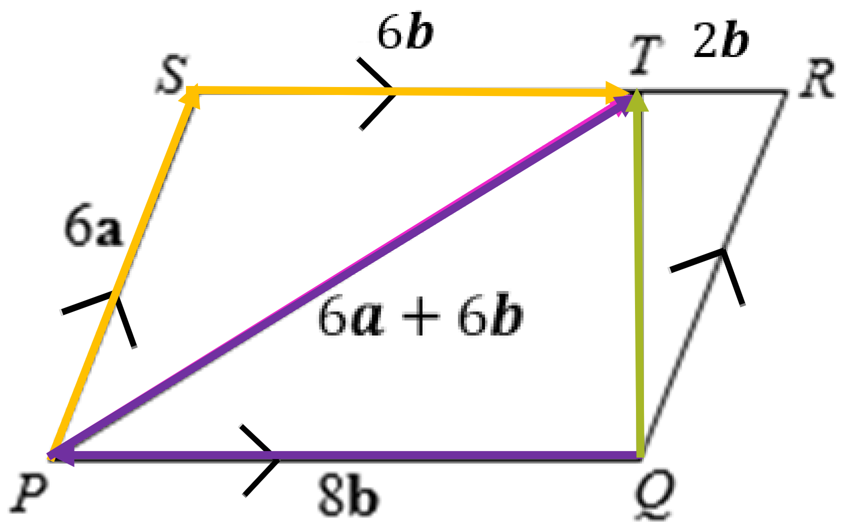
$$\overrightarrow{PT} = 6\boldsymbol{a} + 6\boldsymbol{b}$$

Solution 10b:

$$\overrightarrow{QT} = \overrightarrow{QP} + \overrightarrow{PT}$$

$$\overrightarrow{QT} = -8\boldsymbol{b} + (6\boldsymbol{a} + 6\boldsymbol{b})$$

$$\overrightarrow{QT} = 6\boldsymbol{a} - 2\boldsymbol{b}$$



SUMMARY

VECTORS

8.1 VECTORS

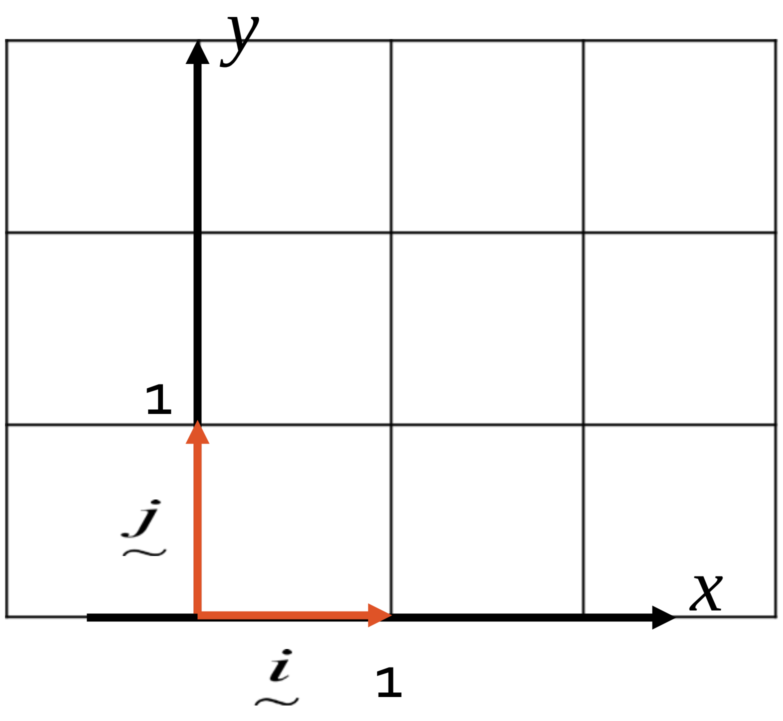
1. What is vector?
2. Notation of vector
3. Zero vector
4. Negative vector
5. Parallel, collinear vector and its different
6. Concept of non-linear vector

8.2 ADDITIONAL AND SUBTRACTION OF VECTORS

1. Addition and subtraction of vectors
2. Triangle law
3. Polygon law
4. Parallelogram law

CONTENT

8.3.1 REPRESENTING VECTORS AND DETERMINING THE MAGNITUDE OF VECTORS IN CARTESIAN PLANE



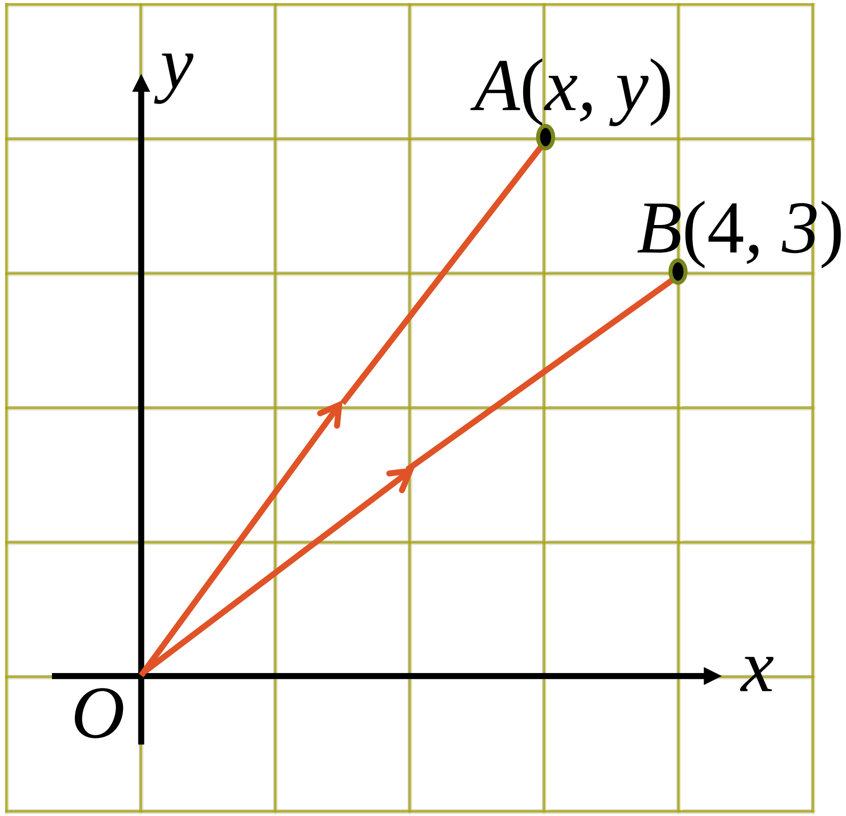
Vector with magnitude 1 unit and parallel with *x*-axis is called vector $\underset{\sim}{i}$ and written as

$$\underset{\sim}{i} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, |\underset{\sim}{i}| = 1$$

