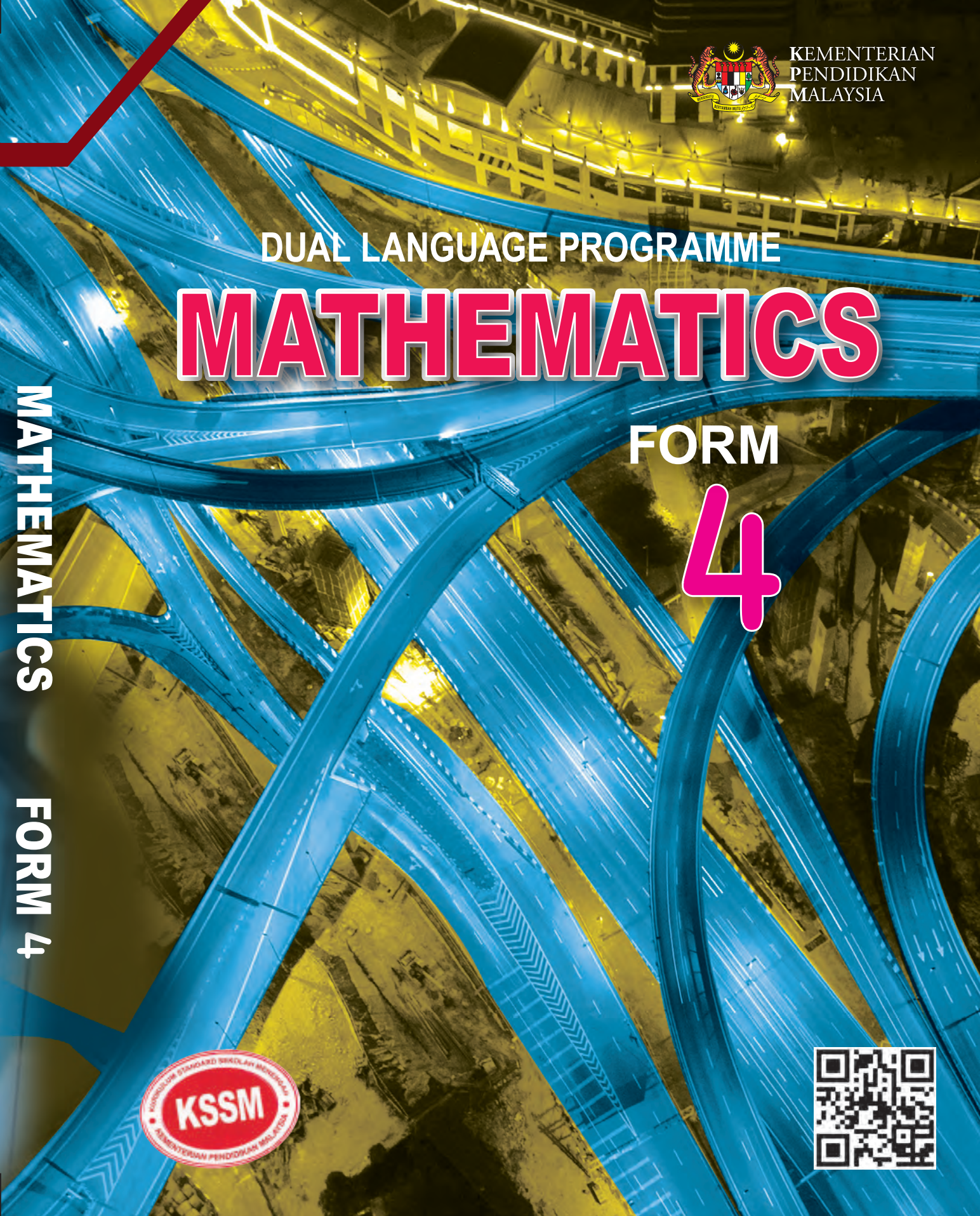

KEMENTERIAN
PENDIDIKAN
MALAYSIA
DUAL LANGUAGE PROGRAMME
MATHEMATICS
FORM
4
MATHEMATICS
FORM 4
KSSM

KURIKULUM STANDARD SEKOLAH MENENGAH

MATHEMATICS
FORM 4

Authors
Yeow Pow Choo
Thavamani A/P Renu
Kamalah A/P Raman
Wong Jin Wen
Vincent De Selva A/L Santhanasamy

Translators
Lien Poh Choo
Chang Tze Hin
Chew Lee Kian
Kho Choong Quan

Editors
Premah A/P Rasamanie
Cynthia Cheok Ching Tuing
Tiew Eyan Keng
Tan Swee Chang

Designer
Ardi Bin Lidding

Illustrators
Asparizal Bin Mohamed Sudin
Mohammad Kamal Bin Ahmad

Sasbadi Sdn. Bhd. 198501006847 (139288-X)
(Wholly-owned subsidiary of Sasbadi Holdings Berhad 201201038178 (1022660-T))

2019

KEMENTERIAN
PENDIDIKAN
MALAYSIA

Book Series No: 0178

KPM2019 ISBN 978-983-77-1531-8
First Published 2019

Published for Ministry of Education Malaysia by:
Sasbadi Sdn. Bhd. 198501006847 (139288-X)
(Wholly-owned subsidiary of Sasbadi Holdings Berhad 201201038178 (1022660-T))
Lot 12, Jalan Teknologi 3/4,
Taman Sains Selangor 1, Kota Damansara,
47810 Petaling Jaya,
Selangor Darul Ehsan, Malaysia.
Tel: +603-6145 1188 Fax: +603-6145 1199
Website: www.sasbadisb.com
E-mail: enquiry@sasbadi.com

Layout and Typesetting:
Sasbadi Sdn. Bhd. 198501006847 (139288-X)
Font type: Times New Roman
Font size: 11 point

Printed by:
Dicetak di Malaysia oleh
C.T. Book Makers Sdn. Bhd. (416129-H)
Lot 530 & 531, Jalan Perusahaan 3,
Bandar Baru Sungai Buloh,
47000 Selangor.

ACKNOWLEDGEMENT

The publishing of this textbook involves cooperation from various parties. Our wholehearted appreciation and gratitude goes out to all involved parties:

- Educational Resources and Technology Division
- Committee members of *Penyemakan Naskhah Sedia Kamera*, Educational Resources and Technology Division, Ministry of Education Malaysia.
- Committee members of *Penyemakan Pembetulan Naskhah Sedia Kamera*, Educational Resources and Technology Division, Ministry of Education Malaysia.
- Officers of the Educational Resources and Technology Division and the Curriculum Development Division, Ministry of Education Malaysia.
- Officers of the English Language Teaching Centre (ELTC), Institute of Teacher Education Malaysia, Ministry of Education Malaysia.
- Chairperson and members of the Quality Control Panel.
- Editorial Team and Production Team, especially the illustrators and designers.
- Everyone who has been directly or indirectly involved in the successful publication of this book.

Contents

Introduction v

Symbols and Formulae vii

CHAPTER 1 **Quadratic Functions and Equations in One Variable** **1**

1.1 Quadratic Functions and Equations 2

CHAPTER 2 **Number Bases** **32**

2.1 Number Bases 34

CHAPTER 3 **Logical Reasoning** **54**

3.1 Statements 56

3.2 Arguments 71

CHAPTER 4 **Operations on Sets** **94**

4.1 Intersection of Sets 96

4.2 Union of Sets 106

4.3 Combined Operations on Sets 116

CHAPTER 5 **Network in Graph Theory** **128**

5.1 Network 130

CHAPTER 6 Linear Inequalities in Two Variables 154

6.1 Linear Inequalities in Two Variables 156
6.2 Systems of Linear Inequalities in Two Variables 165

CHAPTER 7 Graphs of Motion 182

7.1 Distance-Time Graphs 184
7.2 Speed-Time Graphs 195

CHAPTER 8 Measures of Dispersion for Ungrouped Data 210

8.1 Dispersion 212
8.2 Measures of Dispersion 219

CHAPTER 9 Probability of Combined Events 242

9.1 Combined Events 244
9.2 Dependent Events and Independent Events 246
9.3 Mutually Exclusive Events and Non-Mutually Exclusive Events 253
9.4 Application of Probability of Combined Events 262

CHAPTER 10 Consumer Mathematics: Financial Management 270

10.1 Financial Planning and Management 272

Answers 293
Glossary 309
References 311
Index 312

Download a free QR Code scanner application to your mobile device.

Note: Pupils can download a free dynamic geometry software to open the related files.

http://bt.sasbadi.com/m4iv

Introduction

This Form 4 Mathematics Textbook is prepared based on *Kurikulum Standard Sekolah Menengah (KSSM)*. This book contains 10 chapters arranged systematically based on Form 4 Mathematics *Dokumen Standard Kurikulum dan Pentaksiran (DSKP)*.

At the beginning of each chapter, pupils are introduced to materials related to daily life to stimulate their thinking about the content. The learning standards and word lists are included to provide a visual summary of the chapter's content.

Special features of this book are:

	Description
You will learn	contains learning standards that pupils need to achieve in each chapter
Why Study This Chapter?	tells the importance of knowledge and skills to be learned in this chapter
Walking Through Time	contains the historical background or origin of the content
WORD BANK	contains key vocabulary in each chapter
Mind Stimulation	contains activities that help pupils understand the basic mathematical concepts
INFO ZONE	contains additional information about the content
TIPS	contains additional knowledge that pupils need to know
Indicator	contains additional facts and common mistakes that pupils need to know

	Description
	contains challenging tasks to stimulate pupils' critical and creative thinking skills
	develops pupils' mathematical communication skills
	helps pupils to recall what they have learned
	contains mathematical concepts related to Malaysia's achievements
	enables pupils to carry out and present project work
	assesses pupils' understanding on the concepts they have learned
	contains questions of various thinking skill levels
	enables pupils to scan a QR Code using a mobile device for further information
	covers the use of digital tools, calculators, hands-on activities and games to enhance pupils' understanding more effectively
	summarises the chapter
	guides pupils to self-assess their achievement
	contains alternative methods to check the answers
	contains questions to test pupils' higher order thinking skills

Symbols and Formulae

Symbols

$=$	is equal to	$\in$	an element of	$n(A)$	number of elements of set A
$\neq$	is not equal to	$\notin$	not an element of	Σ	sum
$\approx$	is approximately equal to	ξ	universal set	σ^2	variance
$>$	is more than	$\subset$	a subset of	σ	standard deviation
$<$	is less than	$\not\subset$	not a subset of	$\bar{x}$	mean
$\geqslant$	is more than or equal to	A'	complement of set A	G	graph
$\leqslant$	is less than or equal to	$\{\ \}, \phi$	empty set	e	edge
$\sim$	(tilde) negation	$\cap$	intersection	v	vertex
$p \Rightarrow q$	if p, then q	$\cup$	union	d	degree
$p \Leftrightarrow q$	p if and only if q				

Formulae

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$n(A') = n(\xi) - n(A)$

$n(A' \cap B') = n(A \cup B)'$

$n(A' \cup B') = n(A \cap B)'$

$P(A) = \dfrac{n(A)}{n(S)}$

Complement of event A, $P(A') = 1 - P(A)$

$P(A \text{ and } B) = P(A \cap B)$

$P(A \cap B) = P(A) \times P(B)$

$P(A \text{ or } B) = P(A \cup B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Sigma d(v) = 2E;\ v \in V$

Gradient, $m = \dfrac{\text{Vertical distance}}{\text{Horizontal distance}}$

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$m = -\dfrac{y\text{-intercept}}{x\text{-intercept}}$

$\text{Speed} = \dfrac{\text{Distance}}{\text{Time}}$

$\text{Average speed} = \dfrac{\text{Total distance travelled}}{\text{Total time taken}}$

$\text{Acceleration} = \dfrac{\text{Change of speed}}{\text{Change in time}}$

$\bar{x} = \dfrac{\Sigma x}{N}$

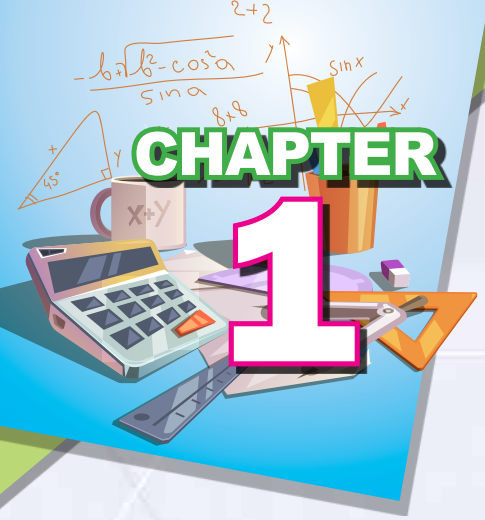
$\bar{x} = \dfrac{\Sigma fx}{\Sigma f}$

Variance, $\sigma^2 = \dfrac{\Sigma(x - \bar{x})^2}{N} = \dfrac{\Sigma x^2}{N} - \bar{x}^2$

Variance, $\sigma^2 = \dfrac{\Sigma f(x - \bar{x})^2}{\Sigma f} = \dfrac{\Sigma fx^2}{\Sigma f} - \bar{x}^2$

Standard deviation, $\sigma = \sqrt{\dfrac{\Sigma(x - \bar{x})^2}{N}} = \sqrt{\dfrac{\Sigma x^2}{N} - \bar{x}^2}$

Standard deviation, $\sigma = \sqrt{\dfrac{\Sigma f(x - \bar{x})^2}{\Sigma f}} = \sqrt{\dfrac{\Sigma fx^2}{\Sigma f} - \bar{x}^2}$



Quadratic Functions and Equations in One Variable

You will learn

► Quadratic Functions and Equations

Pulau Warisan is located in Kuala Terengganu. The island becomes a new tourist attraction because it is a man-made island connected with a bridge. This bridge is similar to the one in Putrajaya.

