# CHAPTER 1

# Functions

# PRIOR KNOWLEDGE

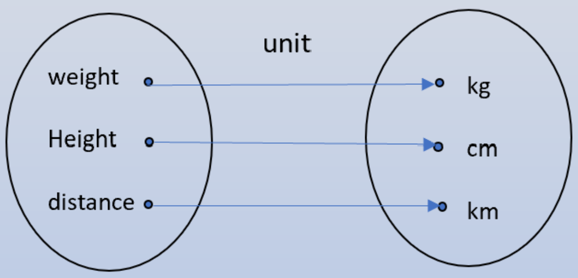
# FORM 2: GRAPHS OF FUNCTION

- 8. 1 FUNCTIONS

# IDENTIFYING FUNCTION

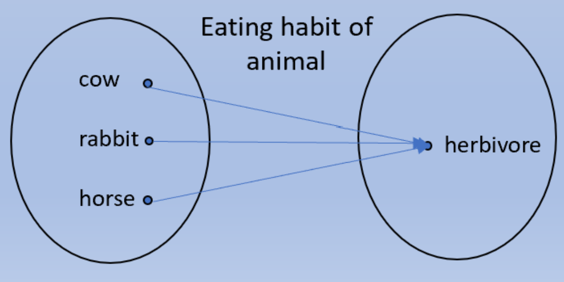
## RELATIONS THAT ARE FUNCTION

### (a) One to one




RELATION WHERE THE OBJECT IN THE DOMAIN HAS ONLY ONE IMAGE

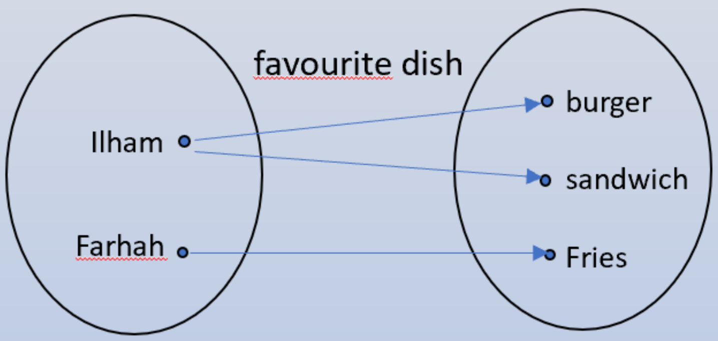
### (b) many to one



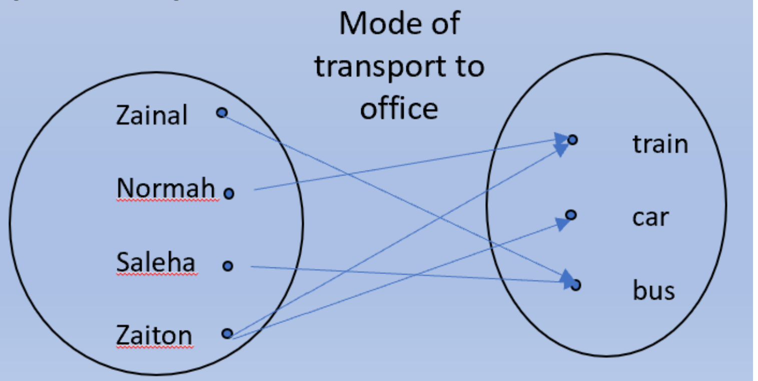

RELATION WHERE MORE THAN ONE OBJECT IS MATCHED TO THE SAME IMAGE

## RELATIONS THAT ARE NOT FUNCTION

### (c) One to many

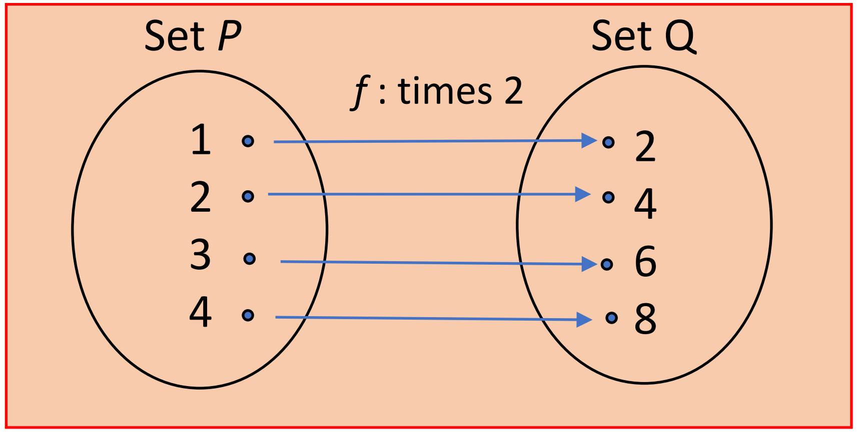



### (d) Many to many

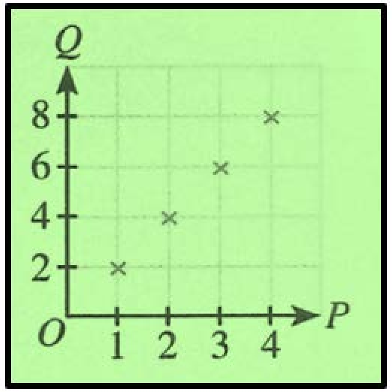


RELATION IS THE MATCHING OF ITEMS FROM SET A TO SET B. RELATIONS CAN BE REPRESENTED BY USING :

(a) Arrow Diagram

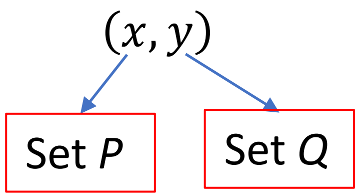



(b) Graph




(c) Ordered pair

Ordered pair, $A = \{(1, 2), (2, 4), (3, 6), (4, 8)\}$

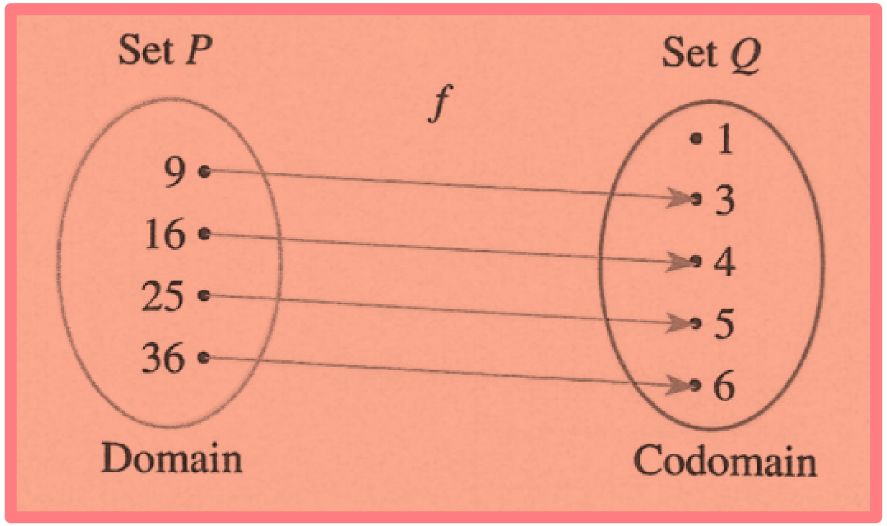



The function above can be written with function notations as follows:

$f : x \rightarrow 2x$ — Read as 'function $f$ maps $x$ to $2x$

$f(x) = 2x$ — Read as ' the function $f$ of $x$ is equal to $2x$

The diagram below shows the function $f$ that maps $x$ to $\sqrt{x}$ which is represented by $f(x) = \sqrt{x}$ .