Vector with magnitude 1 unit and parallel with *y*-axis is called vector $\underset{\sim}{j}$ and written as

$$\underset{\sim}{j} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, |\underset{\sim}{j}| = 1$$

8.3.1 REPRESENTING VECTORS AND DETERMINING THE MAGNITUDE OF VECTORS IN CARTESIAN PLANE



The position vector of point A relative to point O is $\overrightarrow{OA}$

$$\overrightarrow{OA} = x\underset{\sim}{i} + y\underset{\sim}{j} \quad or \quad \overrightarrow{OA} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$Magnitude\ of\ \overrightarrow{OA} = \sqrt{x^2 + y^2}$$

EXAMPLE 11

$$\overrightarrow{OB} = 4\underset{\sim}{i} + 3\underset{\sim}{j} \quad \text{or} \quad \overrightarrow{OB} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$$

$$\left|\overrightarrow{OB}\right| = \sqrt{4^2 + 3^2} = \sqrt{25}$$

$$\left|\overrightarrow{OB}\right| = 5$$

8.3.2 DESCRIBE AND DETERMINE THE UNIT VECTOR IN THE DIRECTION OF A VECTOR

Unit vector is a vector in the direction of a vector **with magnitude of 1 unit**

In general:

If $\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j}$, then vector unit in direction $\underset{\sim}{r}$ is

$$\hat{\underset{\sim}{r}} = \frac{\underset{\sim}{r}}{|\underset{\sim}{r}|} = \frac{x\underset{\sim}{i} + y\underset{\sim}{j}}{\sqrt{x^2 + y^2}}.$$

https://www.geogebra.org/m/nzajbghz

8.3.2 DESCRIBE AND DETERMINE THE UNIT VECTOR IN THE DIRECTION OF A VECTOR

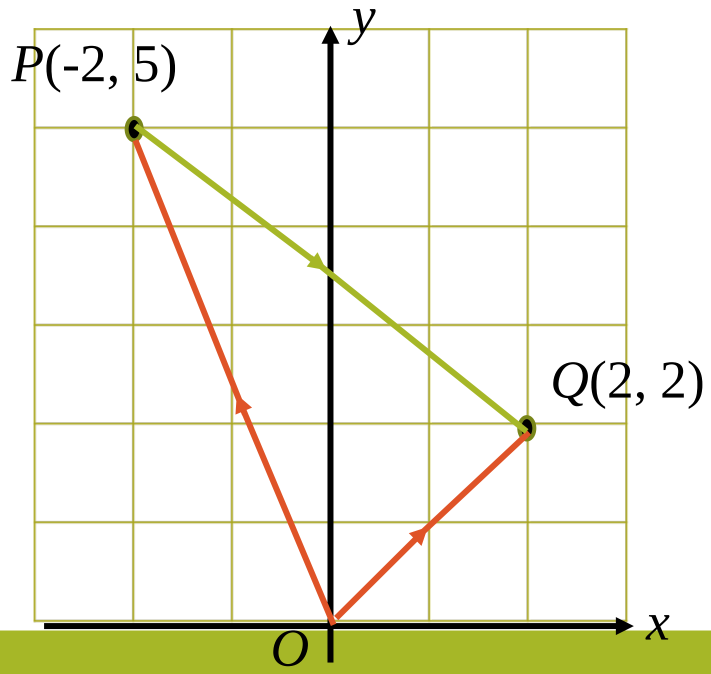
EXAMPLE 12

It is given that $P = (-2, 5)$ and $Q = (2, 2)$. Find,

(a) $\overrightarrow{OP}$, (b) $\overrightarrow{OQ}$,

(c) $\overrightarrow{PQ}$, (d) $|\overrightarrow{PQ}|$,

(e) unit vector in the direction of $\overrightarrow{PQ}$.



Solution 12:

(a) $P(-2, 5)$, Thus, $\overrightarrow{OP} = -2\boldsymbol{i} + 5\boldsymbol{j} = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$

(b) $Q(2, 2)$, Thus, $\overrightarrow{OQ} = 2\boldsymbol{i} + 2\boldsymbol{j} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$

(c) $\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OQ}$

$\overrightarrow{PQ} = -\begin{pmatrix} -2 \\ 5 \end{pmatrix} + \begin{pmatrix} 2 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$

(d) $|\overrightarrow{PQ}| = \sqrt{4^2 + (-3)^2} = 5$

(e) Unit vector in direction of $\overrightarrow{PQ} = \frac{1}{5}(4\boldsymbol{i} - 3\boldsymbol{j}) = \frac{1}{5}\begin{pmatrix} 4 \\ -3 \end{pmatrix}$

8.3.2 DESCRIBE AND DETERMINE THE UNIT VECTOR IN THE DIRECTION OF A VECTOR

EXAMPLE 13

Given that,

$$\boldsymbol{x} = \begin{pmatrix} h+4 \\ 7 \end{pmatrix}, \boldsymbol{y} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}, and\ \boldsymbol{z} = \begin{pmatrix} h \\ 2 \end{pmatrix}$$

Find the values of h if $|\boldsymbol{x} + \boldsymbol{y} - 2\boldsymbol{z}| = 10$

Solution 13:

$$\boldsymbol{x} + \boldsymbol{y} - 2\boldsymbol{z} = \begin{pmatrix} h+4 \\ 7 \end{pmatrix} + \begin{pmatrix} 4 \\ 5 \end{pmatrix} - 2\begin{pmatrix} h \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 8-h \\ 8 \end{pmatrix}$$