Do you know that the shape of this bridge has special mathematics characteristics?

Why Study This Chapter?

Quadratic functions and equations are widely used in science, business, sports and others. In sports, quadratic functions are important in sports events such as shot put, discus and javelin. In architecture, we often see curved structures in the shape of parabola which are related to the mastery of quadratic concepts.

Walking Through Time

Al-Khawarizmi
(780 AD – 850 AD)

Al-Khawarizmi is well-known as the Father of Algebra. He was the founder of a few mathematics concepts. His work in algebra was outstanding. He was responsible for initiating the systematic and logical approach in solving linear and quadratic equations.

http://bt.sasbadi.com/m4001

WORD BANK

• quadratic function	• *fungsi kuadratik*
• axis of symmetry	• *paksi simetri*
• variable	• *pemboleh ubah*
• root	• *punca*
• maximum point	• *titik maksimum*
• minimum point	• *titik minimum*

1.1 Quadratic Functions and Equations

What is a quadratic expression in one variable?

Learning Standard

Identify and describe the characteristics of quadratic expressions in one variable.

Have you ever sketched the movement of a ball kicked by a football player, as shown in the picture?

The shape of this movement is a parabola.

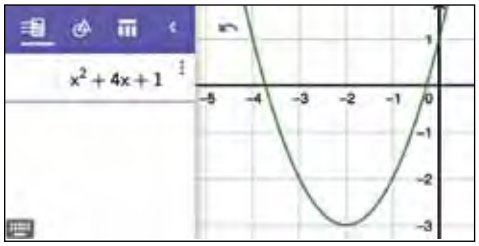
Do you know that this parabola has its own equation, just like a straight line which has its own equation?

Mind Stimulation 1

Aim: To identify and describe the characteristics of quadratic expressions in one variable.

Steps:

1. Based on the table in Step 3, insert all the expressions one by one in the dynamic geometry software as shown below.



Scan the QR Code to carry out this activity.
https://www.geogebra.org/graphing

2. Observe the graph obtained.
3. Complete the table below.

Expression		Characteristic	
		Shape of graph	Coordinates of the lowest or highest point (if any)
(a)	$x^2 + 4x + 1$		
(b)	$x^{-2} - 1$		
(c)	$-2x^2 - 2x + 5$		
(d)	$5x + 4$		
(e)	$3x^2 - 2$		
(f)	$-2x^2 + 4x$		
(g)	$x^3 + 1$		

Discussion:

The graph of a quadratic expression is either $\cup$ or $\cap$ and has the highest point or the lowest point. Which expression is a quadratic expression? Justify your answer.

From the activity in Mind Stimulation 1, it is found that:

A quadratic expression in one variable is an expression whereby the **highest power** for the variable is **two**.

In general,

The general form of a quadratic expression is $ax^2 + bx + c$.

Where;
a, b and c are constants and $a \neq 0$,
x is a variable.

INTERACTIVE ZONE

Why is $a \neq 0$ in a quadratic expression? Discuss.

For example:

$x^2 + 2x - 1$, $-y^2 + 3y$, $\frac{1}{3}m^2 - m + 4$ and $2n^2 + 5$ are quadratic expressions.

TIPS

Besides x, other letters can be used to represent variables.

Example 1

Determine whether each of the following expressions is a quadratic expression in one variable. If not, justify your answer.

(a) $2x^2 + 5$

(b) $x^3 - 6$

(c) $3x^2 + 2y + 1$

(d) $\frac{1}{2}m^2$

(e) $2x^2 - \frac{3}{x^2}$

(f) $4x^2 - x^{\frac{1}{2}}$

TIPS

The values of constants b and c can be zero.

MY MEMORY

$\frac{3}{x^2} = 3x^{-2}$

$x^{\frac{1}{2}} = \sqrt{x}$

Solution:

(a) $2x^2 + 5$ is a quadratic expression in one variable.

(b) $x^3 - 6$ is not a quadratic expression because the highest power of the variable x is 3.

(c) $3x^2 + 2y + 1$ is not a quadratic expression in one variable because there are two variables, x and y.

(d) $\frac{1}{2}m^2$ is a quadratic expression in one variable.

(e) $2x^2 - \frac{3}{x^2}$ is not a quadratic expression because there is a variable with a power which is not a whole number.

(f) $4x^2 - x^{\frac{1}{2}}$ is not a quadratic expression because there is a variable with a power which is not a whole number.

Mind Stimulation 2

Aim: To state the values of a, b and c in a quadratic expression.

Steps:

1. Observe (a) in the table below.
2. Determine the values of a, b and c for the subsequent quadratic expressions.

	Quadratic expression	Comparison
(a)	$2x^2 - 3x + 1$	$2x^2 - 3x + 1$ $ax^2 + bx + c$ $a =$ 2 $b =$ –3 $c =$ 1
(b)	$2x^2 - 4$	$\square x^2 + \square x + \square$ $ax^2 + bx + c$ $a =$ $b =$ $c =$
(c)	$\frac{1}{2}x^2 + 5x - \frac{3}{2}$	$a =$ $b =$ $c =$
(d)	$-x^2 + x$	$a =$ $b =$ $c =$
(e)	$-x^2 - 3x - 9$	$a =$ $b =$ $c =$
(f)	$\frac{1}{2}x^2$	$a =$ $b =$ $c =$

Discussion:
How do you determine the values of a, b and c?

From the activity in Mind Stimulation 2, it is found that:

All quadratic expressions can be written in the form of $ax^2 + bx + c$, where $a \neq 0$.

In a quadratic expression,

a is the coefficient of x^2,
b is the coefficient of x,
c is a constant.

INTERACTIVE ZONE

Why are a and b known as the coefficients and c the constant?

Self Practice 1.1a

1. Determine whether each of the following expressions is a quadratic expression in one variable. If not, justify your answer.

(a) $x^2 - 5$ (b) $2x^2 + x^{-2}$ (c) $3y^2 - 3x + 1$

(d) $-\frac{1}{2}m^2$ (e) $x^3 - x$ (f) $x^{\frac{1}{2}} + 2x - 1$

(g) $\frac{1}{x^2} + 4x - 1$ (h) $p^2 - \frac{1}{2}p + 3$ (i) $n(n-2)$

2. Determine the values of a, b and c for each of the following quadratic expressions.

(a) $2x^2 - 5x + 1$ (b) $x^2 - 2x$ (c) $2y^2 + 1$

(d) $-\frac{1}{2}p^2 + 4p$ (e) $1 - x - 2x^2$ (f) $4x^2$

(g) $h^2 + \frac{3}{2}h - 4$ (h) $\frac{1}{3}k^2 - 2$ (i) $2r(r-3)$

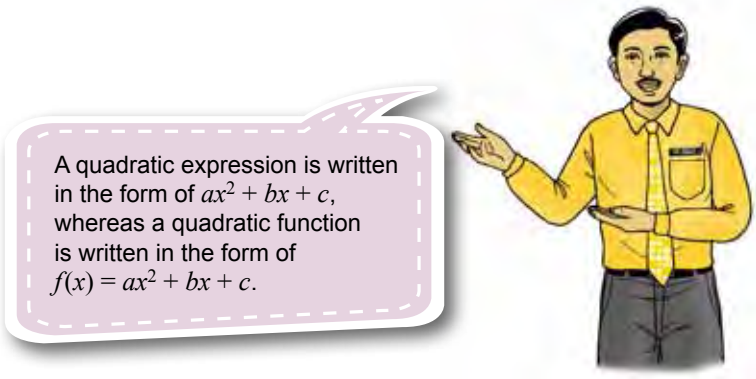
What is the relationship between a quadratic function and many-to-one relation?

Learning Standard

Recognise quadratic function as many-to-one relation, hence, describe the characteristics of quadratic functions.

MY MEMORY

Types of relation
- One-to-one relation
- One-to-many relation
- Many-to-one relation
- Many-to-many relation



INTERACTIVE ZONE

Discuss and give examples of many-to-one relation.

Mind Stimulation 3

Aim: To recognise quadratic functions as many-to-one relation.

Materials: Ruler, pencil

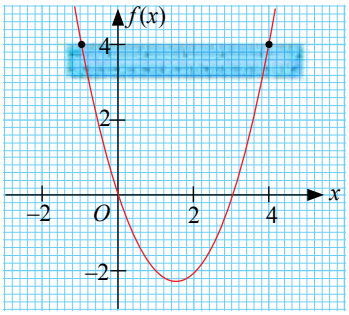
Steps:

1. Based on the graphs of functions $f(x)$ below, draw a line which is parallel to the x-axis on graphs (b) and (c), as in graph (a).
2. Mark the points of intersection between the graph of function $f(x)$ and the straight line.
3. State the number of points of intersection and the coordinates of the points of intersection.
4. Repeat Steps 1 to 3 by placing the ruler at different values of $f(x)$. Ensure the straight lines drawn are parallel to the x-axis.

INFO ZONE

For a quadratic function, $y = f(x)$.

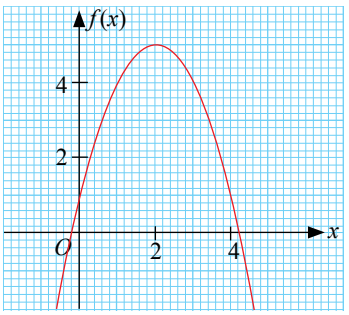
(a) $f(x) = x^2 - 3x$



Number of points of intersection = 2

Points of intersection = (4, 4), (–1, 4)

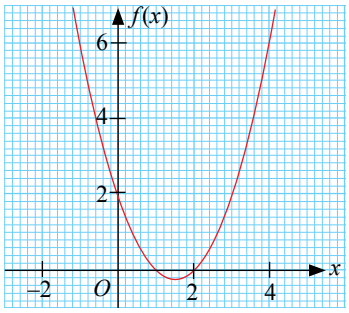
(b) $f(x) = -x^2 + 4x + 1$



Number of points of intersection =

Points of intersection = ( ), ( )

(c) $f(x) = x^2 - 3x + 2$



Number of points of intersection =

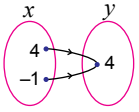
Points of intersection = ( ), ( )

Discussion:

1. What is the relationship between the x-coordinates and y-coordinates of both points of intersection for each function?
2. What is the type of relation of a quadratic function?

MY MEMORY

For a point on a Cartesian plane, the x-coordinate is the object and the y-coordinate is the image.