SET $P$ $= \{9, 16, 25,36\}$ IS THE DOMAIN AND THE ELEMENT IS THE OBJECT,

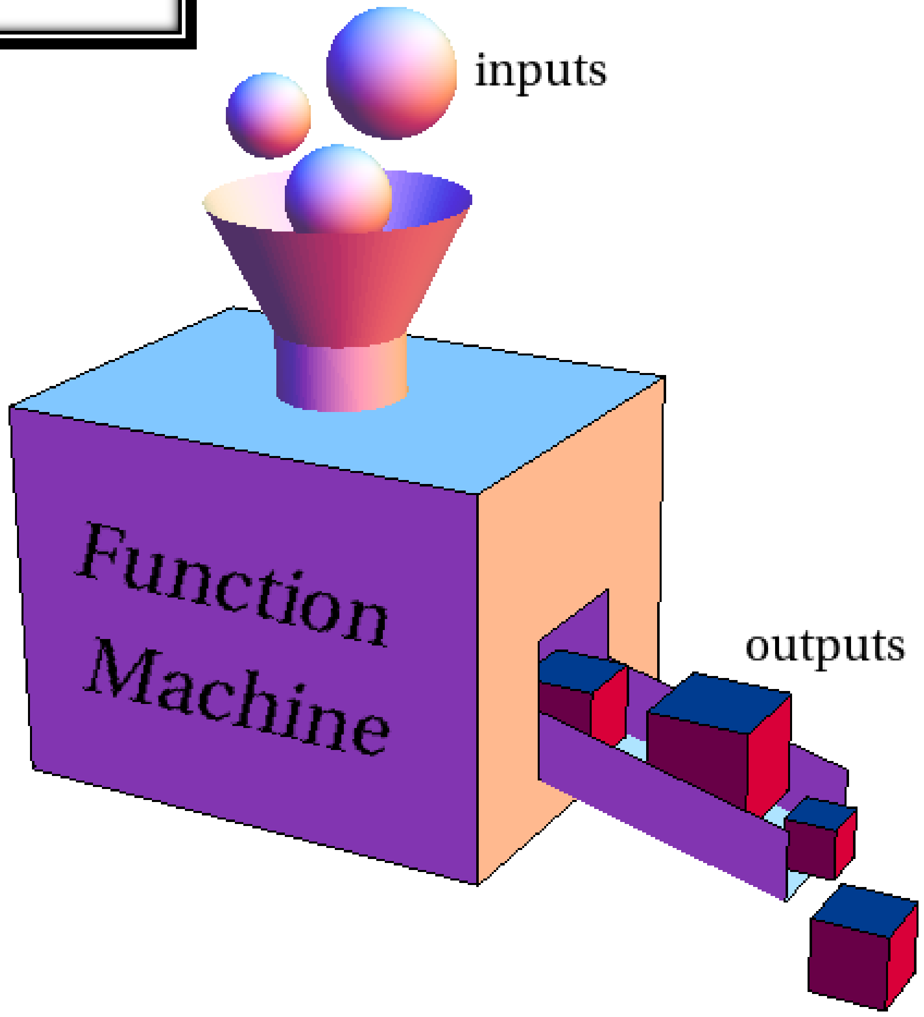
SET $Q$ $= \{1, 3, 4, 5, 6\}$ IS THE CODOMAIN.

THE ELEMENT IN SET $Q$ THAT IS MATCHED TO THE OBJECT IN SET $P$ IS THE IMAGE.

SET $\{3, 4, 5, 6\}$ IS THE RANGE OF THE FUNCTION.

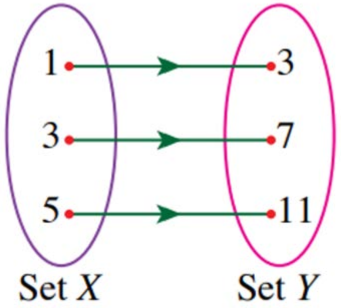
# FORM FOUR

- 1.1 FUNCTIONS
- 1.2 COMPOSITE FUNCTONS
- 1.3 INVERSE FUNCTIONS




# 1.1 FUNCTIONS

A function is a correspondence or mapping between the element of two sets such that each element in the first set corresponds to one and only one element in the second set

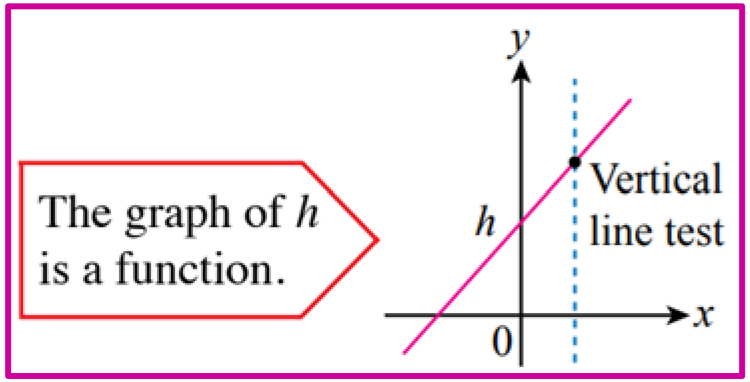


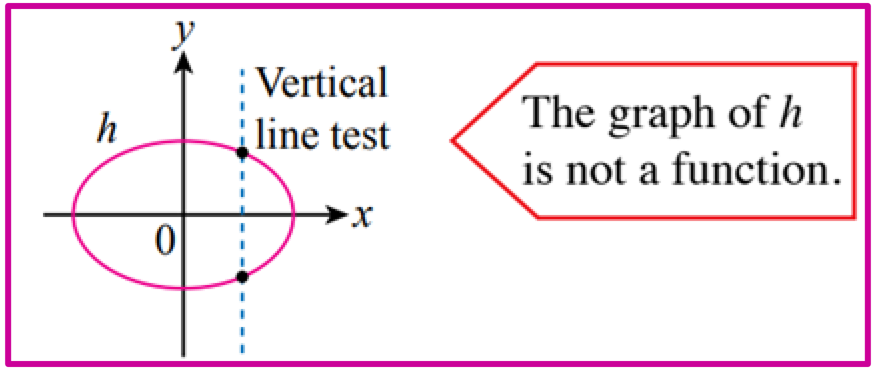

Each element x in set X is mapped to one and only one element y in set Y

# 1.1 FUNCTIONS

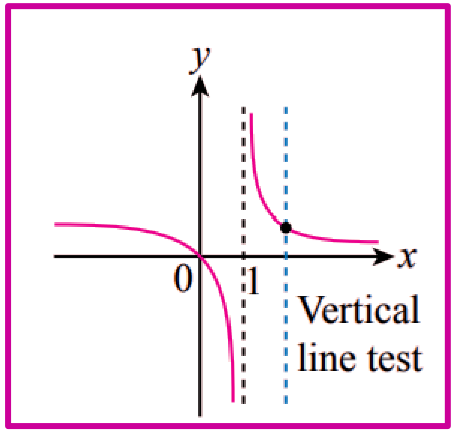
## How to determine whether a graph of a relation is a function or not

**We can use vertical line test**

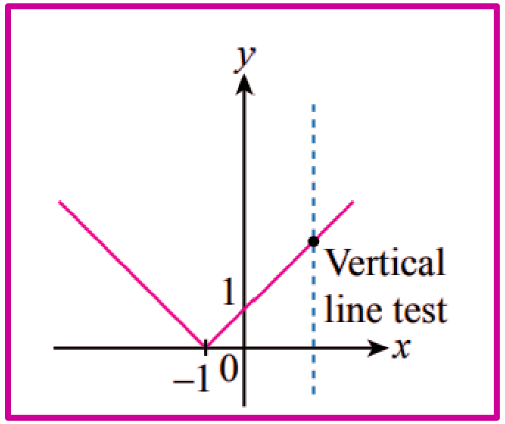


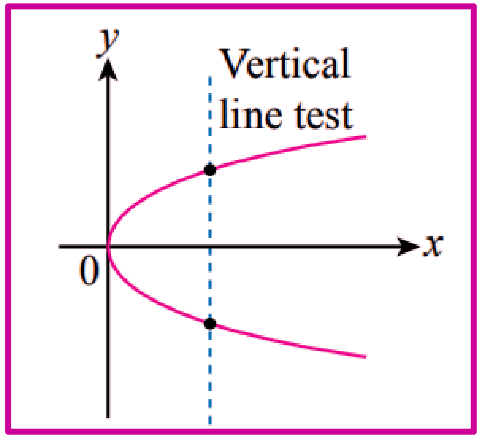

Try these




Function




Function

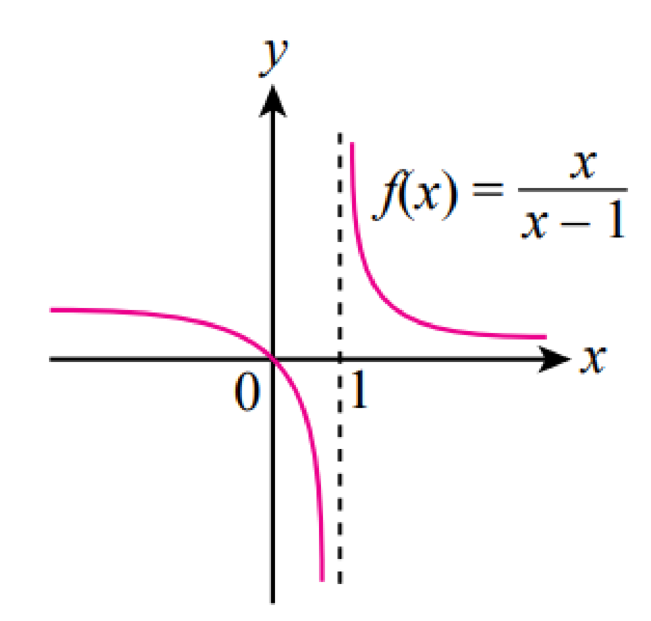



Not function

# 1.1 FUNCTIONS

# EXPLAINING FUNCTION BY GRAPHICAL REPRESENTATION AND NOTATION

Graph of $f(x) = \frac{x}{x-1}$




The diagram shows the graph of the function $f(x) = \frac{x}{1-x}$. When $x \to 1^{-1}$, that is $x$ approaches 1 from the left, $f(x) \to -\infty$, thus the value of $f(x)$ decreases non-terminating.

When $x \to 1^{+1}$, that is $x$ approaches 1 from the right, $f(x) \to \infty$, thus the value of $f(x)$ increases non-terminating.

This implies that the graph will approach but will never touch the line $x = 1$. Therefore, this function is not defined at $x = 1$.