Find the magnitude

Use: $|r| = \sqrt{x^2 + y^2}$

$$|\boldsymbol{x} + \boldsymbol{y} - 2\boldsymbol{z}| = 10$$

$$\sqrt{(8-h)^2 + 8^2} = 10$$

$$(8-h)^2 - 64 = 100$$

$$(8-h)^2 = 36$$

$$64 - 16h + h^2 = 36$$

$$h^2 - 6h + 28 = 0$$

$$(h-2)(h-14) = 0$$

$8 - h = 6$ $\quad$ $8 - h = -6$

$h = 2$ $\quad$ $h = 14$

EXAMPLE 14

Vector $\boldsymbol{p} = (m-1)\boldsymbol{i} + 2\boldsymbol{j}$ is orthogonal with vector $\boldsymbol{q} = 8\boldsymbol{i} + n\boldsymbol{j}$.
Express m in terms of n.

Solution 14:

$O(0,0) \quad P(m-1, 2) \quad Q(8, n)$

$m_1 \times m_2 = -1$

$$m_1 = \frac{2}{m-1}$$

$$m_2 = \frac{n}{8}$$

$$\frac{2}{m-1} \times \frac{n}{8} = -1$$

$$m = \frac{4-n}{4}$$

LAST

EXAMPLE 15

Given $\boldsymbol{p} = m\boldsymbol{i} - n\boldsymbol{j}$ is a unit vector in the direction of vector $\boldsymbol{p}$, express m in terms of n.

Solution 15:

Unit vector of vector $\boldsymbol{p} = \frac{1}{1}(m\boldsymbol{i} - n\boldsymbol{j})$

Magnitude $|\boldsymbol{p}| = 1$

Vector $\boldsymbol{p} = m\boldsymbol{i} - n\boldsymbol{j}$

Magnitude $|\boldsymbol{p}| = \sqrt{m^2 + (-n)^2} = 1$

$$m^2 + n^2 = 1$$

$$m^2 = 1 - n^2$$

$$m = \sqrt{1 - n^2}$$

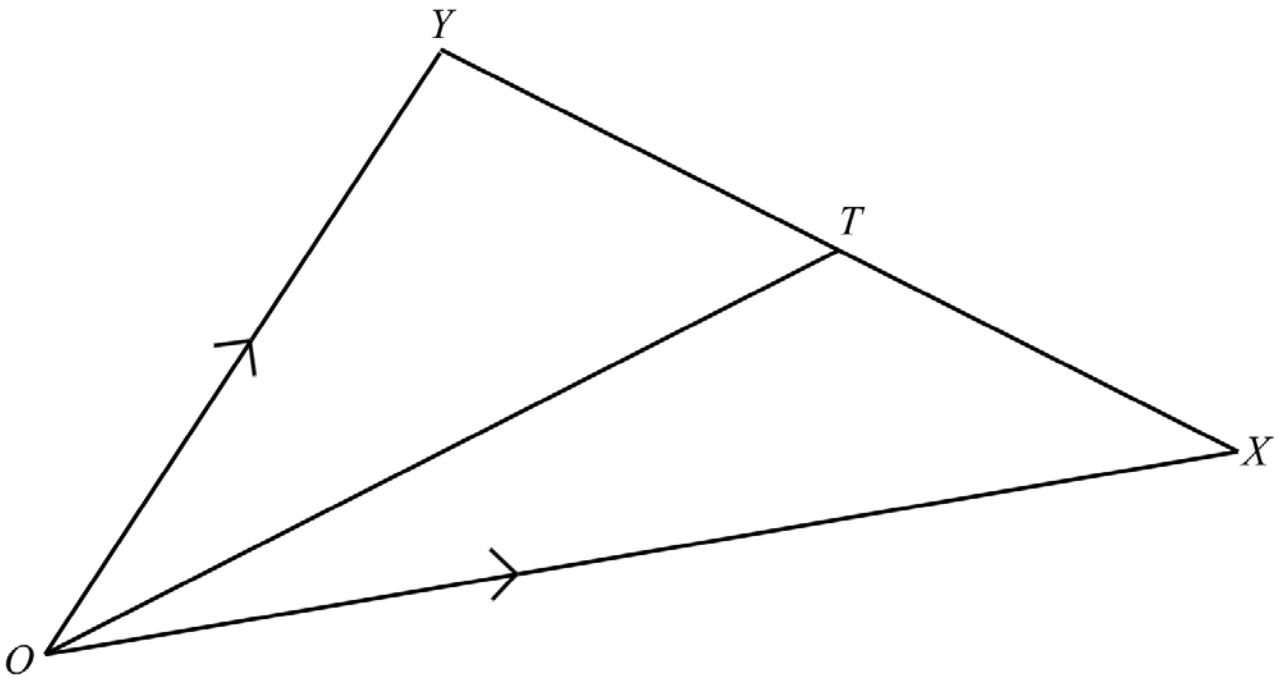
Long questions

SECTION A
6< marks < 8

SECTION B
10 marks

EXAMPLE 16

In the diagram OXY is a triangle with $\overrightarrow{OX} = \boldsymbol{x}$ and $\overrightarrow{OY} = \boldsymbol{y}$