From the activity in Mind Stimulation 3, it is found that:

All quadratic functions have the same image for two different objects.

In general,

The type of relation of a quadratic function is a many-to-one relation.

Scan the QR Code to watch the vertical line test.
http://bt.sasbadi.com/m4006

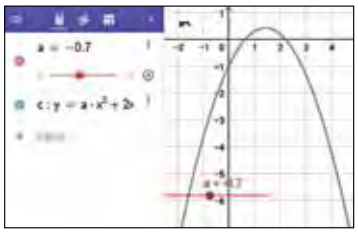
What is the shape of the graph of a quadratic function?

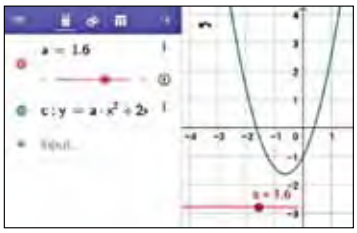
Mind Stimulation 4

Aim: To identify and describe the relationship between the value of a and the shape of the graph of a quadratic function.

Steps:

1. Drag the slider slowly from left to right. Observe the shape of the graph.





Scan the QR Code to carry out this activity.
https://www.geogebra.org/graphing/t5az2zwm

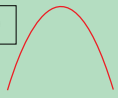
2. Sketch at least two graphs for positive values of a and two graphs for negative values of a.

Discussion:
What is the relationship between the value of a and the shape of a graph?

From the activity in Mind Stimulation 4, it is found that:

For a graph of $f(x) = ax^2 + bx + c$, $a \neq 0$
(a) there are only two shapes of the graphs,
(b) the value of a determines the shape of the graph.

$a > 0$ 

$a < 0$ 

TIPS

The curved shape of the graph of a quadratic function is called a parabola.

What is the maximum or minimum point of a quadratic function?

Each sketch of the graph of a quadratic function has the highest or lowest value of y-coordinate based on the shape of the sketch.

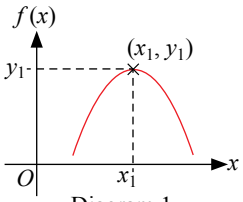


Diagram 1

For the sketch of the graph of a quadratic function with $a < 0$, y_1 is the highest value of y-coordinate and x_1 is the corresponding value for y_1. The point (x_1, y_1) is known as the **maximum point**.

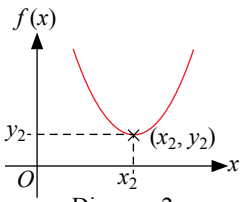


Diagram 2

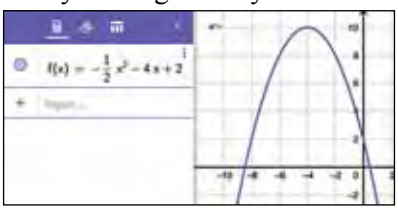
For the sketch of the graph of a quadratic function with $a > 0$, y_2 is the lowest value of y-coordinate and x_2 is the corresponding value for y_2. The point (x_2, y_2) is known as the **minimum point**.

Mind Stimulation 5

Aim: To explore the maximum or minimum point of a quadratic function.

Steps:

1. Based on the table in Step 2, insert the quadratic functions in the dynamic geometry software.



Scan the QR Code to carry out this activity.
https://www.geogebra.org/graphing

2. Complete the table below as in (a).

	Quadratic function	Value of a	Shape of graph	Maximum / Minimum point and coordinates
(a)	$f(x) = -\frac{1}{2}x^2 - 4x + 2$	$a = -\frac{1}{2}$		Maximum point Coordinates = $(-4, 10)$
(b)	$f(x) = x^2 - 4x + 3$			point Coordinates =
(c)	$f(x) = -2x^2 - 4x + 1$			point Coordinates =

3. Repeat Steps 1 and 2 for various quadratic functions.

Discussion:

What is the relationship between the value of a and the maximum or minimum point?

From the activity in Mind Stimulation 5, it is found that:

> For a quadratic function $f(x) = ax^2 + bx + c$,
> the maximum point is obtained when $a < 0$,
> the minimum point is obtained when $a > 0$.

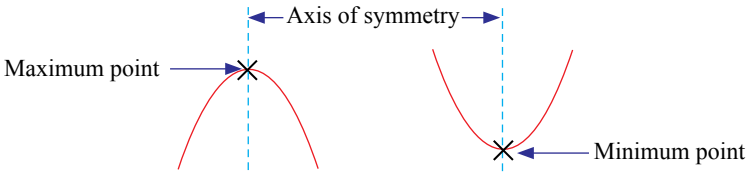
INFO ZONE

The maximum or minimum point is also called a stationary point or a turning point.

What is the axis of symmetry of the graph of a quadratic function?

The axis of symmetry of the graph of a quadratic function is a straight line that is parallel to the y-axis and divides the graph into two parts of the same size and shape.

The axis of symmetry will pass through the maximum or minimum point of the graph of the function as shown in the diagram below.



MY MEMORY

An axis of symmetry is a straight line that divides a geometrical shape or an object into two parts of the same size and shape.

Smart Mind

The equation of the axis of symmetry for a quadratic function is $x = -\frac{b}{2a}$.

Mind Stimulation 6

Aim: To draw and recognise the axis of symmetry of the graph of a quadratic function.

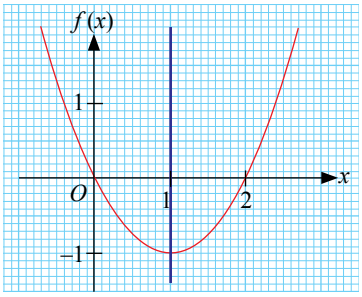
MY MEMORY

The equation of a straight line which is parallel to the y-axis is $x = h$.

Steps:

1. Using a ruler, draw the axis of symmetry for each graph of quadratic function below.
2. Write the equation of the axis of symmetry as in (a).

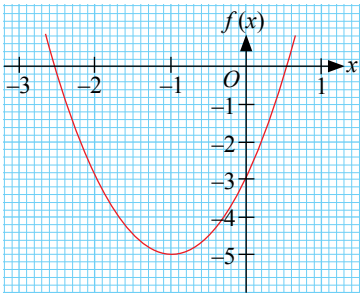
(a) $f(x) = x^2 - 2x$



Equation of axis of symmetry

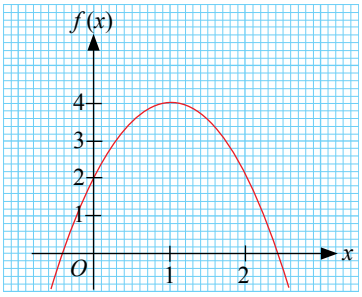
$x = 1$

(b) $f(x) = 2x^2 + 4x - 3$



Equation of axis of symmetry

(c) $f(x) = -2x^2 + 4x + 2$



Equation of axis of symmetry

Discussion:

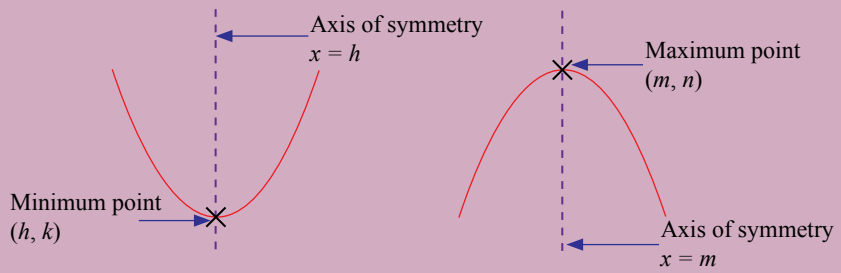
1. What is the relationship between the axis of symmetry of the graph of a quadratic function and the y-axis?
2. What is the relationship between the axis of symmetry of the graph of a quadratic function and the maximum or minimum point?

From the activity in Mind Stimulation 6, it is found that:

> The axis of symmetry of the graph of a quadratic function is parallel to the y-axis and passes through the maximum or minimum point.

In general,

> Each graph of quadratic function has one axis of symmetry which passes through the maximum or minimum point.



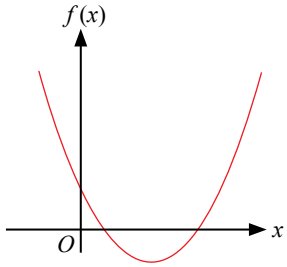
Self Practice 1.1b

1. Determine whether the shapes of the following graphs of quadratic functions is ∪ or ∩.

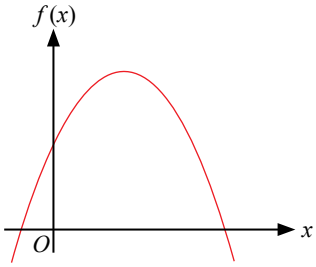
(a) $f(x) = x^2 - 4x + 1$ (b) $g(x) = -x^2 + 2x - 4$

2. For each graph of quadratic function $f(x) = ax^2 + bx + c$ below, state the range of value of a and state whether the graph has a maximum or minimum point.

(a)

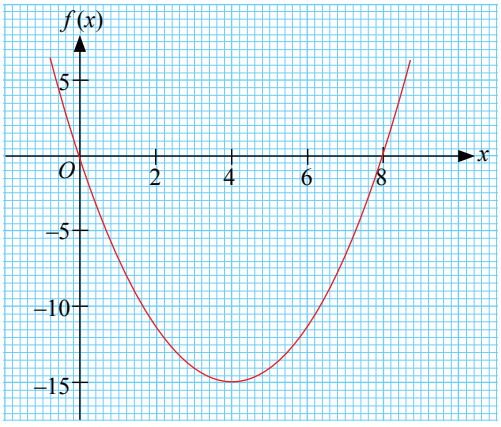


(b)

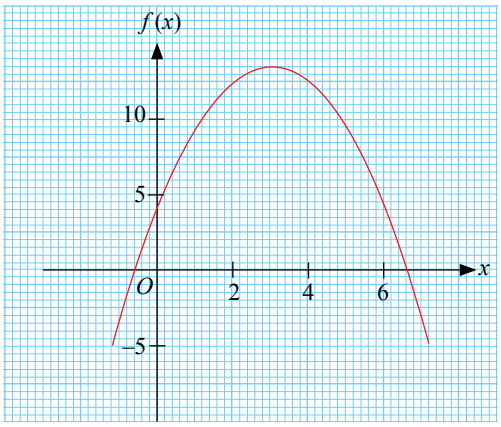


3. Determine the maximum or minimum point and state the equation of the axis of symmetry for each graph of quadratic function below.

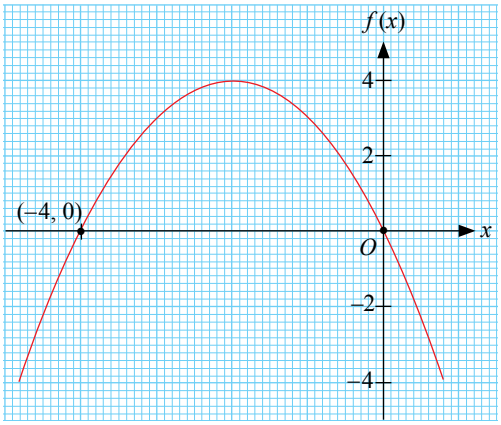
(a)



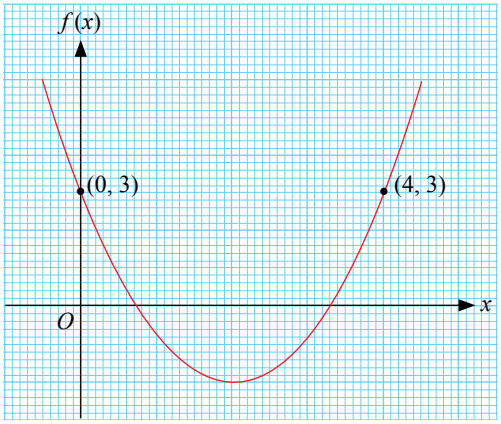
(b)



(c)



(d)



What are the effects of changing the values of a, b and c on graphs of quadratic functions, $f(x) = ax^2 + bx + c$?