# 1.1 FUNCTIONS

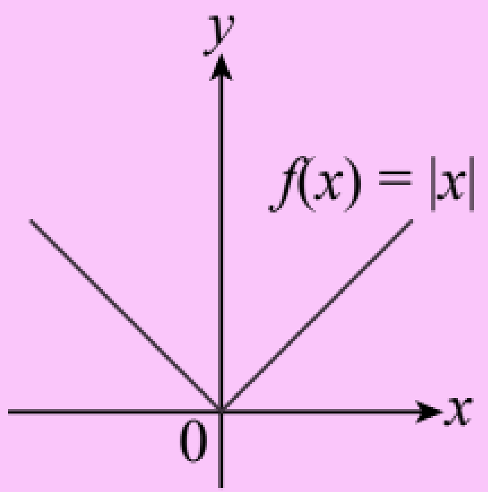
# EXPLAINING FUNCTION BY GRAPHICAL REPRESENTATION AND NOTATION

## Graph of $f(x) = |x|$

$$|x| = \begin{cases} x \text{ if } x \geq 0 \\ -x \text{ if } x < 0 \end{cases}$$

Thus, when $x = -2$, $|-2| = 2$

And when $x = 2$, $|2| = 2$




The graph of a LINEAR OBSOLUTE FUNCTION such as

$f(x) = |x|$

Has a V shaped .

# EXAMPLE 1

$f(x) = 2x - 4$ or $y = 2x - 4$ is the equation of a straight line.

Gradient of a straight line = 2

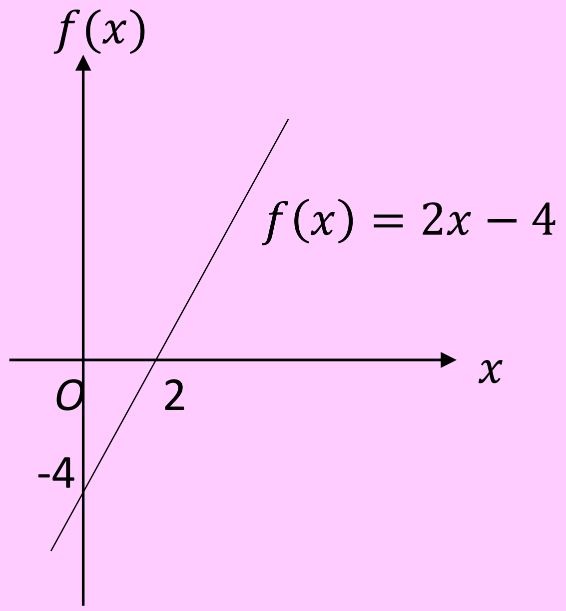
y-intercept = -4

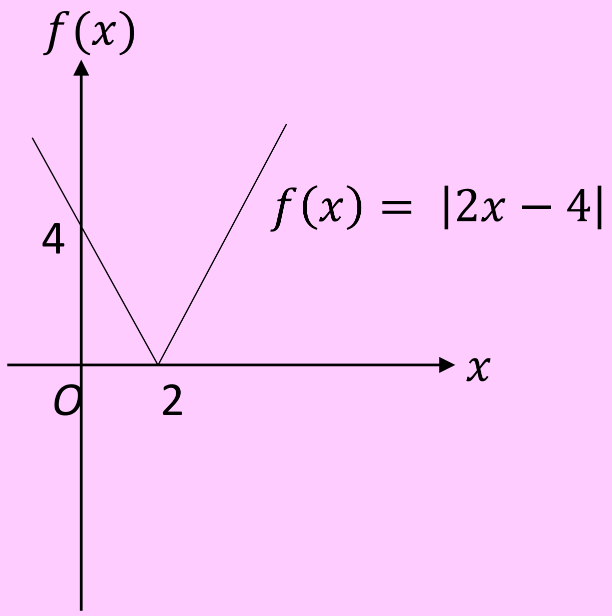
when y = 0 $\quad 2x - 4 = 0$

$$2x = 4$$

$$x = 2$$

| | | |
|---|---|---|
| X | 0 | 2 |
| y | -4 | 0 |



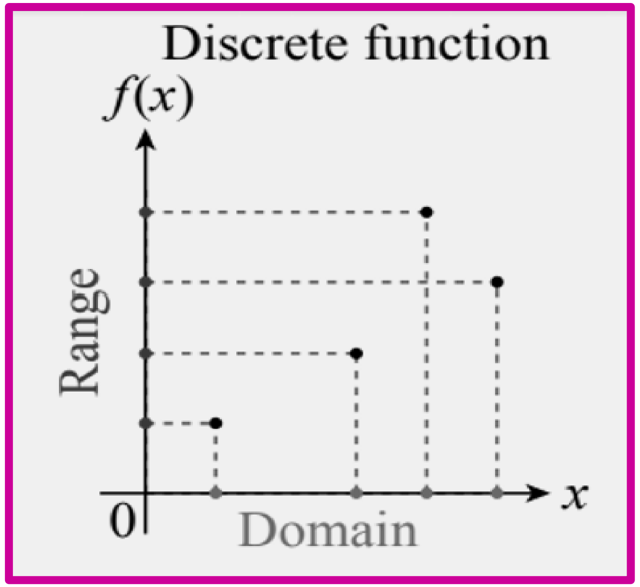

$f(x) = |2x - 4|$ is an absolute value function.

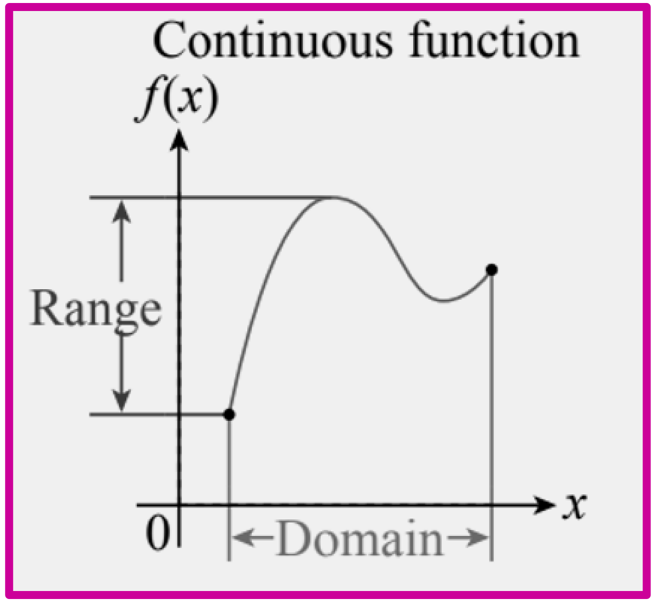
The graph can be obtained by reflecting the Negative part of the graph $f(x) = 2x - 4$ In the x-axis.

# 1.1 FUNCTIONS

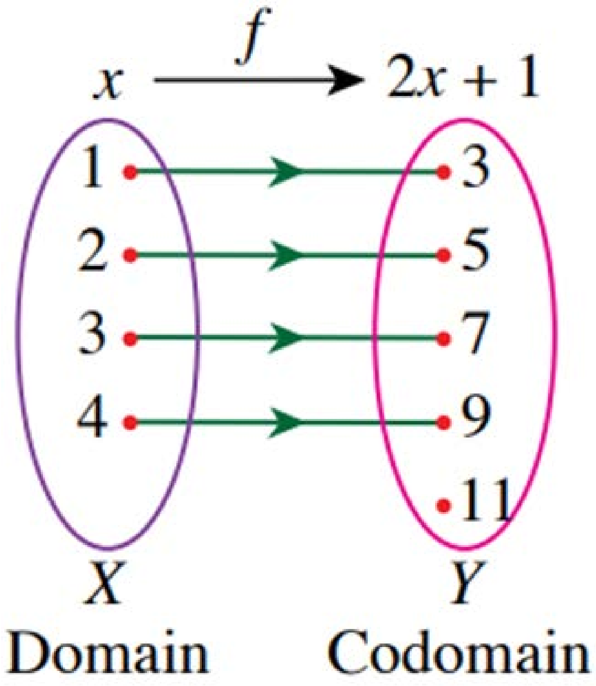
## DITERMINING THE DOMAIN, CODOMAIN AND RANGE OF A FUNCTION

Domain – the set of possible values of x which defines a function.
Range – the set of values of y that are obtaines by substituting all the possible values of x.