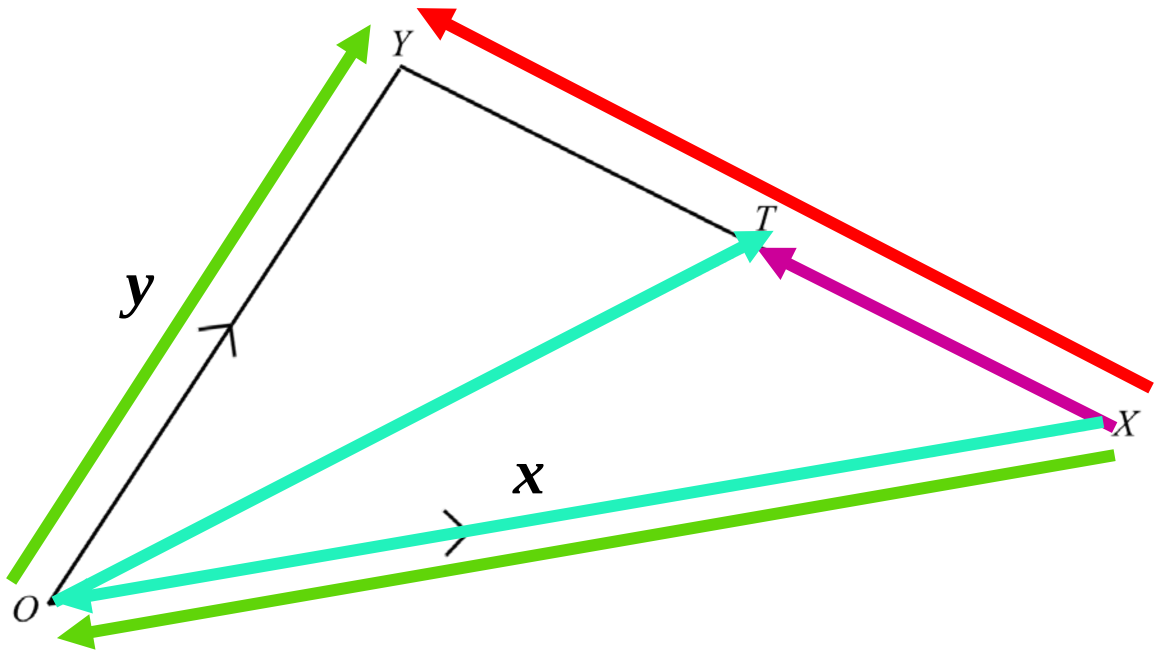


T is a point on XY such that $XT = mXY$, where m is a constant.

(a) Express $\overrightarrow{OT}$ in terms of m, x and y.

(b) Hence, if a is another vector such that $a = \frac{2}{5}\boldsymbol{x} + \frac{7}{20}\boldsymbol{y}$ and a is parallel to $\overrightarrow{OT}$, find the value of m.

EXAMPLE 16a



T is a point on XY such that $XT = mXY$, where m is a constant.

(a) Express $\overrightarrow{OT}$ in terms of m, x and y.

Solution 16a :

$$\overrightarrow{XT} = m\overrightarrow{XY}$$

$$\overrightarrow{XO} + \overrightarrow{OT} = m(\overrightarrow{XO} + \overrightarrow{OY})$$ **K1**

$$-x + \overrightarrow{OT} = m(-x + y)$$ **K1**

$$-\boldsymbol{x} + \overrightarrow{OT} = -m\boldsymbol{x} + m\boldsymbol{y}$$

$$\overrightarrow{OT} = \boldsymbol{x} - m\boldsymbol{x} + m\boldsymbol{y}$$

$$\boxed{\overrightarrow{OT} = \boldsymbol{x} - m\boldsymbol{x} + m\boldsymbol{y}}$$ **N1**

EXAMPLE 16b

(b) Hence, if a is another vector such that $a = \frac{2}{5}\boldsymbol{x} + \frac{7}{20}\boldsymbol{y}$ and a is parallel to $\overrightarrow{OT}$, find the value of m.

$$\overrightarrow{OT} = (1 - m)x + y$$

Solution 16b :

$$\boldsymbol{a} = \lambda \overrightarrow{OT}$$ **K1**

$$\frac{2}{5}x + \frac{7}{20}y = \lambda[(1 - m)x + y]$$ **K1**

$$\frac{2}{5}\boldsymbol{x} + \frac{7}{20}\boldsymbol{y} = (1 - m)\lambda\boldsymbol{x} + m\lambda\boldsymbol{y}$$

$$\frac{2}{5} = \lambda(1 - m)$$

$$m\lambda = \frac{7}{20}$$ **K1**

$$\frac{2}{5} = \lambda - m\lambda$$

$$\frac{2}{5} = \lambda - \frac{7}{20}$$

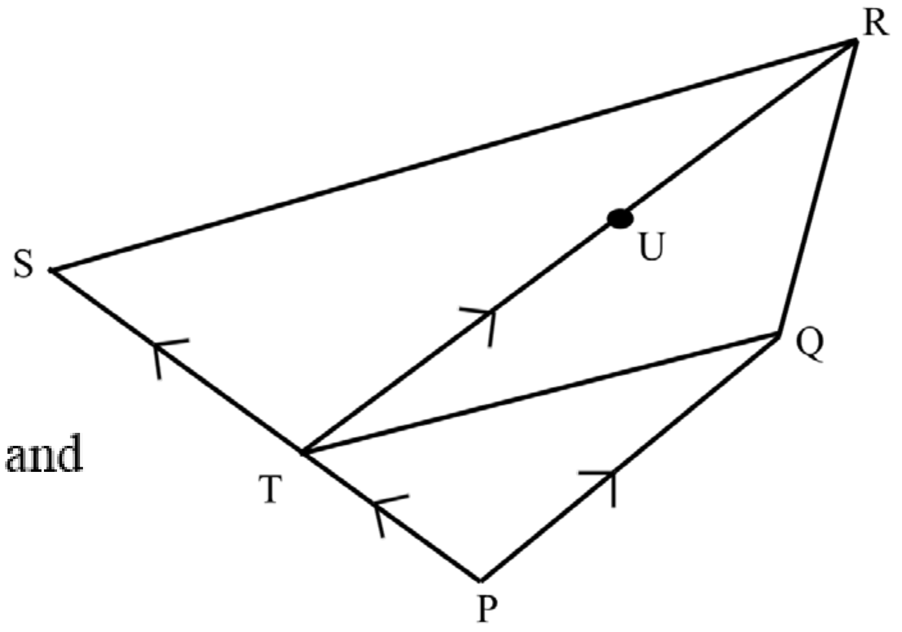
$$\lambda = \frac{3}{4}$$

$$\frac{2}{5} = \frac{3}{4}(1 - m)$$

$$m = \frac{7}{15}$$ **N1**

EXAMPLE 17

In diagram, *PQRS* is a quadrilateral and *QRT* is a triangle. Point *T* lies on *PS* and point *U* lies on *TR*.