Learning Standard

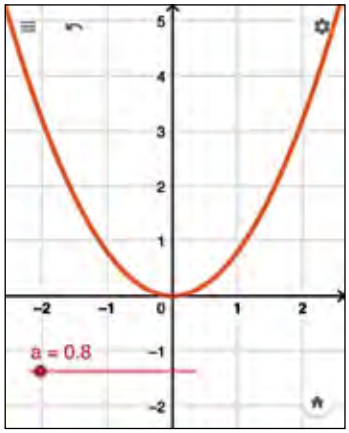
Investigate and make generalisation about the effects of changing the values of a, b and c on graphs of quadratic functions, $f(x) = ax^2 + bx + c$.

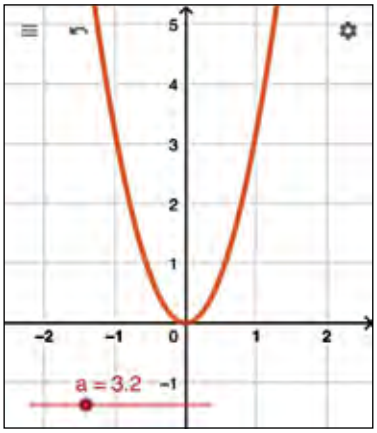
Mind Stimulation 7

Aim: To identify the effects of changing the values of a on graphs of quadratic functions $f(x) = ax^2 + bx + c$.

Steps:

1. Drag the slider from left to right.





Scan the QR Code to carry out this activity.
https://www.geogebra.org/graphing/nhxfjgy3

2. Observe the shape of the graph as the value of a changes.

Discussion:

What are the effects of changing the values of a to the graphs of quadratic functions?

From the activity in Mind Stimulation 7, it is found that:

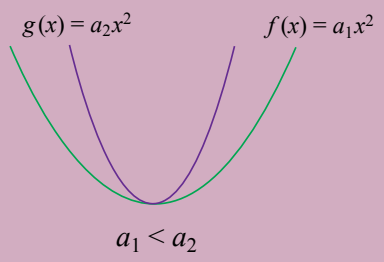
> The value of a determines the shape of the graph.

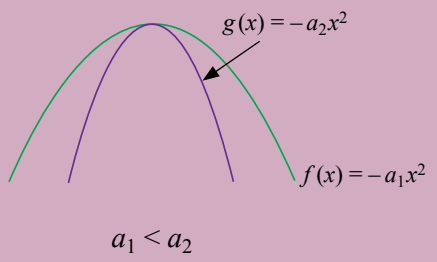
INTERACTIVE ZONE

Discuss the effects on the curve of the graphs of quadratic functions when $a < 0$.

In general,

For the graph of a quadratic function $f(x) = ax^2 + bx + c$, the smaller the value of a, the wider the curved shape of the graph and vice versa.



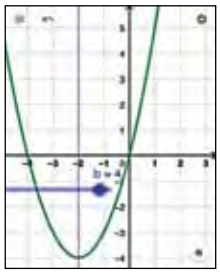
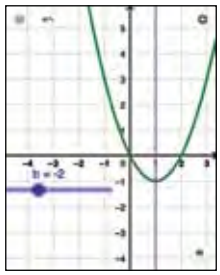


Mind Stimulation 8

Aim: To identify the effects of changing the values of b on graphs of quadratic functions $f(x) = ax^2 + bx + c$.

Steps:

1. Drag the slider from left to right.

Scan the QR Code to carry out this activity.
https://www.geogebra.org/graphing/vpzgvwba

2. Observe the position of the axis of symmetry as the value of b changes.

Discussion:

What are the effects of changing the values of b to the graphs of quadratic functions?

From the activity in Mind Stimulation 8, it is found that:

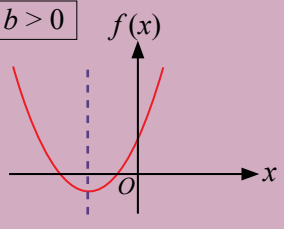
The value of b determines the position of the axis of symmetry.

In general,

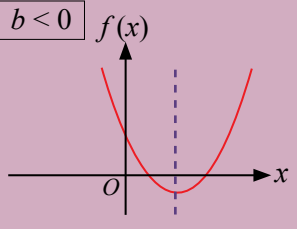
For the graph of a quadratic function $f(x) = ax^2 + bx + c$

if $a > 0$; $b > 0$, then the axis of symmetry lies on the left of the y-axis.
$b < 0$, then the axis of symmetry lies on the right of the y-axis.
$b = 0$, then the axis of symmetry is the y-axis.

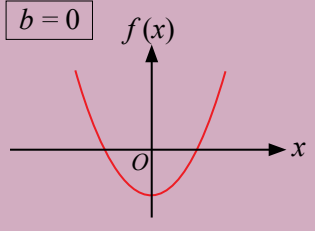
$b > 0$



$b < 0$

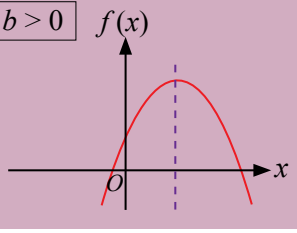


$b = 0$

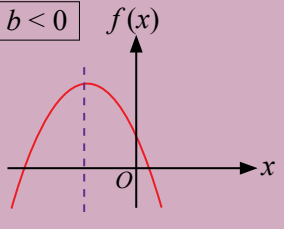


if $a < 0$; $b > 0$, then the axis of symmetry lies on the right of the y-axis.
$b < 0$, then the axis of symmetry lies on the left of the y-axis.
$b = 0$, then the axis of symmetry is the y-axis.

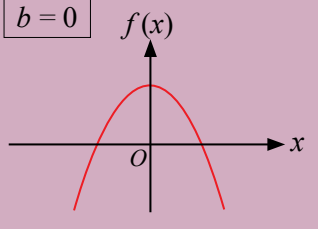
$b > 0$



$b < 0$



$b = 0$

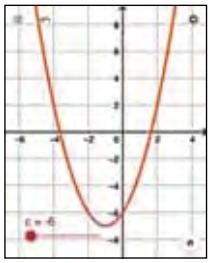


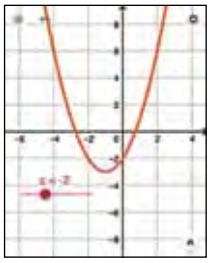
Mind Stimulation 9

Aim: To identify the effects of changing the values of c on graphs of quadratic functions $f(x) = ax^2 + bx + c$.

Steps:

1. Drag the slider from left to right.





Scan the QR Code to carry out this activity.
https://www.geogebra.org/graphing/rv7njx84

MY MEMORY

The y-intercept is a point of a graph that intersects the y-axis.

2. Observe the position of the y-intercept as the value of c changes.

Discussion:

What are the effects of changing the values of c to the graphs of quadratic functions $f(x) = ax^2 + bx + c$?

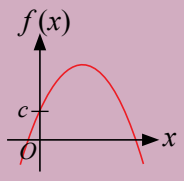
From the activity in Mind Stimulation 9, it is found that:

The value of c determines the position of the y-intercept.

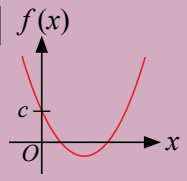
In general,

For the graph of a quadratic function $f(x) = ax^2 + bx + c$, the value of c determines the y-intercept of the graph.

$a < 0$



$a > 0$



Example 2

The quadratic function $f(x) = x^2 - 3x + c$ passes through a point A as given below. Calculate the value of c for each of the following cases.

(a) $A(0, 4)$ (b) $A(-1, 3)$

Solution:

(a) The point $A(0, 4)$ lies on the y-axis, thus $c = 4$.

(b) $f(x) = x^2 - 3x + c$

Substitute the values of $x = -1$ and $f(x) = 3$ into the quadratic function.

$3 = (-1)^2 - 3(-1) + c$

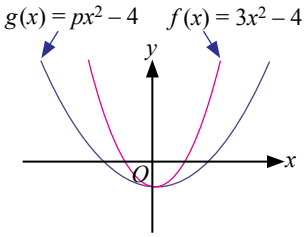
$c = -1$

TIPS

c is the y-intercept of a quadratic function $f(x) = ax^2 + bx + c$.

Example 3

The diagram shows two graphs of quadratic functions, $y = f(x)$ and $y = g(x)$, drawn on the same axes. State the range of the values of p. Explain your answer.



Solution:

$0 < p < 3$.

Since the curve of the graph $g(x)$ is wider, thus $p < 3$.

For a graph with the shape $\cup$, $p > 0$.

Thus, $0 < p < 3$.

Self Practice 1.1c

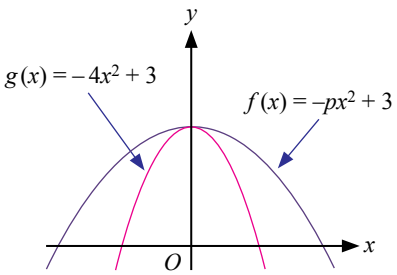
1. The quadratic functions below pass through the points stated. Calculate the value of c for each case.

(a) $f(x) = x^2 + 7x + c$, passes through point (0, 5).

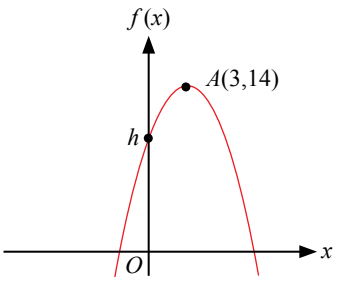
(b) $f(x) = 2x^2 - 4x + c$, passes through point (2, –3).

(c) $f(x) = -2x^2 + x + c$, y-intercept = 4.

2. The diagram on the right shows two graphs of quadratic functions, $y = f(x)$ and $y = g(x)$, drawn on the same axes. State the range of the values of p. Explain your answer.



3. The diagram on the right shows the graph of a quadratic function $f(x) = kx^2 + 6x + h$. Point $A(3, 14)$ is the maximum point of the graph of quadratic function.



(a) Given k is an integer where $-2 < k < 2$, state the value of k.

(b) Using the value of k from (a), calculate the value of h.

(c) State the equation of the quadratic function formed when the graph is reflected in the x-axis. Give your answer in the form of $f(x) = ax^2 + bx + c$.

How do you form a quadratic equation based on a situation?

Learning Standard

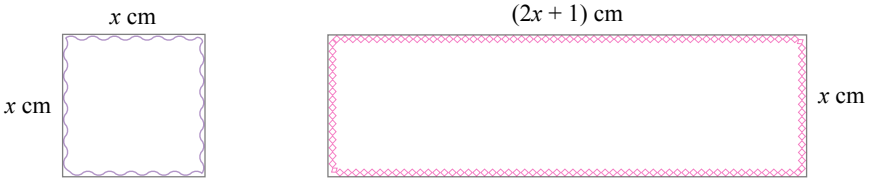
Form quadratic functions based on situations, and hence relate to the quadratic equations.

A quadratic function is written in the form of $f(x) = ax^2 + bx + c$ while a quadratic equation is written in the general form, $ax^2 + bx + c = 0$.

Try to guess my age. First I multiply my age with my own age. Next 21 times my age is subtracted from it. The result is 72.

Example 4

Mr Ganesan plans to make two different types of cards for Mathematics Club activities. The measurements of the cards are shown in the diagram below.



(a) Form a quadratic expression for the total area of the two cards, A cm^2, in terms of x.

(b) The total area of the two cards is 114 cm^2. Form a quadratic equation in terms of x.

Solution:

(a) $A = x^2 + x(2x + 1)$
$= x^2 + 2x^2 + x$
$= 3x^2 + x$

(b) $3x^2 + x = 114$
$3x^2 + x - 114 = 0$

Self Practice 1.1d

1. The diagram on the right shows a piece of land with a length of $(x + 20)$ m and a width of $(x + 5)$ m.

(a) Write a function for the area, A m^2, of the land.

(b) If the area of the land is 250 m^2, write a quadratic equation in terms of x. Give your answer in the form of $ax^2 + bx + c = 0$.