# DITERMINING THE DOMAIN, CODOMAIN AND RANGE OF A FUNCTION




Domain = {1, 2, 3, 4}

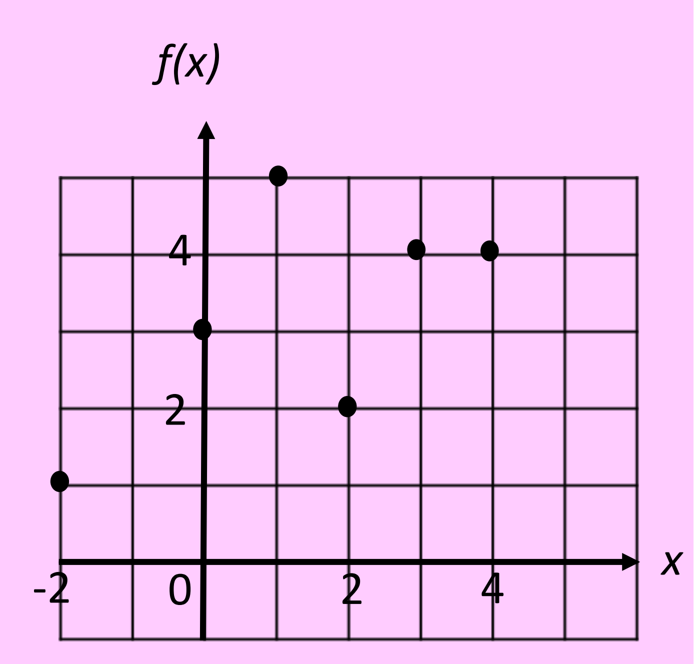
Codomain = {3, 5, 7, 9, 11}

Range = {3, 5, 7, 9}

# DITERMINING THE DOMAIN, CODOMAIN AND RANGE OF A FUNCTION

## 1.1 FUNCTIONS

### Discrete function

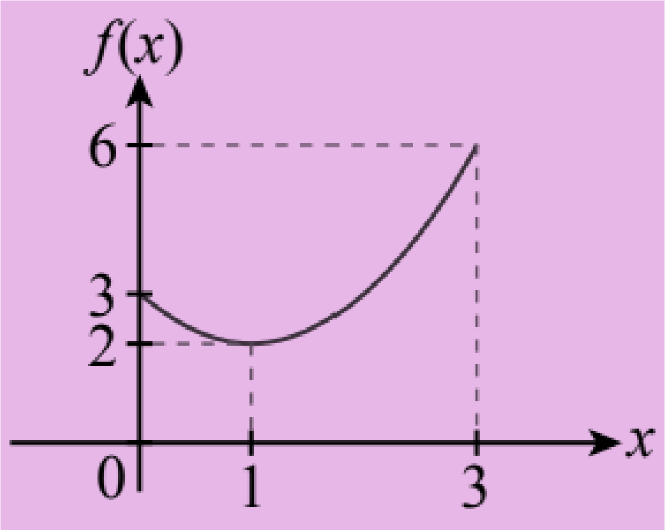



Domain $= \{-2, 0, 1, 2, 3, 4\}$

Codomain $\{1, 2, 3, 4, 5\}$

Range$= \{1,2,3,4,5\}$

### Continuous function




Domain of $f$ is $0 \leq x \leq 3$

Codomain of $f$ is $2 \leq f(x) \leq 6$

Range of $f$ is $2 \leq f(x) \leq 6$

## EXAMPLE 2

Given the function $f: x \rightarrow 3x - 8$, find

(a)the image of 4,

(b)the object that have the image -14

## SOLUTION

$f(x) = 3x - 8$

(a)When $x = 4$, $f(4) = 3(4) - 8$

$= 12 - 8$

$= 4$

(b)When the function $f$ have an image -14, it means that $f(x) = -14$

When $f(x) = -14$

$3x - 8 = -14$

$3x = -6$

$x = -2$

# 1.1 FUNCTIONS

## EXAMPLE 3

Given the function $f: x \rightarrow x^2 - 4x - 1$, find

(a)the image of 2,

(b) the objects that have the image 4

## SOLUTION

(a)$f : x \rightarrow x^2 - 4x - 1$

$f(x) = x^2 - 4x - 1$

When $x = 2$

$f(2) = 2^2 - 4(2) - 1$

$= 4 - 8 - 1$

$= -5$

(b) When the function $f$ has an image 4, it means that $f(x) = 4$

When $f(x) = 4$

$x^2 - 4x - 1 = 4$

$x^2 - 4x - 5 = 0$ CHANGE TO GENERAL FORM

$(x + 1)(x - 5) = 0$

$x + 1 = 0 \quad , x - 5 = 0$

$x = -1 , 5$

# 1.1 FUNCTIONS

## EXAMPLE 4

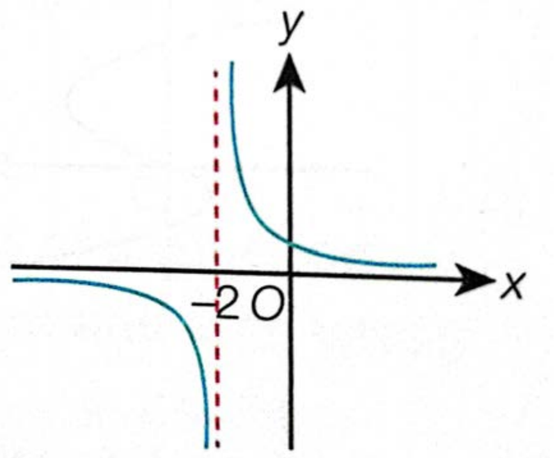
The diagram shows the graph of the function $f(x) = \frac{1}{x-k}, x \neq k.$

(a) State the value of k.

(b) Hence, find

(i) the image when x = 0.

(ii) the object when $f(x) = 5$.




## SOLUTION

(a) $-2 - k = 0$

$k = -2$

(b) $f(x) = \frac{1}{x+2}$

$f(0) = \frac{1}{2}$

(ii) $f(x) = 5$

$\frac{1}{x+2} = 5$

$5x + 10 = 1$

$x = -\frac{9}{5}$

# 1.1 FUNCTIONS

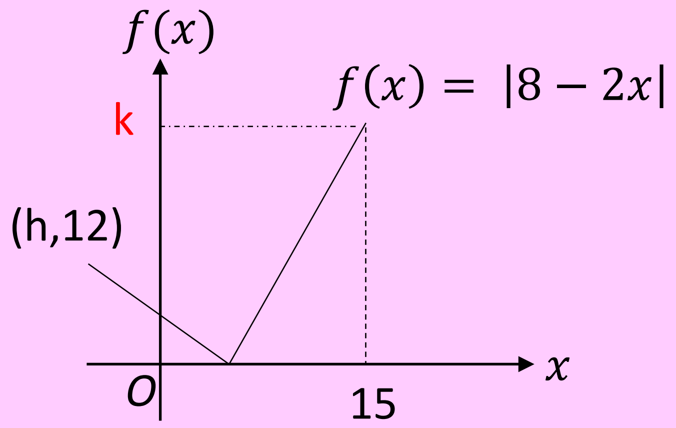
## EXAMPLE 5

The diagram shows the graph of the function $f : x \rightarrow |8 - 2x|$ for the domain $h \leq x \leq 15$. Given $f(15) = k$ .

(a) Find the value of h and of k.

(b) Find the value of x when f(x) = 0.

(c) State the domain of $0 \leq f(x) \leq 12$.




## SOLUTION

(a) $12 = 8 - 2h$

$2h = -4$

$h = -2$

$|8 - 2(15)| = k$

$|8 - 30| = k$

$k = 22$

(b) $|8 - 2x| = 0$

$2x = 8$

$x = 4$

(c) $|8 - 2x| = 12$

$8 - 2x = \pm 12$

$8 - 2x = 12$

$x = -2$

$8 - 2x = -12$

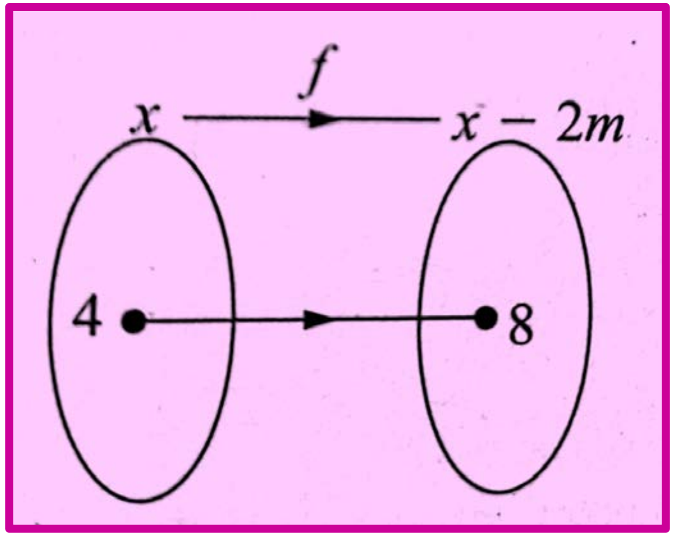
$x = 10$

$-2 \leq x \leq 10$

# 1.1 FUNCTIONS

EXAMPLE 6

Diagram shows the function $f : x \rightarrow x - 2m$, where m is a constant.
Find the value of m.




SOLUTION

$$f(x) = x - 2m$$
$$8 = 4 - 2m$$
$$2m = -4$$
$$m = -2$$

# EXAMPLE 7

Given $f: x \to x^2 - 2$ . Find the values of x which map on to itself.