It is given that $\overrightarrow{PQ} = 12\boldsymbol{x}$, $\overrightarrow{PT} = \boldsymbol{y}$, $TR : UR = 5 : 2$, $\overrightarrow{TU} = \overrightarrow{PQ}$ and $\overrightarrow{TS} = h\boldsymbol{y}$, where h is constant.

(a) Express in terms of x and y:

(i) $\overrightarrow{QT}$

(ii) $\overrightarrow{RT}$

(iii) $\overrightarrow{QR}$

(b) Find $\overrightarrow{RS}$ in terms of h, x and y.

Hence, if *RS* is parallel to *QT*, Find the value of h.

EXAMPLE 17a

It is given that $\overrightarrow{PQ} = 12\boldsymbol{x}$, $\overrightarrow{PT} = \boldsymbol{y}$, $TR : UR = 5 : 2$, $\overrightarrow{TU} = \overrightarrow{PQ}$ and $\overrightarrow{TS} = h\boldsymbol{y}$, where h is constant.

(a) Express in terms of x and y:

(i) $\overrightarrow{QT}$

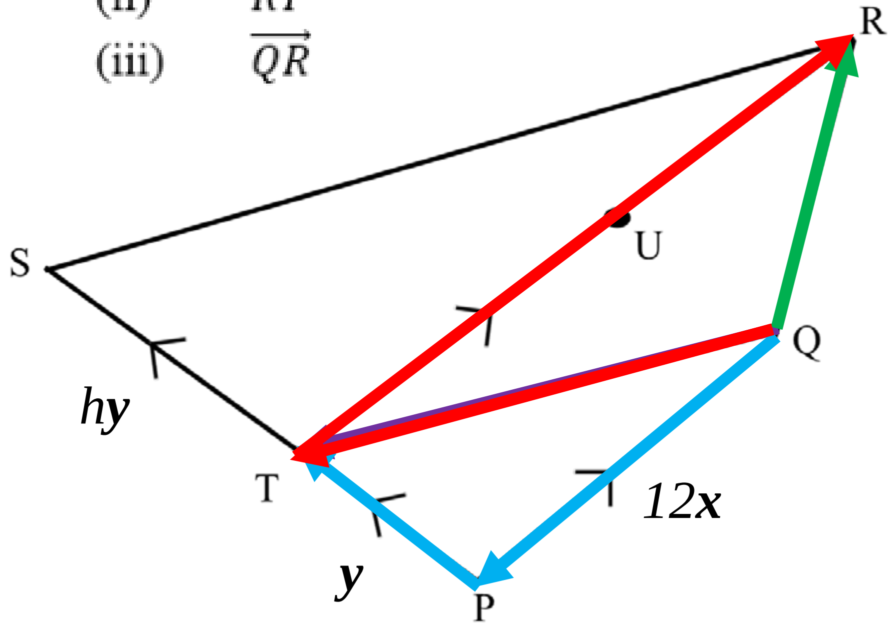
(ii) $\overrightarrow{RT}$

(iii) $\overrightarrow{QR}$

NOTE:

$$\frac{TR}{UR} = \frac{5}{2}$$

$$UR = \frac{2}{5}TR, TU = \frac{3}{5}TR$$



Solution 17a :

(a)

(i)

$$\overrightarrow{QT} = \overrightarrow{QP} + \overrightarrow{PT}$$ **K1**

$$\overrightarrow{QT} = -12\boldsymbol{x} + \boldsymbol{y}$$ **N1**

(ii)

$$\overrightarrow{TU} = \overrightarrow{PQ} = 12\boldsymbol{x}$$

$$\frac{3}{5}\overrightarrow{TR} = 12\boldsymbol{x}$$

$$\overrightarrow{TR} = 20\boldsymbol{x}$$

$$\overrightarrow{RT} = -\overrightarrow{TR} = -20\boldsymbol{x}$$ **N1**

(iii)

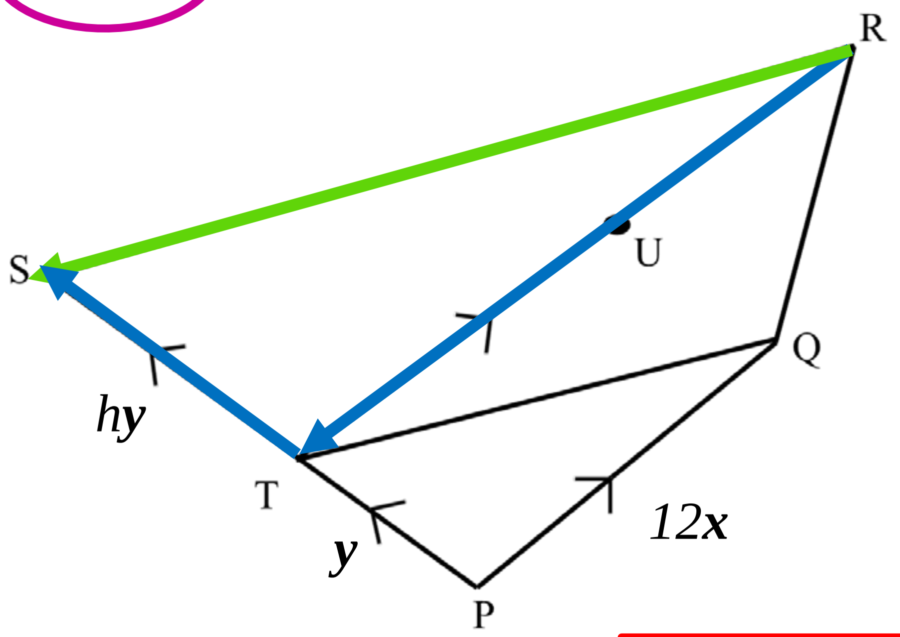
$$\overrightarrow{QR} = \overrightarrow{QT} + \overrightarrow{TR}$$

$$\overrightarrow{QR} = -12\boldsymbol{x} + \boldsymbol{y} + 20\boldsymbol{x}$$

$$\overrightarrow{QR} = 8\boldsymbol{x} + \boldsymbol{y}$$ **N1**

Solution 17b :