2. Aiman is 4 years older than his younger brother. The product of Aiman and his younger brother's ages is equal to their father's age. The father is 48 years old and Aiman's younger brother is p years old. Write a quadratic equation in terms of p.

What do you understand about the roots of a quadratic equation?

The roots of a quadratic equation $ax^2 + bx + c = 0$ are the values of the variable, x, which satisfy the equation.

Do you know how the roots of a quadratic equation are determined?

Learning Standard

Explain the meaning of roots of a quadratic equation.

INTERACTIVE ZONE

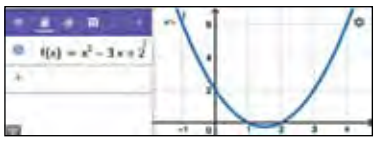
What is the meaning of "satisfy an equation"? Discuss.

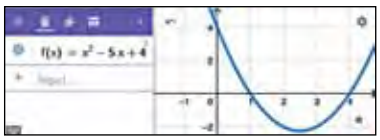
Mind Stimulation 10

Aim: To determine the values of a variable that satisfy a quadratic equation.

Steps:

1. Divide the class into two groups, A and B.
2. Group A will complete the table below without using the dynamic geometry software.
3. Group B will carry out this activity using the dynamic geometry software. Type each quadratic expression into the software. For each graph, determine the value of the quadratic expression for each given value of x.





Scan the QR Code to carry out this activity.
https://www.geogebra.org/graphing

4. Complete and determine the values of x that satisfy the quadratic equation in the table below.

$x^2 - 3x + 2 = 0$	
Value of x	Value of $x^2 - 3x + 2$
0	$0^2 - 3(0) + 2 = 2$
1	0
2	0
3	2
4	6
x are 1, 2	

$x^2 - 5x + 4 = 0$	
Value of x	Value of $x^2 - 5x + 4$
0	
1	
2	
3	
4	
x are	

$x^2 - 2x + 1 = 0$	
Value of x	Value of $x^2 - 2x + 1$
–2	
–1	
0	
1	
2	
x are	

$x^2 + x - 2 = 0$	
Value of x	Value of $x^2 + x - 2$
–2	
–1	
0	
1	
2	
x are	

$x^2 - 4x + 5 = 2$	
Value of x	Value of $x^2 - 4x + 5$
0	
1	
2	
3	
4	
x are	

$x^2 + 2x - 2 = 1$	
Value of x	Value of $x^2 + 2x - 2$
–3	
–2	
–1	
0	
1	
x are	

Discussion:
How can you determine the values of the variable that satisfy a quadratic equation?

From the activity in Mind Stimulation 10, it is found that:

(a) There are one or two values of the variable that satisfy a quadratic equation.
(b) The values of the variable that satisfy a quadratic equation are known as the roots of the quadratic equation.

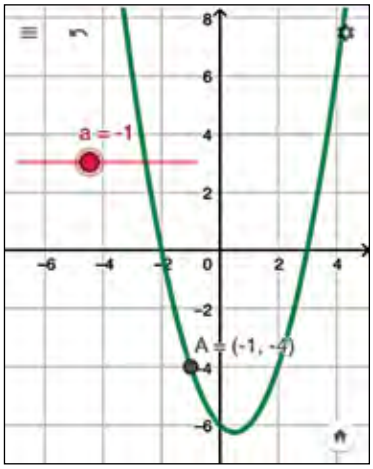
What is the relationship between the roots of a quadratic equation and the positions of the roots?

Mind Stimulation 11

Aim: To explore the positions of the roots of a quadratic equation on the graph of a quadratic function, $f(x) = 0$.

Steps:

1. Drag the slider to observe the changes of the x-coordinate and y-coordinate on the graph.
2. The roots of quadratic equation $x^2 - x - 6 = 0$ can be determined when $y = 0$. Drag the slider from left to right. Observe the coordinates of A.



Scan the QR Code to carry out this activity.
https://www.geogebra.org/graphing/bykrknjx

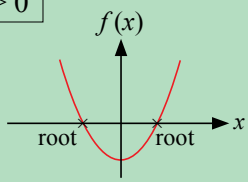
3. Determine the position of point A when y is 0.
4. Mark the point on the above diagram.

Discussion:
What do you notice about the positions of the roots of a quadratic equation on the graph of the quadratic function?

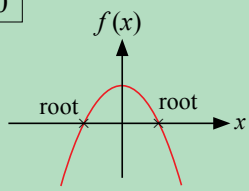
From the activity in Mind Stimulation 11, it is found that:

The roots of a quadratic equation $ax^2 + bx + c = 0$ are the points of intersection of the graph of the quadratic function $f(x) = ax^2 + bx + c$ and the x-axis, which are also known as the x-intercepts.

$a > 0$



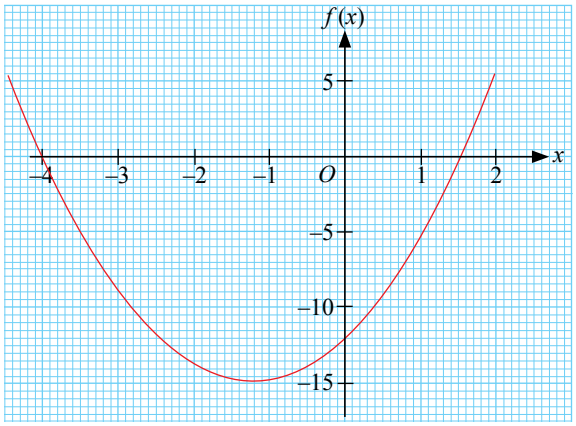
$a < 0$



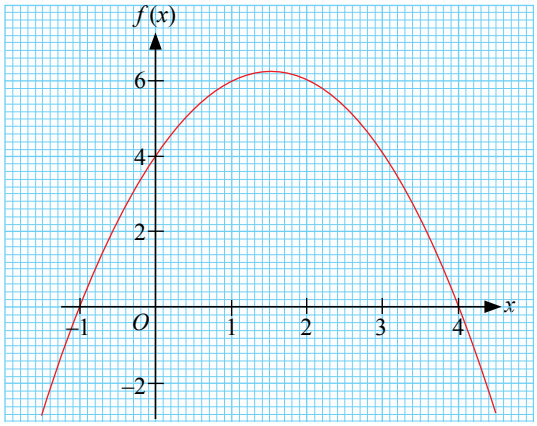
Example 5

For each graph of quadratic equation below, mark and state the roots of the given quadratic equation.

(a) $2x^2 + 5x - 12 = 0$

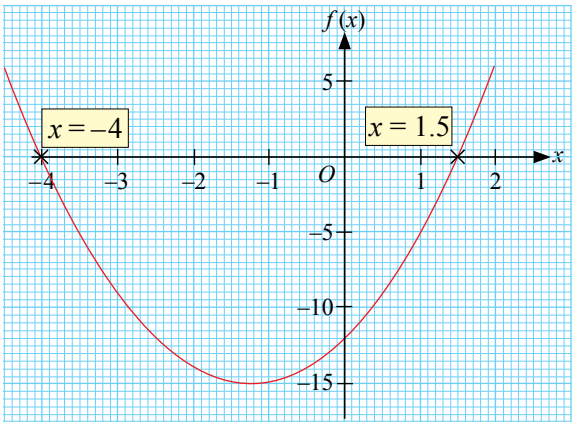


(b) $-x^2 + 3x + 4 = 0$



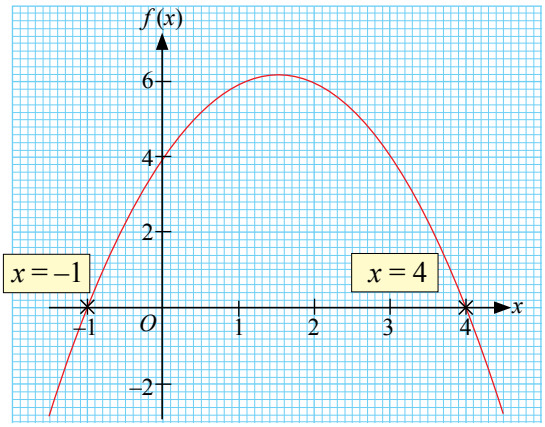
Solution:

(a) $2x^2 + 5x - 12 = 0$



The roots are -4 and 1.5.

(b) $-x^2 + 3x + 4 = 0$



The roots are -1 and 4.

Example 6

Determine whether each of the following values is a root of the given quadratic equation.

(a) $2x^2 - 7x + 3 = 0$; $x = 1$, $x = 3$

(b) $3x^2 - 7x + 5 = 3$; $x = 1$, $x = \frac{1}{3}$

Solution:

(a) $2x^2 - 7x + 3 = 0$

When $x = 1$,

Left:	Right:
$2x^2 - 7x + 3 = 2(1)^2 - 7(1) + 3$	0
$= 2 - 7 + 3$	
$= -2$ ← not the same	

Thus, $x = 1$ is not a root of the equation $2x^2 - 7x + 3 = 0$.

When $x = 3$,

Left:	Right:
$2x^2 - 7x + 3 = 2(3)^2 - 7(3) + 3$	0
$= 18 - 21 + 3$	
$= 0$ ← same	

Thus, $x = 3$ is a root of the equation $2x^2 - 7x + 3 = 0$.

(b) $3x^2 - 7x + 5 = 3$

When $x = 1$,

Left:	Right:
$3x^2 - 7x + 5 = 3(1)^2 - 7(1) + 5$	3
$= 3 - 7 + 5$	
$= 1$ ← not the same	

Thus, $x = 1$ is not a root of the equation $3x^2 - 7x + 5 = 3$.

When $x = \frac{1}{3}$,

Left:	Right:
$3x^2 - 7x + 5 = 3(\frac{1}{3})^2 - 7(\frac{1}{3}) + 5$	3
$= \frac{1}{3} - \frac{7}{3} + 5$	
$= 3$ ← same	

Thus, $x = \frac{1}{3}$ is a root of the equation $3x^2 - 7x + 5 = 3$.

MY MEMORY

The roots of a quadratic equation are the values of x that satisfy the equation.

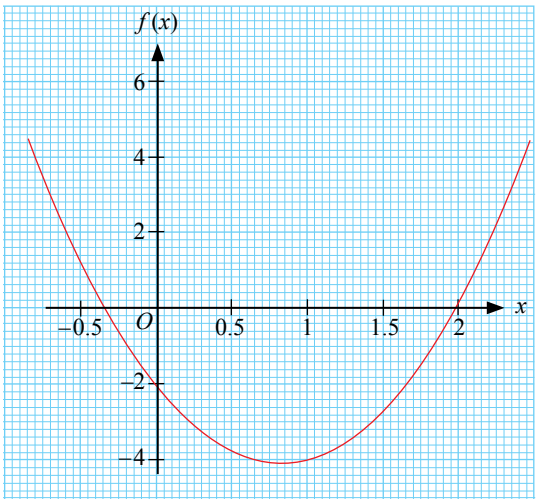
Checking Answer

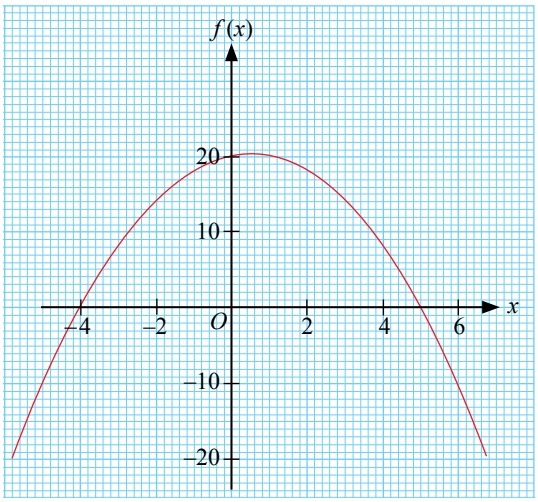
1. Press 2, Alpha, X, x^2, −, 7, Alpha, X, +, 3
2. Press CALC
   Display: x? 0.
3. Press 1 =
   Display: $2x^2 - 7x + 3$ −2.
4. Press CALC
   Display: x? 1.
5. Press 3 =
   Display: $2x^2 - 7x + 3$ 0.