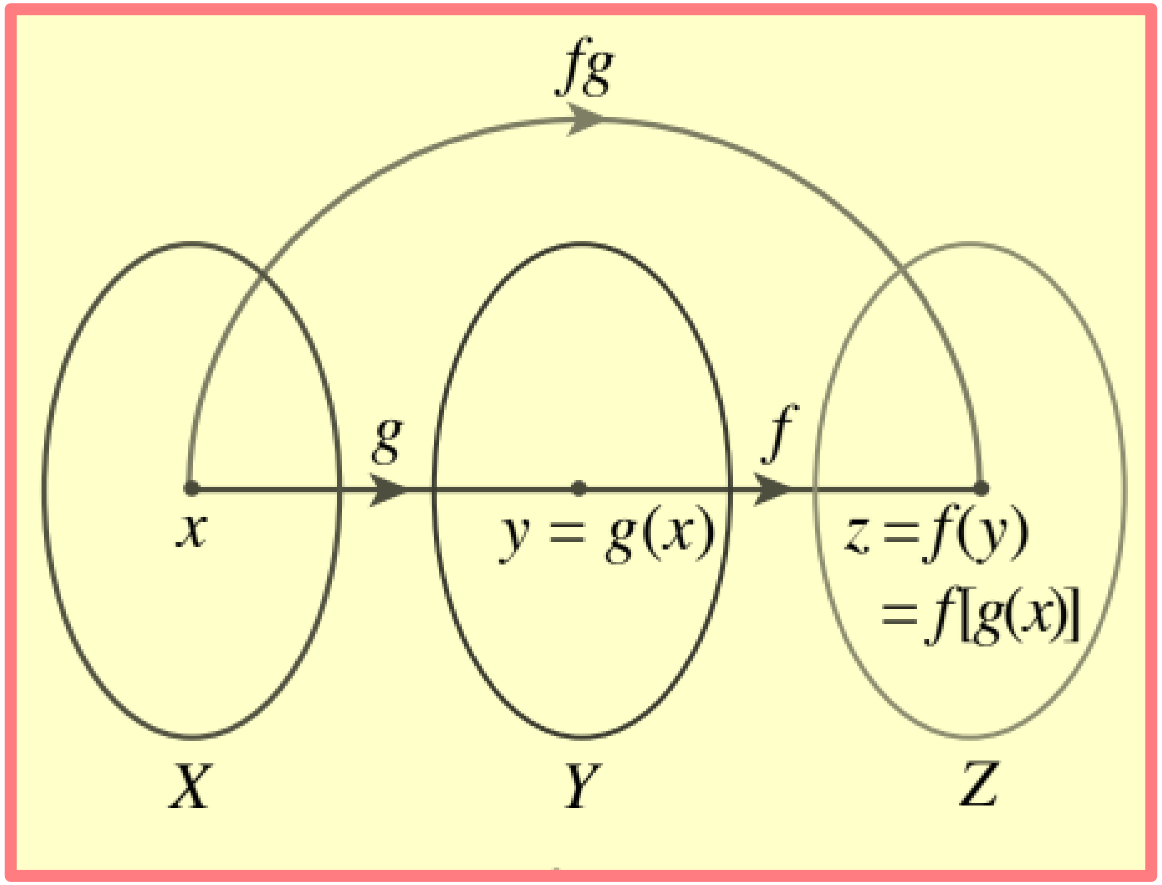
SOLUTION

$$f(x) = x$$
$$x^2 - 2 = x$$
$$x^2 - x - 2 = 0$$
$$(x+1)(x-2) = 0$$
$$x = -1, x = 2$$

# 1.2 COMPOSITE FUNCTIONS

# 1.1 COMPOSITE FUNCTIONS

If $g$ is a function which maps set X onto set Y and $f$ is a function which maps set Y onto set Z, then $fg$ is a COMPOSITE FUNCTION OF $g$ FOLLOWED BY $f$ which maps set X onto set Z



IMPORTANT NOTES

$fg(x)$ means $f[g(x)]$

In general $fg \neq gf$

$f^2 = ff, f^3 = fff$ or $ff^2$ and so on.

# 1.1 COMPOSITE FUNCTIONS

## EXAMPLE 8

The functions $f$ and $g$ are defined by $f: x \rightarrow 2x + 1$ and $g: x \rightarrow x^2 -$ respectively. Find

(a) the value of $fg(3)$,

(b) the composite functions

(i) $fg$ (ii) $gf$

## SOLUTION

(a) $fg(3) = f[g(x)]$

$= f(3^2 - 2)$

$= f(7)$

$= 2(7) + 1$

$= 15$

(b) $f: x \rightarrow 2x + 1$ $g: x \rightarrow x^2 - 2$

$f(x) = 2x + 1$ $g(x) = x^2 - 2$

(i) $fg(x)$

$= f[g(x)]$

$= f(x^2 - 2)$

$= 2(x^2 - 2) + 1$

$= 2x^2 - 3$

$fg(x) = 2x^2 - 3$

(ii) $gf(x)$

$= g[f(x)]$

$= g(2x + 1)$

$= (2x + 1)^2 - 2$

$= 4x^2 + 4x + 1 - 2$

$= 4x^2 + 4x - 1$

$gf(x) = 4x^2 + 4x - 1$

$fg \neq gf$

# 1.1 COMPOSITE FUNCTIONS

## EXAMPLE 8

The functions $f$ and $g$ are defined by $f: x \to 2x + 1$ and $g: x \to x^2 - 2$ respectively. Find

(b) the composite functions

(iii) $f^2$ (iv) $g^2$

(c) the value of $x$ if $gf(x) = 23$

## SOLUTION

(iii) $f^2(x)$

$= ff(x)$

$= f(2x + 1)$

$= 2(2x + 1) + 1$

$= 4x + 3$

$f^2(x) = 4x + 3$

(iv) $g^2(x)$

$= gg(x)$

$= g(x^2 - 1)$

$= (x^2 - 2)^2 - 2$

$= x^4 - 4x^2 + 4 - 2$

$= x^4 - 4x^2 + 2$

$g^2(x) = x^4 - 4x^2 + 2$

$(c) gf(x) = 23$

$4x^2 + 4x - 1 = 23$

$4x^2 + 4x - 24 = 0$

$x^2 + x - 6 = 0$

$(x - 2)(x + 3) = 0$

$x = 2, -3$

# 1.1 COMPOSITE FUNCTIONS

## EXAMPLE 9

It is given that $g : x \to \frac{1}{x}, x \neq 0$.

(a) Find

(i) $g^2$, (ii) $g^3$, (iii) $g^4$.

(b) Hence, deduce $g^{19}$.

## SOLUTION

(a)(i) $g^2(x)$

$$= g[g(x)]$$
$$= g\left(\frac{1}{x}\right)$$
$$= \frac{1}{\frac{1}{x}}$$
$$= x$$

$g^2(x) = x$

(a)(ii) $g^3(x)$

$$= g[g^2(x)]$$
$$= g(x)$$
$$= \frac{1}{x}$$

$g^3(x) = \frac{1}{x}$

(a) (iii) $g^4(x)$

$$= g[g^3(x)]$$
$$= g\left(\frac{1}{x}\right)$$
$$= \frac{1}{\frac{1}{x}}$$
$$= x$$

$g^4(x) = x$

(b) $g^{19}(x) = g[g^{18}(x)]$

$$= g(x)$$
$$= \frac{1}{x}$$

The function that maps to x is an IDENTITY FUNCTION

$$g^2(x) = x$$

is an identity function.

# 1.1 COMPOSITE FUNCTIONS

**EXAMPLE 10**

Given the function $f(x) = 2x - 5$ and $g(x) = px + q$, where p and q are constant.
Express p in terms of q such that $gf(2) = 7$.

**SOLUTION**

$gf(x) = g[f(x)]$

$= g(2x - 5)$

$= p(2x - 5) + q$

$gf(x) = 2px - 5p + q$

Given $gf(2) = 7$

$2p(2) - 5p + q = 7$

$4p - 5p + q = 7$

$p = q - 7$

Alternative Method

$gf(2) = 7$

$g[f(2)] = 7$

$f(2) = 2(2) - 5$

$g(2(2) - 5) = 7$

$g(-1) = 7$

$-p + q = 7$

$p = q - 7$

# 1.1 COMPOSITE FUNCTIONS

## EXAMPLE 11

It is given that $f: x \rightarrow 3x - 4$ and $g: x \rightarrow 2 - x$.

(a) Find

(i) $g(7)$,

(ii) the value of k if $f(k-2) = \frac{1}{2}g(10)$,

(iii) $fg(x)$.

(b) Hence, sketch the graph of $y = |fg(x)|$ for $-2 \leq x \leq 5$. State the range of $y$.

# 1.1 COMPOSITE FUNCTIONS

## EXAMPLE 11

It is given that $f : x \to 3x - 4$ and $g : x \to 2 - x$.

(a) Find

(i) $g(7)$,

(ii) the value of k if $f(k-2) = \frac{1}{2}g(10)$,

(iii) $fg(x)$.

## SOLUTION

(a) (i)

$$g(x) = 2 - x$$
$$g(7) = 2 - 7$$
$$= -5$$

(ii)

$$f(x) = 3x - 4$$
$$f(k-2) = \frac{1}{2}g(10)$$
$$3(k-2) - 4 = \frac{1}{2}(2 - 10)$$
$$3k - 6 - 4 = -4$$
$$3k = 6$$
$$k = 2$$

(iii)

$$fg(x) = f[g(x)]$$
$$= 3(2 - x) - 4$$
$$= 2 - 3x$$
$$fg(x) = 2 - 3x.$$

## EXAMPLE 11

(b) Hence, sketch the graph of $y = |fg(x)|$ for $-2 \leq x \leq 5$. State the range of *y*.