(b) Find $\overrightarrow{RS}$ in terms of h, x and y.
Hence, if RS is parallel to QT, Find the value of h.



$$\overrightarrow{RT} = -\overrightarrow{TR} = -20\boldsymbol{x}$$

$$\overrightarrow{QT} = -12\boldsymbol{x} + \boldsymbol{y}$$

$$\overrightarrow{QR} = 8\boldsymbol{x} + \boldsymbol{y}$$

K1

$$\overrightarrow{RS} = \overrightarrow{RT} + \overrightarrow{TS} \quad \text{or} \quad \overrightarrow{RS} = \overrightarrow{RQ} + \overrightarrow{QP} + \overrightarrow{PS}$$

$$\overrightarrow{RS} = -20\boldsymbol{x} + h\boldsymbol{y}$$ **N1**

$$\overrightarrow{RS} = \lambda\overrightarrow{QT}$$ **K1**

$$-20\boldsymbol{x} + h\boldsymbol{y} = \lambda(-12\boldsymbol{x} + \boldsymbol{y})$$ **K1**

$$-20\boldsymbol{x} + h\boldsymbol{y} = -12\lambda\boldsymbol{x} + \lambda\boldsymbol{y}$$

$$-20 = -12\lambda \qquad h = \lambda$$ **K1**

$$h = \frac{20}{12} = \frac{5}{3}$$ **N1**

LAST

EXAMPLE 18

The diagram shows quadrilateral *PQRS*. Given $\overrightarrow{PQ} = 12\boldsymbol{x}$, $\overrightarrow{PS} = 8\boldsymbol{y}$ and $\overrightarrow{SR} = 2\boldsymbol{x} + 2\boldsymbol{y}$. Straight lines *PR* and *QS* intersects at point T.

(a) Express in terms of x and y

(i) $\overrightarrow{PR}$

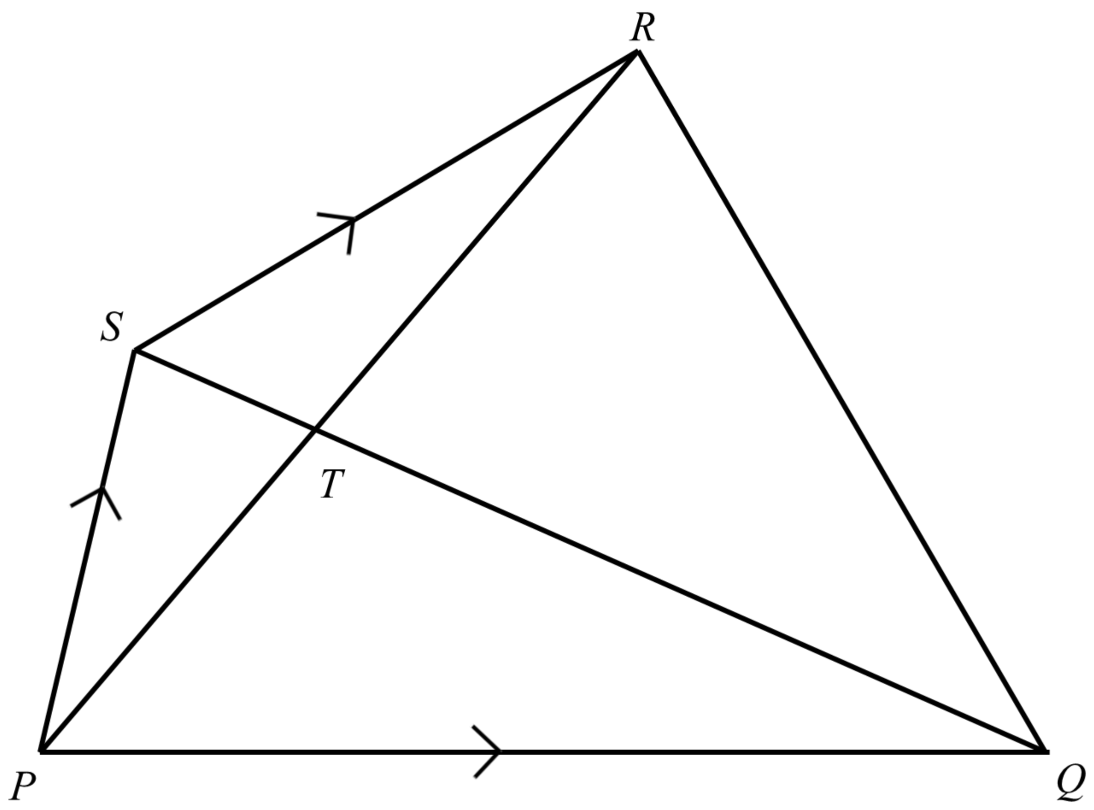
(ii) $\overrightarrow{QS}$

(b) Given $\overrightarrow{PT} = h\overrightarrow{PR}$ and $\overrightarrow{QT} = k\overrightarrow{QS}$, express $\overrightarrow{PT}$

(i) In terms of h, x and y,

(ii) In terms of k, x and y.

Hence, find the values of h and k.