Self Practice 1.1e

1. For each graph of quadratic function below, state the roots of the given quadratic equation.

(a) $3x^2 - 5x - 2 = 0$ (b) $-x^2 + x + 20 = 0$





2. Determine whether each of the following values is a root of the given quadratic equation.

(a) $x^2 - 5x + 6 = 0$; $x = 3$, $x = 2$

(b) $2x^2 - x - 1 = 0$; $x = 1$, $x = \frac{1}{2}$

(c) $3x^2 - 5x - 2 = 0$; $x = -\frac{1}{3}$, $x = -2$

(d) $3x^2 + 4x + 2 = 6$; $x = 2$, $x = \frac{2}{3}$

3. Determine whether each of the following values is a root of the given quadratic equation.

(a) $(x - 1)(x + 4) = 0$; $x = -4$, $x = 2$, $x = 1$

(b) $2(x - 3)(x - 5) = 0$; $x = -3$, $x = 3$, $x = 5$

(c) $3(2 + x)(x - 4) = 0$; $x = -2$, $x = 2$, $x = 4$

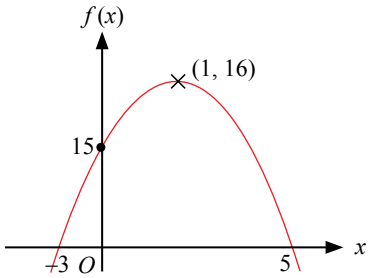
4. For the graph of quadratic function on the right, determine whether the given value of x is a root of the quadratic equation $f(x) = 0$.

(a) $x = 1$

(b) $x = -3$

(c) $x = 15$

(d) $x = 5$



How do you determine the roots of a quadratic equation by factorisation method?

Factorisation method is one of the methods used to determine the roots of a quadratic equation.

A quadratic equation needs to be written in the form of $ax^2 + bx + c = 0$ before we carry out factorisation.

Learning Standard

Determine the roots of a quadratic equation by factorisation method.

MY MEMORY

$2x^2 + 5x - 3$
$= (2x - 1)(x + 3)$

INFO ZONE

A quadratic equation can also be solved by using:
- method of completing the squares.
- formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 7

Determine the roots of the following quadratic equations by factorisation method.

(a) $x^2 - 5x + 6 = 0$

(b) $x^2 + \frac{7}{2}x = 2$

(c) $\frac{x}{2} = \frac{5x - 24}{x - 4}$

(d) $(y + 2)(y + 1) = 2(y + 11)$

Solution:

(a) $x^2 - 5x + 6 = 0$
$(x - 3)(x - 2) = 0$
$x = 3$ or $x = 2$

(b) $x^2 + \frac{7}{2}x = 2$
$2x^2 + 7x = 4$
$2x^2 + 7x - 4 = 0$
$(2x - 1)(x + 4) = 0$
$x = \frac{1}{2}$ or $x = -4$

(c) $\frac{x}{2} = \frac{5x - 24}{x - 4}$
$x(x - 4) = 2(5x - 24)$
$x^2 - 4x = 10x - 48$
$x^2 - 14x + 48 = 0$
$(x - 8)(x - 6) = 0$
$x = 8$ or $x = 6$

(d) $(y + 2)(y + 1) = 2(y + 11)$
$y^2 + 3y + 2 = 2y + 22$
$y^2 + y - 20 = 0$
$(y + 5)(y - 4) = 0$
$y = -5$ or $y = 4$

Checking Answer

Steps to solve $x^2 - 5x + 6 = 0$.

1. Press [mode] 3 times until the following display is shown.
   [EQN] [1] [MAT] [2] [VCT] [3]
2. Press [1] to choose [EQN], which is equation.
3. Display shows [unknowns?] [2] [3] press ▷
4. Display shows [Degree?] [2] [3] press [2], for power of 2
5. Display shows a? Enter the value 1, then press [=]
6. Display shows b? Enter the value -5, then press [=]
7. Display shows c? Enter the value 6, then press [=]
8. $x_1 = 3$ is displayed, press [=]
9. $x_2 = 2$ is displayed.

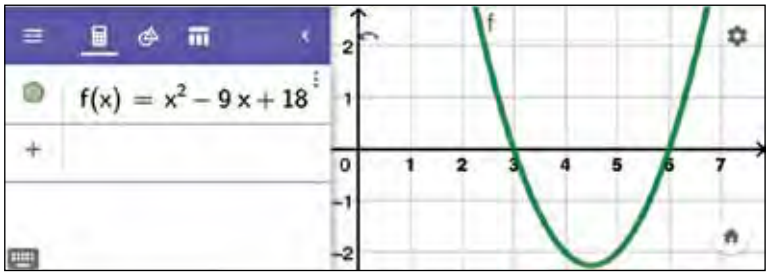
How do you determine the roots of a quadratic equation by the graphical method?

Mind Stimulation 12

Aim: To determine the roots of a quadratic equation on the graph of a quadratic function using the dynamic geometry software.

Steps:

1. Insert the quadratic equations in the dynamic geometry software.



Scan the QR Code to carry out this activity.
https://www.geogebra.org/graphing

2. Determine the roots of the quadratic equations and complete the following table.

Activity Sheet:

	Quadratic Equation	Roots
(a)	$x^2 - 9x + 18 = 0$	$x = 3, x = 6$
(b)	$4x^2 + 4x - 3 = 0$	
(c)	$-x^2 + 9x - 20 = 0$	
(d)	$-4x^2 - 11x + 3 = 0$	

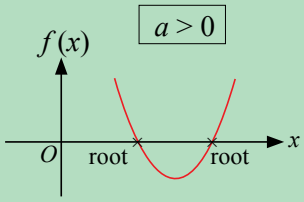
TIPS

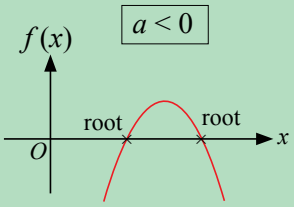
The root of a quadratic equation $ax^2 + bx + c = 0$ is the value of x which satisfies the quadratic equation.

Discussion:
How do you determine the roots of a quadratic equation using the graphical method?

From the activity in Mind Stimulation 12, it is found that:

The roots of a quadratic equation $ax^2 + bx + c = 0$ can be obtained using the graphical method by reading the values of x which are the points of intersection of the graph of the quadratic function $f(x) = ax^2 + bx + c$ and the x-axis.





Self Practice 1.1f

1. Determine the roots of each of the following quadratic equations using the factorisation method.

(a) $x^2 - 3x - 10 = 0$ (b) $x^2 - 10x + 16 = 0$ (c) $3x^2 - 5x + 2 = 0$

(d) $2x^2 + 8x - 24 = 0$ (e) $2x^2 + 3x - 9 = 0$ (f) $4x^2 - 3x - 10 = 0$

(g) $-3x^2 - x + 14 = 0$ (h) $x^2 - 5x = 0$ (i) $x^2 - 4 = 0$

2. Write each of the following quadratic equations in the general form. Hence, solve the quadratic equation.

(a) $m(m + 2) = 3$ (b) $3p(11 - 2p) = 15$ (c) $\frac{1}{2}y^2 = 12 - y$

(d) $a + \frac{5}{a} = 6$ (e) $\frac{8}{k} = 2 + k$ (f) $2h + \frac{6}{h} = 7$

(g) $(h - 2)(h - 1) = 12$ (h) $(2x - 1)^2 = 3x - 2$ (i) $(r + 1)(r + 9) = 16r$

How do you sketch the graphs of quadratic functions?

Learning Standard

Sketch graphs of quadratic functions.

When sketching the graph of a quadratic function, the following characteristics should be shown on the graph.

1. The correct shape of the graph.
2. y-intercept.
3. x-intercept or one point that passes through the graph.

Case 1

The graph of a quadratic function intersects the x-axis.

Example 8

Sketch the following graphs of quadratic functions.

(a) $f(x) = x^2 - 4x + 3$

(b) $f(x) = x^2 - 6x + 9$

(c) $f(x) = -x^2 + 2x + 15$

(d) $f(x) = -2x^2 + 18$

MY MEMORY

$f(x) = x^2 - 4x + 3$
$a = 1, b = -4, c = 3$

MY MEMORY

The constant c of a quadratic function is the y-intercept of the graph of the quadratic function.

Solution:

(a) $f(x) = x^2 - 4x + 3$

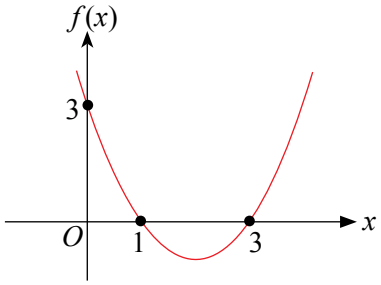
Value of $a = 1 > 0$, shape ∪

Value of $c = 3$, y-intercept $= 3$

When $f(x) = 0$, $x^2 - 4x + 3 = 0$

$(x - 3)(x - 1) = 0$

$x = 1$ or $x = 3$



(b) $f(x) = x^2 - 6x + 9$

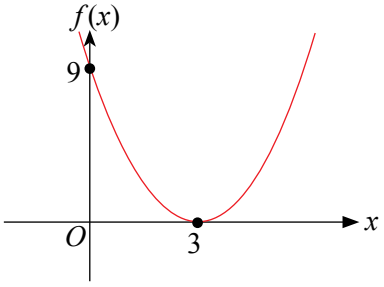
Value of $a = 1 > 0$, shape ∪

Value of $c = 9$, y-intercept $= 9$

When $f(x) = 0$, $x^2 - 6x + 9 = 0$

$(x - 3)(x - 3) = 0$

$x = 3$



(c) $f(x) = -x^2 + 2x + 15$

Value of $a = -1 < 0$, shape ∩

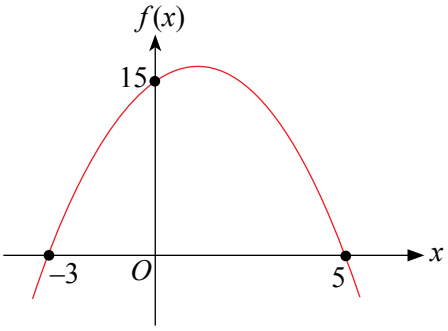
Value of $c = 15$, y-intercept $= 15$

When $f(x) = 0$, $-x^2 + 2x + 15 = 0$

$x^2 - 2x - 15 = 0$

$(x - 5)(x + 3) = 0$

$x = -3$ or $x = 5$



(d) $f(x) = -2x^2 + 18$

Value of $a = -2 < 0$, shape ∩

Value of $b = 0$, axis of symmetry is the y-axis

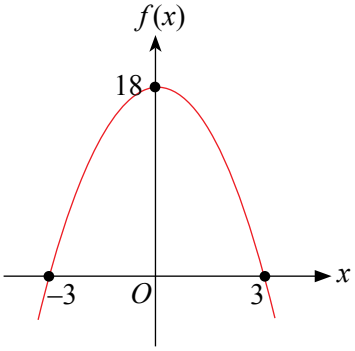
Value of $c = 18$, y-intercept $= 18$

When $f(x) = 0$, $-2x^2 + 18 = 0$

$x^2 - 9 = 0$

$(x + 3)(x - 3) = 0$

$x = -3$ or $x = 3$



Case 2

The graph of a quadratic function does not intersect the x-axis.

MY MEMORY

(a) $f(x) = x^2 + 1$
$a = 1, b = 0, c = 1$
(b) $f(x) = -x^2 - 3$
$a = -1, b = 0, c = -3$

MY MEMORY

If $b = 0$ for a quadratic function, then the y-axis is the axis of symmetry of the graph of the quadratic function.

Example 9

Sketch each of the following graphs of quadratic functions.