## SOLUTION

$y = |fg(x)| = |2 - 3x|$

When $y = 0$, $|2 - 3x| = 0$

$$2 - 3x = 0$$

$$x = \frac{2}{3}$$

The graph intersects the $x-$axis at $x = \frac{2}{3}$

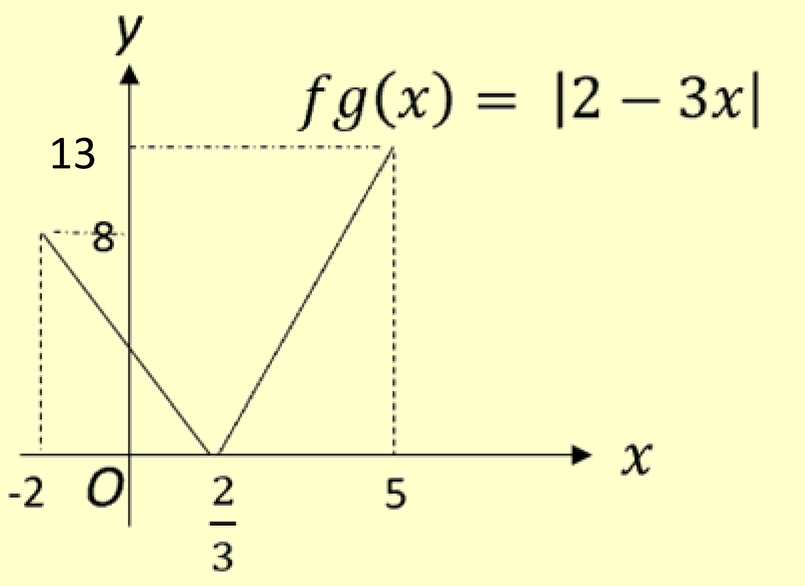
When $x = -2, y = |2 - 3(-2)|$

$$= 8$$

When $x = 5, y = |2 - 3\,(5)|$

$$= |-13|$$

$$= 13$$




The range of y is $0 \leq y \leq 13$

## EXAMPLE 12

Given $f: x \rightarrow 3 - 4x$ and $fg: x \rightarrow 2x + 1$, find the function $g$

**SOLUTION**

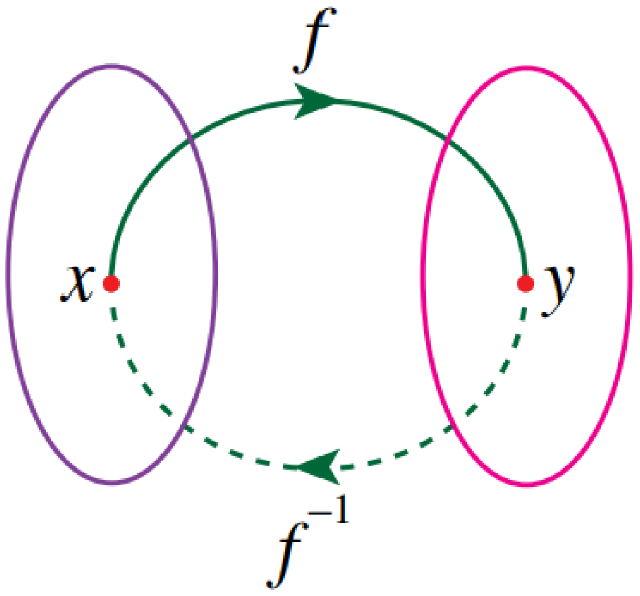
$$fg(x) = f[g(x)]$$
$$= 3 - 4g(x)$$

Thus, $3 - 4g(x) = 2x + 1$

$$4g(x) = 3 - 2x - 1$$
$$g(x) = \frac{1}{4}(2 - 2x)$$
$$g(x) = \frac{1}{2}(1 - x)$$

# 1.3 INVERSE FUNCTIONS

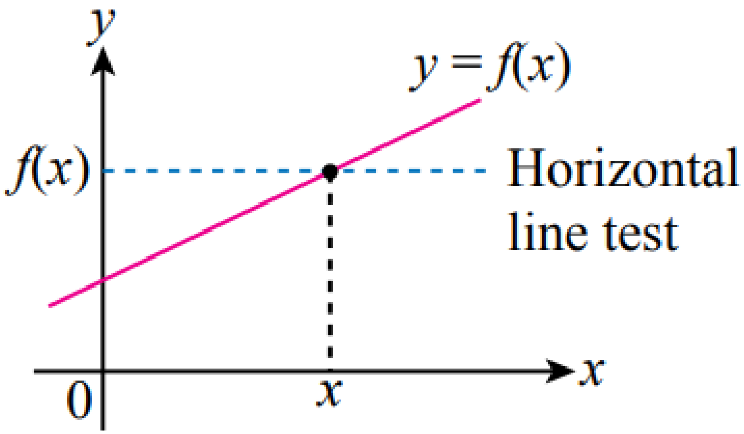
If $f : x \rightarrow y$ is a function that maps x onto $y$, its INVERSE FUNCTION is denoted by $f^{-1}$. Inverse function that maps $y$ back to $x$.




If $f(x) = y$ then $f^{-1}(y) = x$

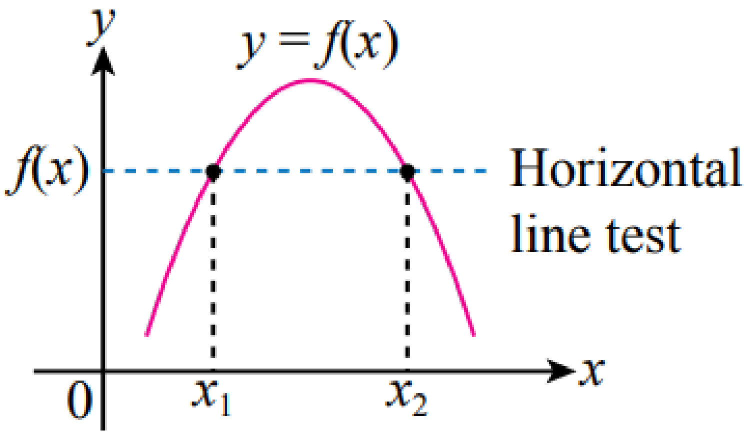
# 1.3 INVERSE FUNCTION

The horizontal line test is used to determine whether a function is one-to-one relation or many-to-one relation




*f* has an inverse function

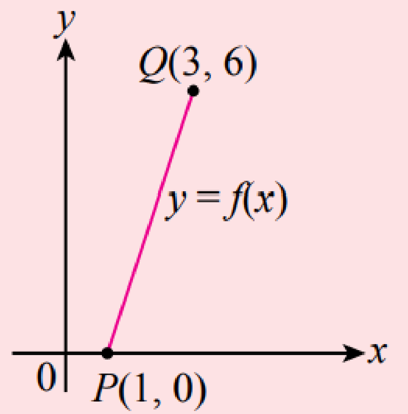
If there is one point of intersection, then the function is ONE-TO-ONE relation and the INVERSE FUNCTION EXISTS



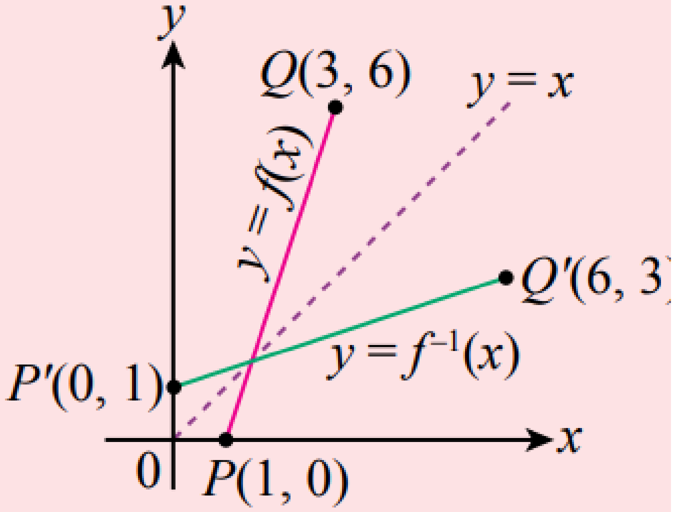

*f* does not have an inverse function

If there are MORE THAN ONE POINT OF INTERSECTIONS, then the function is a MANY-TO-ONE relation and the INVERSE FUNCTION DOES NOT EXIST

The diagram on the right shows the graph of $y = f(x)$ passing through the points $P(1, 0)$ and $Q(3, 6)$. On the same diagram, sketch the graph of $y = f^{-1}(x)$ by showing the points corresponding to point $P$ and point $Q$.




The graph of $y = f^{-1}(x)$ is the reflection of the graph of $y = f(x)$ in the line $y = x$. The points $P'$ and $Q'$ on the graph of $y = f^{-1}(x)$ correspond to the points $P$ and $Q$ as shown in the diagram on the right.