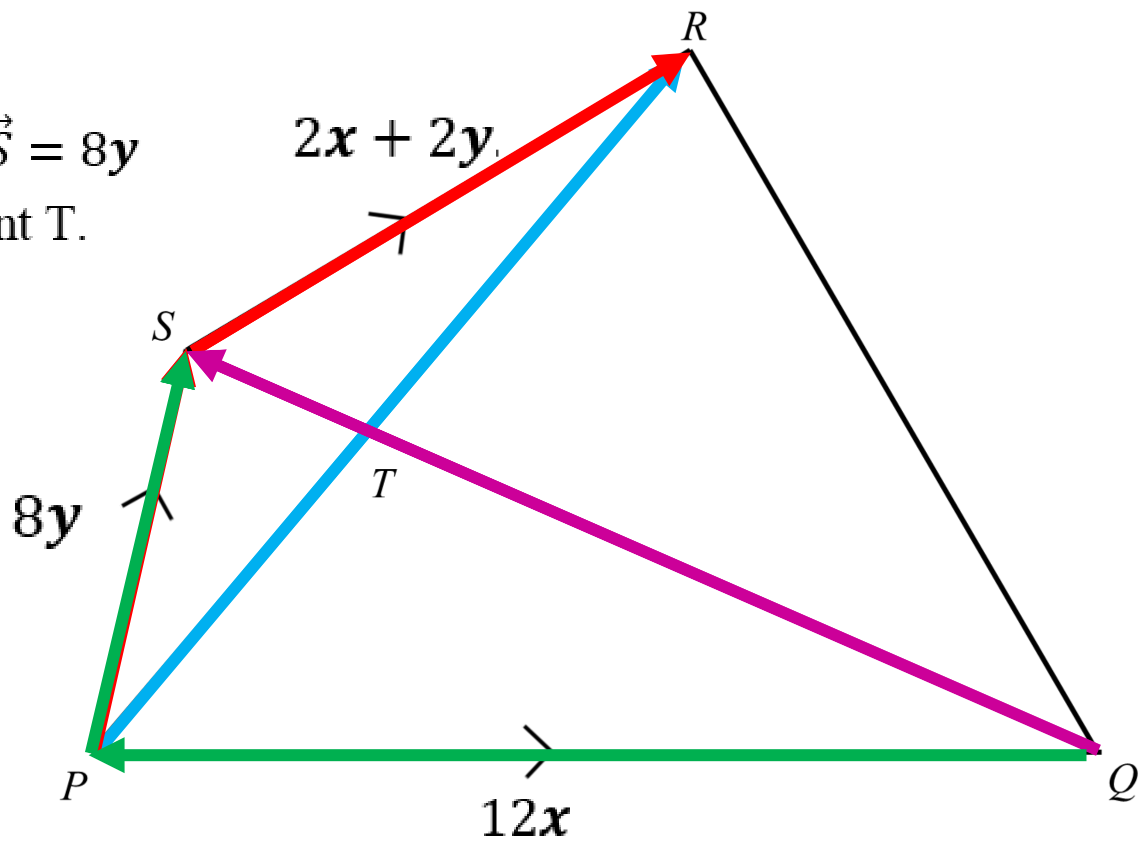
EXAMPLE 18a

The diagram shows quadrilateral $PQRS$. Given $\overrightarrow{PQ} = 12\boldsymbol{x}$, $\overrightarrow{PS} = 8\boldsymbol{y}$ and $\overrightarrow{SR} = 2\boldsymbol{x} + 2\boldsymbol{y}$. Straight lines PR and QS intersects at point T.

(a) Express in terms of x and y

(i) $\overrightarrow{PR}$

(ii) $\overrightarrow{QS}$



Solution 18a (i) :

$\overrightarrow{PR} = \overrightarrow{PS} + \overrightarrow{SR}$ **K1**

$\overrightarrow{PR} = 8\boldsymbol{y} + 2\boldsymbol{x} + 2$

$\overrightarrow{PR} = 2\boldsymbol{x} + 10\boldsymbol{y}$ **N1**

Solution 18a (ii) :

$\overrightarrow{QS} = \overrightarrow{QP} + \overrightarrow{PS}$

$\overrightarrow{QS} = -12\boldsymbol{x} + 8\boldsymbol{y}$ **N1**

LAST

EXAMPLE 18b

(b) Given $\overrightarrow{PT} = h\overrightarrow{PR}$ and $\overrightarrow{QT} = k\overrightarrow{QS}$, express $\overrightarrow{PT}$

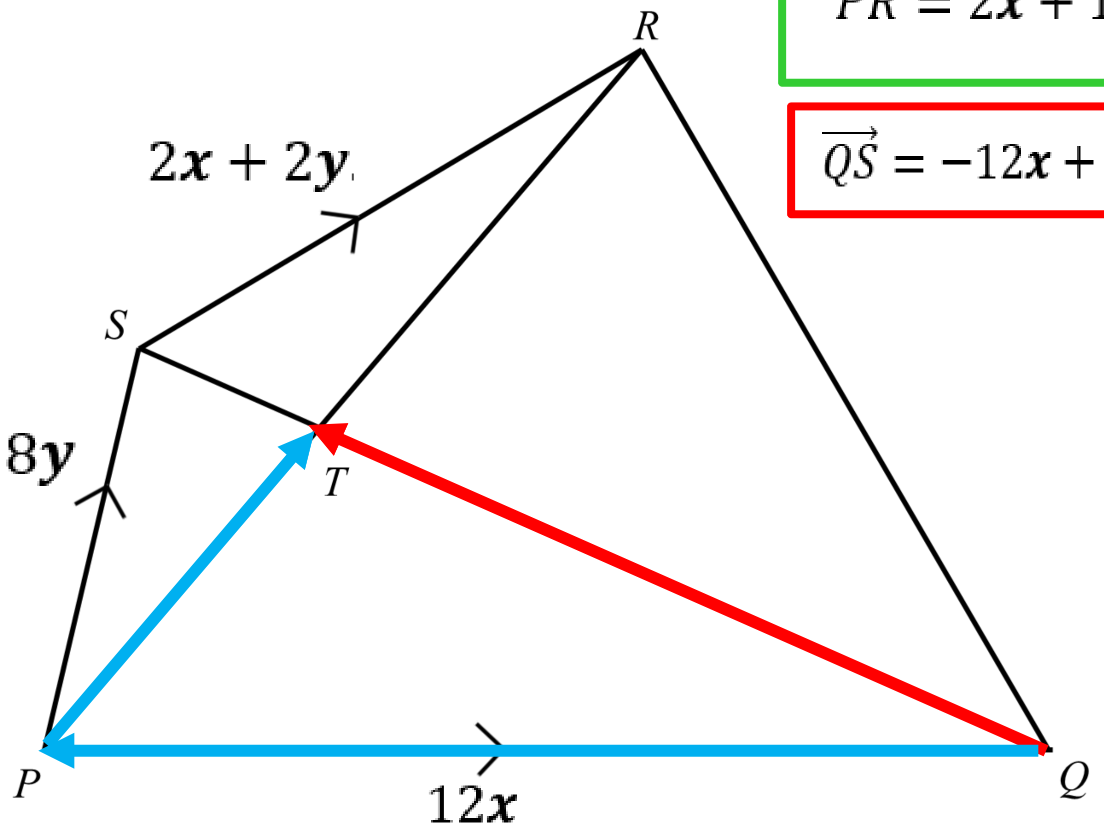
(i) In terms of h, x and y,

(ii) In terms of k, x and y.