(a) $f(x) = x^2 + 1$

(b) $f(x) = -x^2 - 3$

Solution:

(a) $f(x) = x^2 + 1$

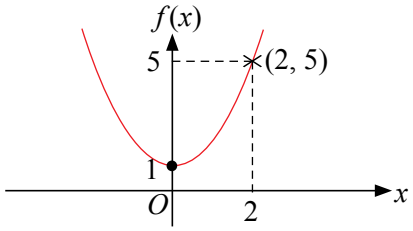
Value of $a = 1 > 0$, shape $\cup$

Value of $b = 0$, axis of symmetry is the y-axis

Value of $c = 1$, y-intercept is 1

thus the minimum point is (0, 1)

When $x = 2$, $f(2) = 2^2 + 1$
$= 5$



(b) $f(x) = -x^2 - 3$

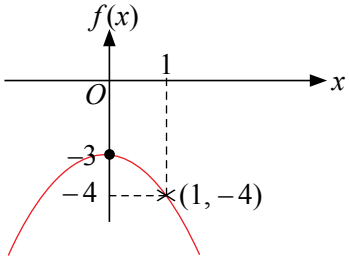
Value of $a = -1 < 0$, shape $\cap$

Value of $b = 0$, axis of symmetry is the y-axis

Value of $c = -3$, y-intercept is -3

thus the maximum point is (0, –3)

When $x = 1$, $f(1) = -(1)^2 - 3$
$= -4$



Self Practice 1.1g

1. Sketch each of the following graphs of quadratic functions.

(a) $f(x) = 2x^2 + 2x - 24$
(b) $f(x) = x^2 - 8x + 16$
(c) $f(x) = -2x^2 + 2x + 40$
(d) $f(x) = -2x^2 + 8$

2. Sketch each of the following graphs of quadratic functions.

(a) $f(x) = x^2 + 5$
(b) $f(x) = 2x^2 + 1$
(c) $f(x) = -x^2 + 2$

How do you solve problems involving quadratic equations?

Learning Standard

Solve problems involving quadratic equations.

Example 10

Joseph wants to make the framework of a box in the shape of a cuboid using wooden rods. The price of the wooden rod is RM5 per metre. The base of the cuboid is a square. The height of the cuboid is 30 cm more than the length of its base. The total surface area of the box is 4 800 cm^2. Joseph's budget to build the frame of a box is RM15. Determine whether Joseph has enough budget.

Solution:

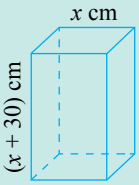
Understanding the problem

Length of the base = x cm

Height of the cuboid = $(x + 30)$ cm

Total surface area = 4 800 cm^2

Budget = RM15 for a box



Planning a strategy

- Determine the expression for the surface area of the cuboid.
- Form a quadratic equation.
- Solve the quadratic equation.
- Determine the measurements of the box and the actual cost.

Implementing the strategy

Total surface area $= 2(x)(x) + 4(x)(x + 30)$
$= 2x^2 + 4x^2 + 120x$
$= 6x^2 + 120x$

$6x^2 + 120x = 4\,800$
$6x^2 + 120x - 4\,800 = 0$
$x^2 + 20x - 800 = 0$
$(x + 40)(x - 20) = 0$
$x = -40$ or $x = 20$

$x = -40$ is not acceptable, thus $x = 20$ cm

The measurements of the box are 20 cm × 20 cm × 50 cm.

Total length of the edges of the box = 8 × 20 cm + 4 × 50 cm
= 160 cm + 200 cm
= 360 cm
= 3.6 m

Actual cost = 3.6 × RM5
= RM18

The actual cost for a box is RM18.

INFO ZONE

The measurement of the length cannot be negative.

Checking Answer

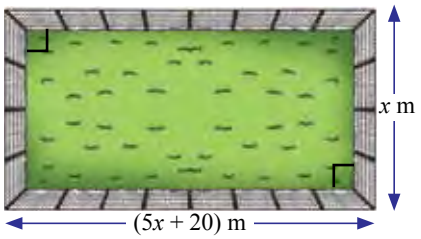
When $x = 20$
Area $= 6(20)^2 + 120(20)$
$= 2\,400 + 2\,400$
$= 4\,800$

Conclusion

Joseph does not have enough budget to build the framework of the box.

Self Practice 1.1h

1. A rectangular field needs to be fenced up using mesh wire. The length of the field is $(5x + 20)$ m and its width is x m.



(a) Express the area of the field, A m^2, in terms of x.

(b) Given the area of the field is 5 100 m^2, calculate the cost of fencing the field if the cost of the mesh wire used is RM20 per metre.

2. Encik Kamarul drove his car at an average speed of $(20t - 20)$ km h^{-1} for $(t - 3)$ hours along a highway. The distance travelled by Encik Kamarul was 225 km. The highway speed limit is 110 km h^{-1}. Did Encik Kamarul follow the highway speed limit?

Comprehensive Practice

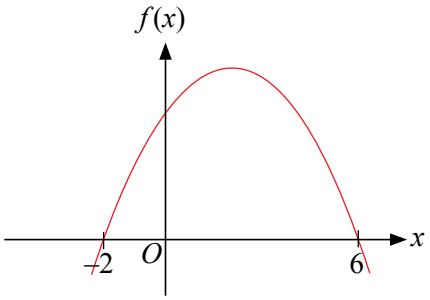
1. Determine whether each of the following expressions is a quadratic expression in one variable.

(a) $p^2 - 4p + 1$ (b) $\frac{1}{2}y^2 - 4y + 9$ (c) $\frac{1}{3} - 2b + a^2$

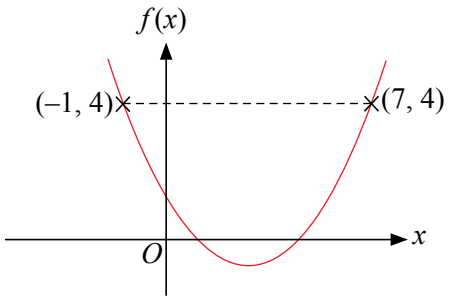
(d) $-m + 1$ (e) $b^2 + 2$ (f) $\frac{a^2 + 2a + 1}{3}$

2. State the equation of the axis of symmetry for each graph of quadratic function below.

(a)



(b)



3. Solve each of the following quadratic equations.

(a) $4x^2 - 1 = 0$ (b) $x^2 - 81 = 0$ (c) $y^2 - 4y = 0$

(d) $x^2 + 3x + 2 = 0$ (e) $2x^2 - x - 10 = 0$ (f) $(x - 2)^2 = 16$

(g) $m^2 + 3m - 4 = 0$ (h) $2p^2 - 13p + 20 = 0$ (i) $(k - 4)(k - 1) = 18$

(j) $\frac{h - 1}{3} = \frac{1}{h + 1}$ (k) $2(x - 2)^2 = 5x - 7$

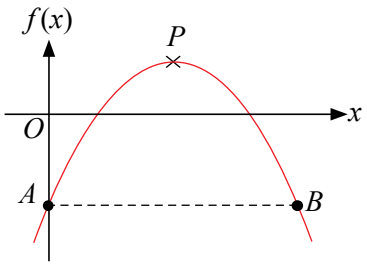
4. Given one of the roots of the quadratic equation $x^2 + px - 18 = 0$ is 2, calculate the value of p.

5. Show that the quadratic equation $(m - 6)^2 = 12 - 2m$ can be written as $m^2 - 10m + 24 = 0$. Hence, solve the equation $(m - 6)^2 = 12 - 2m$.

6. Determine the coordinates of the minimum point from the graph of the quadratic function $f(x) = x^2 - 6x + 5$.

7. Given $x = 4$ is the axis of symmetry of the graph of the quadratic function $f(x) = 7 + 8x - x^2$, determine the coordinates of the maximum point.

8. The diagram shows part of the graph of the quadratic function $f(x) = -x^2 + 6x - 5$. The straight line AB is parallel to the x-axis. Determine



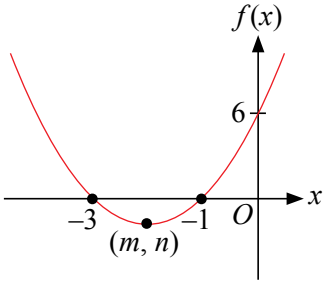
(a) the coordinates of point A,

(b) the equation of the axis of symmetry,

(c) the coordinates of point B,

(d) the coordinates of the maximum point P.

9. The diagram shows the graph of the quadratic function $f(x) = ax^2 + 8x + c$. Calculate the value of each of the following.



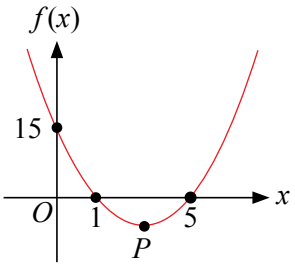
(a) c

(b) m

(c) a

(d) n

10. The diagram shows part of the graph of the quadratic function $f(x) = a(x - h)(x - k)$ where $h < k$. Point P is the minimum point of the graph of the quadratic function.



(a) Calculate the value of

(i) h, (ii) k, (iii) a.

(b) Determine the equation of the axis of symmetry.

(c) State the coordinates of point P.

11. The length of a rectangle is $(x + 1)$ cm and its width is 5 cm less than its length.

(a) Express the area of the rectangle, A cm^2, in terms of x.

(b) The area of the rectangle is 24 cm^2. Calculate the length and width of the rectangle.

12. Diagram 1 shows an isosceles triangle with a base of $4y$ cm and a height of $(y + 5)$ cm. Diagram 2 shows a square with sides of y cm.
The area of the triangle is more than the area of the square by 39 cm^2. Calculate the difference in perimeter between both shapes.

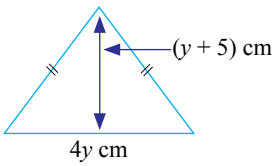


Diagram 1

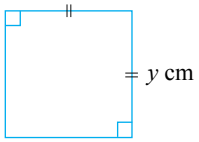
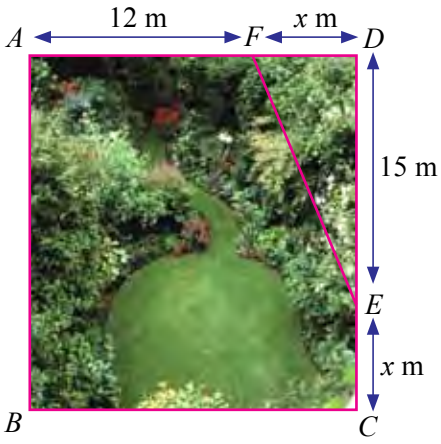


Diagram 2

13. The diagram shows a rectangular garden $ABCD$. E and F are two points on CD and AD respectively such that $CE = DF = x$ m. The lengths of $AF = 12$ m and $DE = 15$ m.



(a) Form an expression for the area of the rectangle, A m^2, in terms of x.

(b) The area of the rectangle is 460 m^2. Calculate the value of x.

(c) Aiman wants to build a small footpath from point E to point F with tiles which costs RM50 per metre. Aiman has a budget of RM1 000, determine whether Aiman has enough budget to build the footpath.

14. The History Club of SMK Seri Jaya has drawn two rectangular murals in conjunction with Malaysia's Independence Day.

(a) Express the difference in area between the two murals, A m^2, in terms of x.

(b) The difference in area between the two murals is 10 m^2. Calculate the value of x.

(c) Calculate the perimeter of the smaller mural.

$(3x + 1)$ m

$(x - 3)$ m

$(2x - 1)$ m

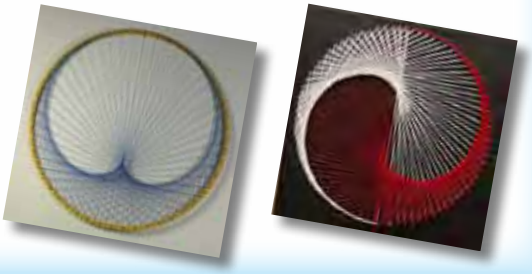
$(x - 1)$ m

PROJECT

Use your creativity to build different shapes based on the examples below. Display your work at the Mathematics Corner.