EXAMPLE 13

Given the function $f: x \to 5x - 7$, find $f^{-1}(x)$.



Change function $y = f(x)$ to the form $x = f(y)$.

Write $x$ as $f^{-1}(y)$.

Replace variable $y$ with variable $x$.

(a) $f(x) = 5x - 7$

Let $y = 5x - 7$

$x = \frac{1}{5}(y + 7)$

Make x be the subject

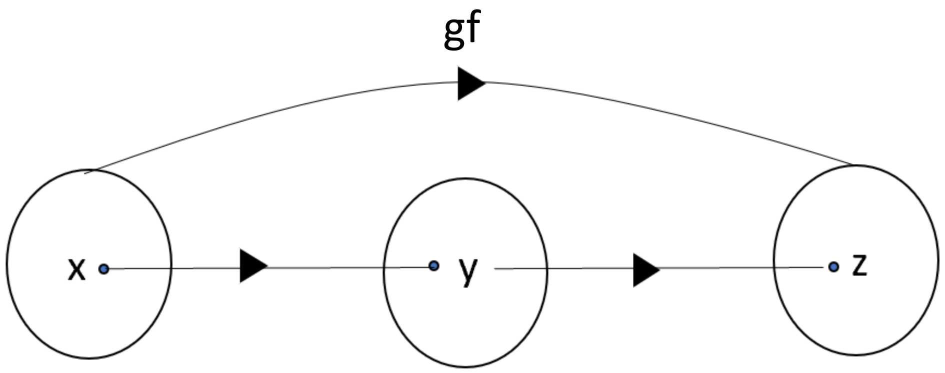
Then, $f^{-1}(y) = \frac{1}{5}(y + 7)$

Thus, $f^{-1}(x) = \frac{1}{5}(x + 7)$ Substitute y with x

If $f(x) = y$ then $f^{-1}(y) = x$

# EXAMPLE 14

The diagram shows the composite function $gf$ that maps $x$ to $z$




State
(a) the function that maps x to y,
(b) $g^{-1}(z)$

SOLUTION

(a) f

(b) y

# EXAMPLE 15

- Given $f(x) = 2x + 1$, find $ff^{-1}$(x).

SOLUTION

$$f(x) = 2x + 1$$
Let $y = 2x + 1$
$$2x = y - 1$$
$$x = \frac{y-1}{2}$$
$$f^{-1}(x) = \frac{x-1}{2}$$

$$ff^{-1}(x) = f[f^{-1}(x)]$$
$$= f\left(\frac{x-1}{2}\right)$$
$$= 2\left(\frac{x-1}{2}\right) + 1$$
$$= x$$

$$ff^{-1}(x) = x$$

# EXAMPLE 16



Given $g(x) = x + 1$ and $fg : x \to x^2 + 3$, find the function $f$

SOLUTION

$$fg(x) = x^2 + 3$$

Let $y = x + 1$

$$x = y - 1$$

$$g^{-1}(x) = x - 1$$

$$f(x) = f\underbrace{gg^{-1}}_{x}$$

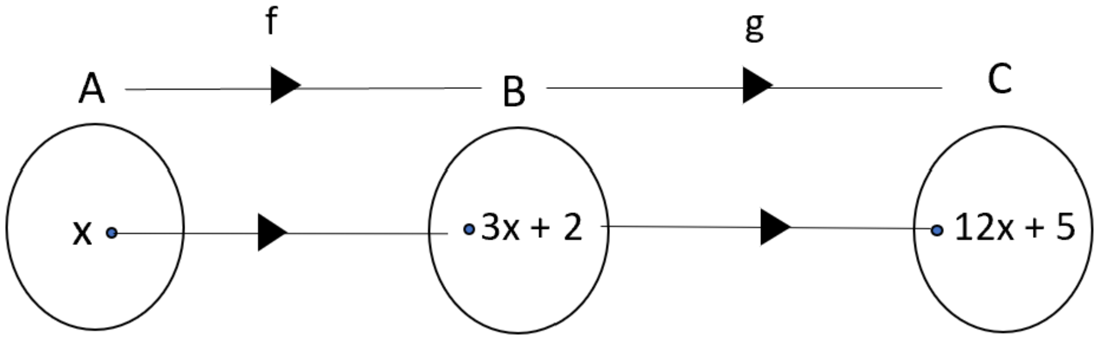
$$f(x) = (x-1)^2 + 3$$

$$f(x) = x^2 - 2x + 1 + 3$$

$$f(x) = x^2 - 2x + 4$$

# EXAMPLE 17

The diagram shows the function $f$ maps set $A$ to set $B$ and the function $g$ maps set $B$ to set $C$




Find

(a) In terms of x, the function

(i) which maps set B to set A,

(ii) g(x).

SOLUTION

a)(i) $f(x) = 3x + 2$

Let $y = 3x + 2$

$3x = y - 2$

$x = \frac{y-2}{3}$

$f^{-1}(x) = \frac{x-2}{3}$

(a)(ii) $gf(x) = 12x + 5$

Guna $f^{-1}(x)$ dari (a)(i)

$g(x) = gff^{-1}(x)$

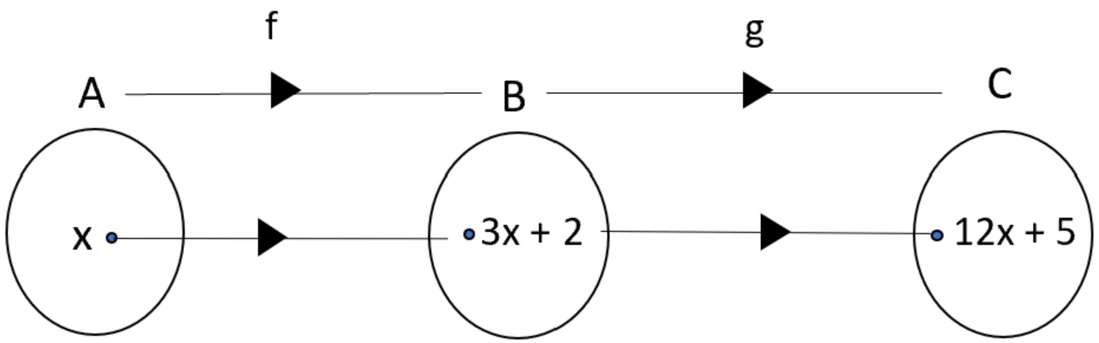
$g(x) = 12\left(\frac{x-2}{3}\right) + 5$

$g(x) = 4(x - 2) + 5$

$g(x) = 4x - 3$

# EXAMPLE 17

The diagram shows the function $f$ maps set $A$ to set $B$ and the function $g$ maps set $B$ to set $C$




Find

(b) The value of $x$ such that $fg(x) = 8x + 1$.

SOLUTION

(b) $f(x) = 3x + 2 \quad g(x) = 4x - 3$

$$fg(x) = 3(4x - 3) + 2$$

$$fg(x) = 12x - 7$$

$$12x - 7 = 8x + 1$$

$$4x = 8$$

$$x = 2$$

# EXAMPLE 18



Given that the function $f(x) = \frac{ax-b}{x+4}, x \neq -4$ and its inverse function,

$f^{-1}(x) = \frac{-4x-3}{x-2}, x \neq 2.$

(a) Find the values of a and of b.

(b) Hence, if $f(x) = 3x$, find the values of x.

SOLUTION

(a) $f^{-1}(x) = \frac{-4x-3}{x-2}$

Let $y = \frac{-4x-3}{x-2}$

$$y(x-2) = -4x-3$$

$$xy - 2y = -4x - 3$$

$$xy + 4x = 2y - 3$$

$$x(y+4) = 2y - 3$$

$$x = \frac{2y-3}{y+4}$$

$$f(x) = \frac{2x-3}{x+4}$$

$$\frac{2x-3}{x+4} = \frac{ax-b}{x+4}$$

$a = 2$ $\quad$ $b = 3$

(b)

$$f(x) = 3x$$

$$\frac{2x-3}{x+4} = 3x$$

$$2x - 3 = 3x(x+4)$$

$$2x - 3 = 3x^2 + 12x$$

$$3x^2 + 10x + 3 = 0$$

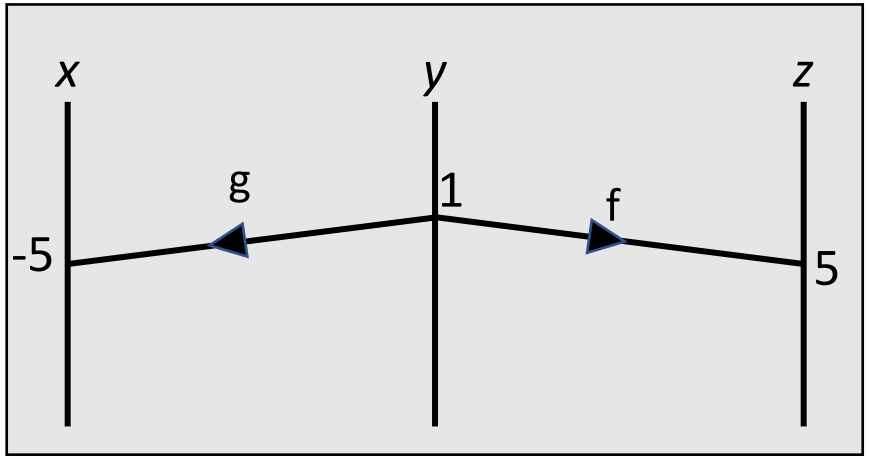
$$(3x+1)(x+3) = 0$$

$$x = -\frac{1}{3}, x = -3$$

# EXAMPLE 19

The diagram represents the mapping of y onto $x$ by $g(y) = \frac{5}{1-by}, y \neq \frac{1}{b}$ and the mapping of $y$ onto $z$ by the function $f(y) = ay + b$.