Hence, find the values of h and k.

$\overrightarrow{PR} = 2\boldsymbol{x} + 10\boldsymbol{y}$

$\overrightarrow{QS} = -12\boldsymbol{x} + 8\boldsymbol{y}$



Solution 18b(i) :

$\overrightarrow{PT} = h\overrightarrow{PR}$

$\overrightarrow{PT} = h(2\boldsymbol{x} + 10\boldsymbol{y})$ **N1**

Solution 18b(ii) :

$\overrightarrow{QT} = k\overrightarrow{QS}$

$\overrightarrow{QP} + \overrightarrow{PT} = k\overrightarrow{QS}$ **K1**

$-12\boldsymbol{x} + \overrightarrow{PT} = k(-12\boldsymbol{x} + 8\boldsymbol{y})$

$\overrightarrow{PT} = 12\boldsymbol{x} + k(-12\boldsymbol{x} + 8\boldsymbol{y})$ **N1**

$$h(2\boldsymbol{x} + 10\boldsymbol{y}) = 12\boldsymbol{x} + k(-12\boldsymbol{x} + 8\boldsymbol{y})$$ **K1**

$$2h\boldsymbol{x} + 10h\boldsymbol{y} = 12\boldsymbol{x} - 12k\boldsymbol{x} + 8k\boldsymbol{y}$$

$$2h = 12 - 12k \quad \text{and} \quad 10h = 8k$$ **K1**

$$h = \frac{4}{5}k$$

$$2\left(\frac{4}{5}k\right) = 12 - 12k$$ **K1**

$$\frac{2}{5}k + 3k = 3$$

$$k = \frac{15}{17}$$ **N1**

$$h = \frac{4}{5}\left(\frac{15}{17}\right) = \frac{12}{17}$$ **N1**

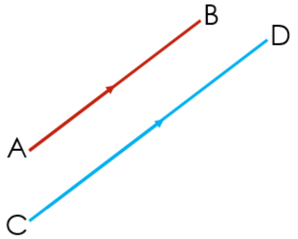
$$k = \frac{15}{17}, \qquad h = \frac{12}{17}$$

MIND MAPPING

$\overrightarrow{AB}$ and $\overrightarrow{CD}$ are **parallel vectors**

$$\overrightarrow{AB} = \lambda\overrightarrow{CD}$$

where $\lambda \neq 0$



IMPORTANT!!!!

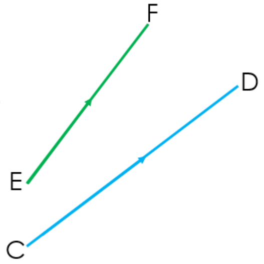
Given three points P,Q and R.
The followings are the conditions for the points to be collinear

(a) $\overrightarrow{PQ} = k\overrightarrow{QR}$
(b) PQ parallel to QR
(c) Q is COMMON Point

$\overrightarrow{CD}$ and $\overrightarrow{EF}$ are **non-parallel vectors**

$$\overrightarrow{CD} = \lambda\overrightarrow{EF}$$

where $\lambda = 0$



$\overrightarrow{CD}$ and $\overrightarrow{EF}$ cannot be expressed in Equality Form because of non parallel vectors.

Therefore,

If $h\overrightarrow{CD} = k\overrightarrow{EF}$, then $h = k = 0$

VECTORS

PARALLEL AND COLLINEAR

NOT PARALLEL

VECTOR IN CARTESIAN PLANE

RESULTANT VECTORS

- Any vector $\boldsymbol{r}$ on a Cartesian Plane can be written as:

$$\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j} = \begin{pmatrix} x \\ y \end{pmatrix}$$

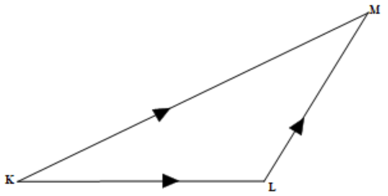
- The **magnitude of a vector** is the length of the vector. The magnitude of the vector is denoted:

$$magnitude\ of\ \boldsymbol{r} = |\boldsymbol{r}| = \sqrt{x^2 + y^2}.$$

- The ***Unit Vector in the direction of vector*** $\boldsymbol{r}$ is denoted by $\hat{\boldsymbol{r}}$ and is given by:

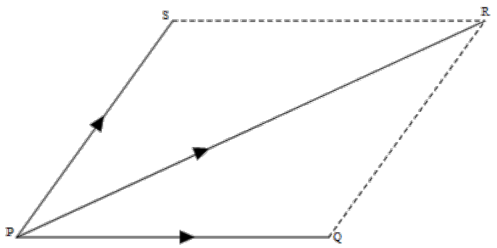
$$\hat{\boldsymbol{r}} = \frac{\boldsymbol{r}}{|\boldsymbol{r}|} = \frac{x\boldsymbol{i} - y\boldsymbol{j}}{\sqrt{x^2 + y^2}}$$

TRIANGLE LAW



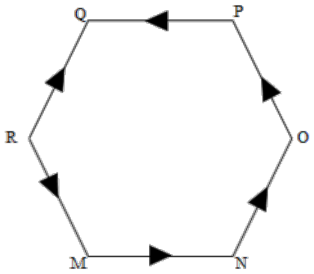
$\overrightarrow{KM} = \overrightarrow{KL} + \overrightarrow{LM}$

PARALLELOGRAM LAW



$\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QR} = \overrightarrow{PS} + \overrightarrow{SR}$

POLYGON LAW



$\overrightarrow{RQ} = \overrightarrow{RM} + \overrightarrow{MN} + \overrightarrow{NO} + \overrightarrow{OP} + \overrightarrow{PQ}$