(a) Find the value of a and of b.




SOLUTION

$$(a)g(y) = \frac{5}{1-by}$$

$$g(1) = -5$$

$$-5 = \frac{5}{1-b(1)}$$

$$-5(1-b) = 5$$

$$5b = 10$$

$$b = 2$$

$$f(y) = ay + b$$

$$f(1) = 5$$

$$5 = a(1) + b$$

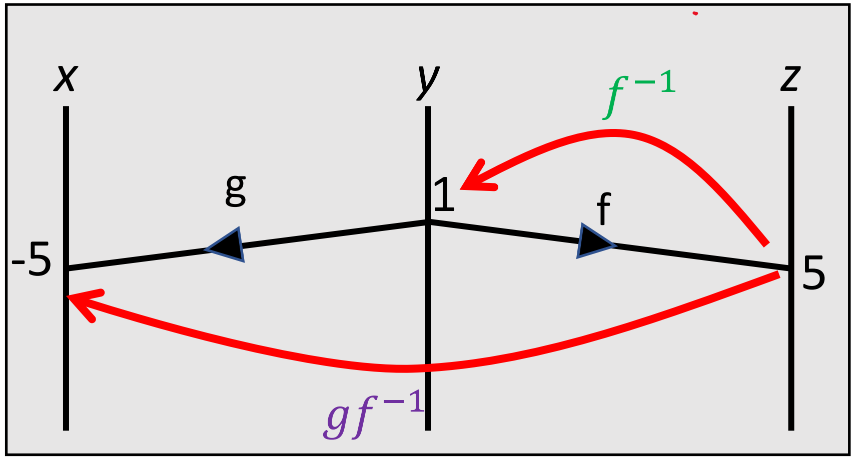
$$5 = a + 2$$

$$a = 3$$

# EXAMPLE 19

The diagram represents the mapping of y onto *x* by $g(y) = \frac{5}{1-by}, y \neq \frac{1}{b}$ and the mapping of *y* onto *z* by the function $f(y) = ay + b$.

(b) Show the function which maps *z* onto *x* is $\frac{15}{7-2z}$, $z \neq \frac{7}{2}$.




SOLUTION

$g(z) = \frac{5}{1-2z}$ $\quad f(z) = 3z + 2$

Let $w = 3z + 2$

$$3z = w - 2$$

$$z = \frac{w-2}{3}$$

$$f^{-1}(z) = \frac{z-2}{3}$$

$$gf^{-1}(z) = \frac{5}{1-2\left(\frac{z-2}{3}\right)}$$

$$gf^{-1}(z) = \frac{5}{\frac{7-2z}{3}}$$

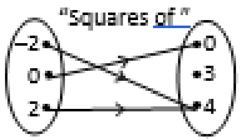
$gf^{-1}(z) = \frac{15}{7-2z}, z \neq \frac{7}{2}$

# FUNCTIONS

## Relations

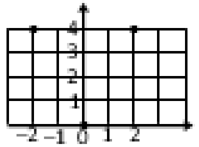
1. Representations of Relations:

(a) **Arrow Diagram**




(b) **Ordered pairs** $\{(-2,4),(0,0),(2,4)\}$

(c) **Graph**

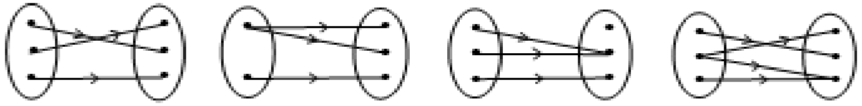


2. Other Terms:

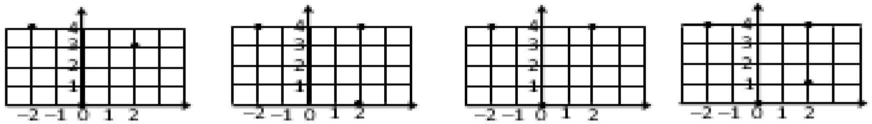
(a) **Objects** of 4 are −2 and 2.
(b) **Image** of 2 is 4.
(c) **Domain** $= \{-2,0,2\}$
(d) **Codomain** = Second set. Eg: Codomain of 1(a) $= \{0,3,4\}$
(e) **Range** $= \{0,4\}$

3. Types of Relations:

(a) **One to one** (b) **One to many** (c) **Many to one** (d) **Many to many**



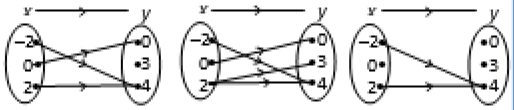
$\{(a,b),(c,d),(e,f)\}$ $\{(a,b),(a,d),(e,f)\}$ $\{(a,b),(c,b),(e,f)\}$ $\{(a,b),(c,b),(a,f)\}$



## Function

1. − is a relation
   − every object is mapped onto **only one** image.

2. Examples:




3. Notation:

Example: Given $f: x \to x^2$ or $f(x) = x^2$. Find

(a) the image of 9 (b) the objects of 9.

(a) $f(9) = 9^2$
$= 81$

(b) $f(x) = 9$
$x^2 = 9$
$x = \pm 3$

## Absolute Value Function

Example:

**Given** $f(x) = |2x - 5|$.

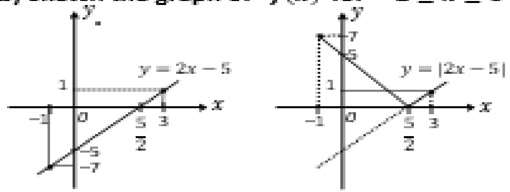
**(a) Find (i) the image of 2 and of 3,**
**(ii) the values of $x$ when $f(x) = 3$.**

(a)(i) $f(2)$
$= |2(2) - 5|$
$= 1$

$f(3)$
$= |2(3) - 5|$
$= 1$

(ii) $f(x) = 3$
$|2x - 5| = 3$
$2x - 5 = \pm 3$
$x = 1,\ x = 4$

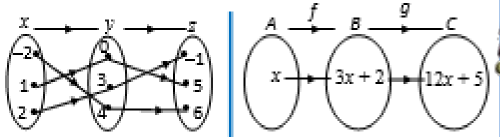
**(b) Sketch the graph of $f(x)$ for $-1 \le x \le 3$**




## Composite Function

1. Examples: $gf, fg, g^2, f^{-1}g$

2.



$hg(2) = -1$
$g^{-1}h^{-1}(5) = 1$

$f(x) = 3x + 2$
$gf(x) = 12x + 5$

3. More Example:

**Given** $g(x) = \frac{x+1}{3x+2}, x \neq -\frac{2}{3}$ **and** $h(x) = x^2$, **find**

**(a)** $gh$

$g(h(x)) = \frac{x^2+1}{3x^2+2}$,

**(b)** $hg$

$h(g(x)) = \left(\frac{x+1}{3x+2}\right)^2$

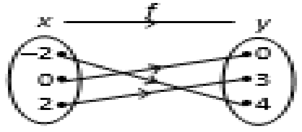
**(c)** $g^2$

$g^2(x) = gg(x)$
$= \frac{\frac{x+1}{3x+2}+1}{3\left(\frac{x+1}{3x+2}\right)+2}$
$= \frac{4x+3}{9x+7}, x \neq -\frac{7}{9}$

## Inverse Function

1. $f^{-1}$ = inverse function of $f$.

2.



$f^{-1}: y \to x$
$f^{-1}(3) = 2$

3. More Examples:

Given $g(x) = \frac{x+1}{3x+2}, x \neq -\frac{2}{3}$, find $g^{-1}(x)$.

Let $u = \frac{x+1}{3x+2}$
Then $u(3x+2) = x + 1$
$x = \frac{1-2u}{3u-1}$
$\therefore g^{-1}(x) = \frac{1-2x}{3x-1}, x \neq \frac{1}{3}$

## Given a function & a composite function, find the other function

Notes:

1. $g(x) = gh(h^{-1}(x))$
2. $g(x) = h^{-1}(hg(x))$

More Example:

**Given** $gh(x) = \frac{x+1}{3x+2}, x \neq -\frac{2}{3}$ **and** $h(x) = 2x$, **find** $g$.

$h^{-1}(x) = \frac{x}{2}$
$g(x) = gh(h^{-1}(x))$
$= gh\left(\frac{x}{2}\right)$
$= \frac{\frac{x}{2}+1}{3\left(\frac{x}{2}\right)+2}$
$= \frac{x+2}{3x+4}, x \neq -\frac{4}{3}$