Materials:

1. Graph paper/blank paper.
2. Protractor, a pair of compasses.
3. Coloured pens.



CONCEPT MAP

Quadratic Functions and Equations in One Variable

Quadratic Expression

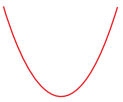
(a) Highest power is 2
(b) Involves one variable

General form $ax^2 + bx + c$
a, b and c
are constants, $a \neq 0$

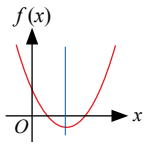
Quadratic Function

General form
$f(x) = ax^2 + bx + c$

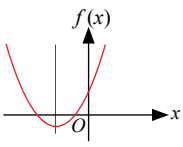
Shape of graph, $a > 0$



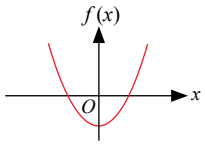
Axis of symmetry
$b < 0$



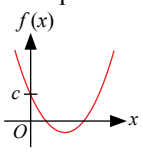
Axis of symmetry
$b > 0$



Axis of symmetry
$b = 0$

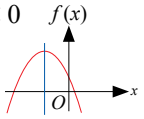


y-intercept

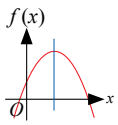


Shape of graph, $a < 0$

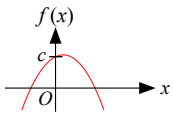
Axis of symmetry
$b < 0$



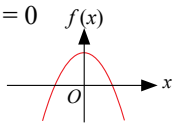
Axis of symmetry
$b > 0$



y-intercept



Axis of symmetry
$b = 0$



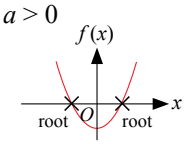
Quadratic Equation

General form
$ax^2 + bx + c = 0$

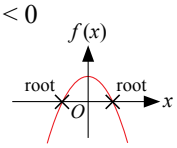
The roots of a quadratic equation are the values of the variable that satisfy the equation

The roots of a quadratic equation can be determined using
(a) factorisation method
(b) graphical method

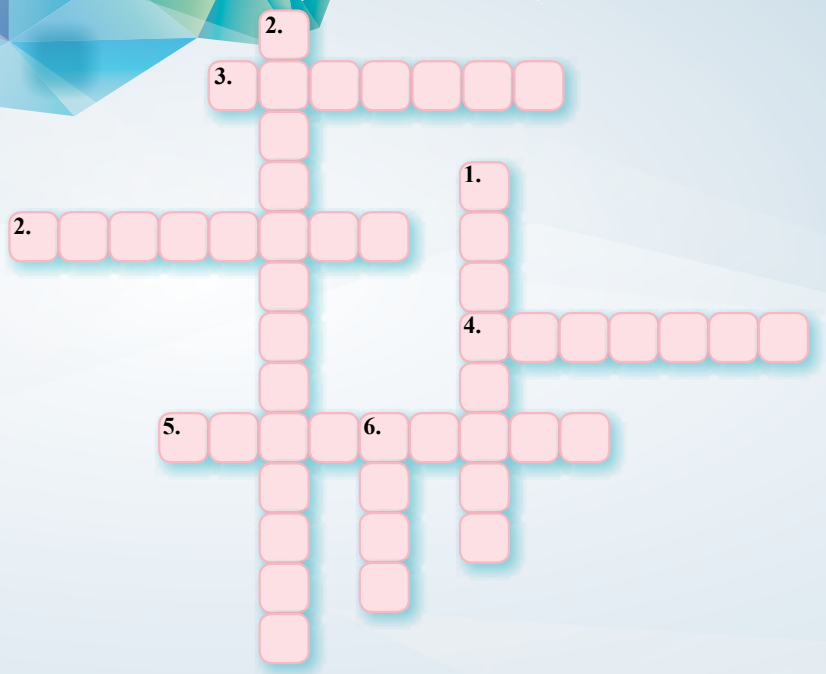
$a > 0$



$a < 0$



Self Reflection



Across

2. The shape of the graph of a quadratic function.

3. The highest point of the graph of a quadratic function.

4. The lowest point of the graph of a quadratic function.

5. A function which its highest power is two.

Down

1. The vertical axis that passes through the maximum or minimum point of the graph of a quadratic function.

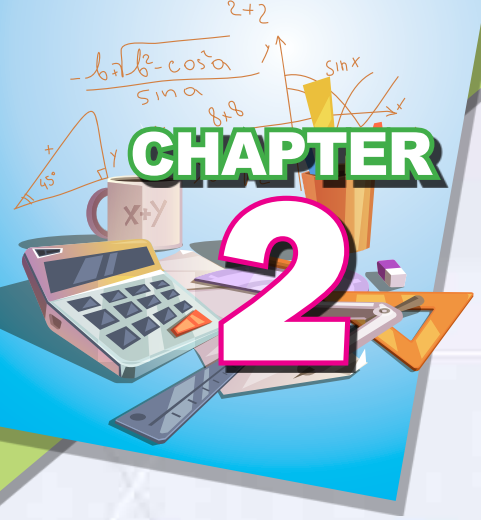
2. A method used to determine the roots of a quadratic equation.

6. The values of variable that satisfy a quadratic equation.

Mathematics Exploration

The shape of the graph of a quadratic function is one of the most common shapes found in our daily life. Observe the following photos.

Use your creativity to draw a quadratic structure.



CHAPTER 2

Number Bases

You will learn

- Number Bases

Malaysia has become a major focus of various technological developments which are capable of transforming the people's lifestyle in this 21st century. These advances in technology enable Malaysians to enjoy fast download rates, hologram technology in education, medicine, industries, self-driving cars and more. A society that is proficient in information technology and telecommunication needs to be well versed in number bases as they have become the basis of all technologies.

Do you know the relationship between the number bases and technology?

Why Study This Chapter?

Number bases are the keys to all calculations in daily life. Among the fields involved are computer science and other areas that use information technology as the basis of research and development such as biotechnology, design technology, aerospace design, pharmacy and others.

Walking Through Time

Brahmagupta
(598 AD – 668 AD)

Brahmagupta was an astronomer from the state of Rajasthan in the north-west of India. He introduced the digit 0 to the number system which has become the basis for all the number bases used in olden times and today.

http://bt.sasbadi.com/m4033

WORD BANK

• number base	• *asas nombor*
• binary	• *binari*
• index	• *indeks*
• place value	• *nilai tempat*
• digit value	• *nilai digit*
• number system	• *sistem nombor*

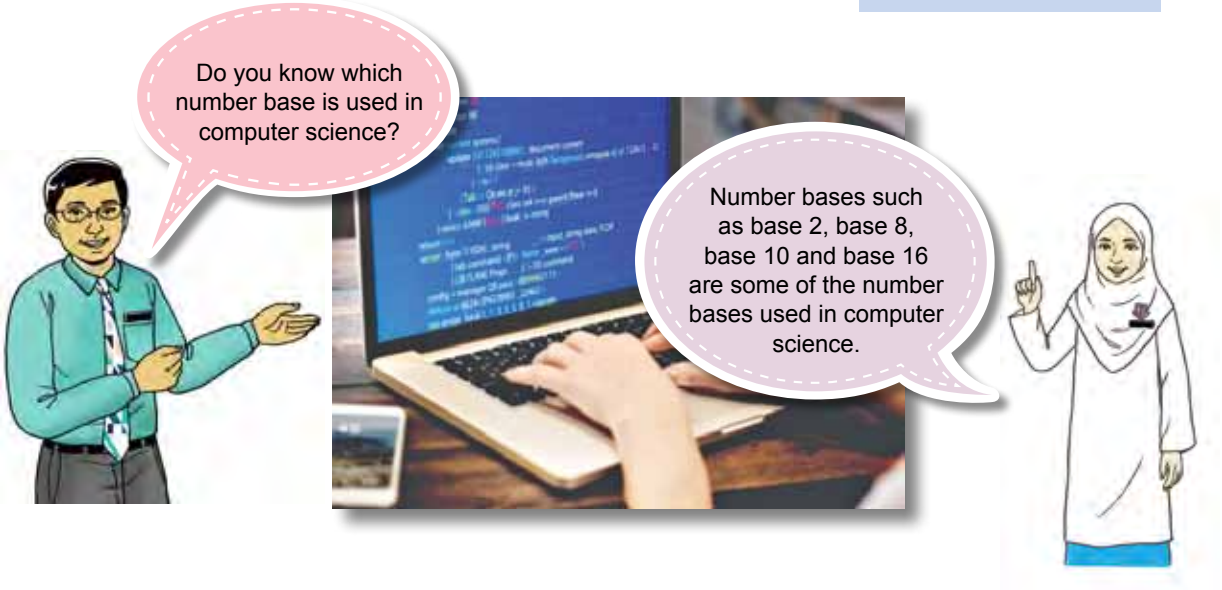
2.1 Number Bases

How do you represent and explain numbers in various bases in terms of numerals, place values, digit values and number values based on the collection process?

Learning Standard

Represent and explain numbers in various bases in terms of numerals, place values, digit values and number values based on the collection process.

Number bases are number systems consisting of digits from 0 to 9. The number systems are made up of numbers with various bases. Base ten is a decimal number system used widely in daily life.



The table below shows the digits used in base two up to base ten.

Number base	Digit
Base 2	0, 1
Base 3	0, 1, 2
Base 4	0, 1, 2, 3
Base 5	0, 1, 2, 3, 4
Base 6	0, 1, 2, 3, 4, 5
Base 7	0, 1, 2, 3, 4, 5, 6
Base 8	0, 1, 2, 3, 4, 5, 6, 7
Base 9	0, 1, 2, 3, 4, 5, 6, 7, 8
Base 10	0, 1, 2, 3, 4, 5, 6, 7, 8, 9

INFO ZONE

Digits are the symbols used or combined to form a number in the number system. 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 are the ten digits used in the decimal number system.
For example, 2 145 has 4 digits.

Example 1

Give two examples of numbers that represent numbers in base two up to base ten.

Solution:

Number base	Number	
2	10_2	1001_2
3	21_3	1201_3
4	23_4	213_4
5	41_5	342_5
6	35_6	4510_6
7	64_7	463_7
8	17_8	472_8
9	78_9	385_9
10	69_{10}	2893_{10}

TIPS

Each base has digits from 0 to a digit which is less than its base. For example, base two has only digits 0 and 1.

INFO ZONE

number 32_5 base

is read as

"Three two base five"

What are the place values involved in numbers in base two up to base ten?

Each base has place values according to each respective base. The place values of a base are the repeated multiplication of that base. Let's say a is a base, then its place values start with a^0, a^1, a^2, …, a^n as shown in the table below.

MY MEMORY

a - base
n - power
$a^4 = a \times a \times a \times a$

a^n

Number base	a^n	Place value							
		a^7	a^6	a^5	a^4	a^3	a^2	a^1	a^0
Base 2	2^n	128	64	32	16	8	4	2	1
Base 3	3^n	2187	729	243	81	27	9	3	1
Base 4	4^n	16384	4096	1024	256	64	16	4	1
Base 5	5^n	78125	15625	3125	625	125	25	5	1
Base 6	6^n	279936	46656	7776	1296	216	36	6	1
Base 7	7^n	823543	117649	16807	2401	343	49	7	1
Base 8	8^n	2097152	262144	32768	4096	512	64	8	1
Base 9	9^n	4782969	531441	59049	6561	729	81	9	1
Base 10	10^n	10000000	1000000	100000	10000	1000	100	10	1

Example 2

State the place value of each digit in the numbers below.
(a) 6231_8 (b) 111101_2

Solution:

(a)

Number in base 8	6	2	3	1
Place value	8^3	8^2	8^1	8^0

(b)

Number in base 2	1	1	1	1	0	1
Place value	2^5	2^4	2^3	2^2	2^1	2^0

Walking Through Time

Konrad Zuse (1910 – 1995) was the inventor and pioneer of modern computers from Germany. He was the founder of the programmable computer. He designed the first high-level programming language known as *Plankalkuel*.

How do you state the value of a particular digit in a number in various bases?

The value of a particular digit in a number is the multiplication of a digit and the place value that represents the digit.

Multiplication of digit and place value

$10\underline{1}0_2$

Number	1	0	$\underline{1}$	0
Place value	2^3	2^2	2^1	2^0
Digit value			$1 \times 2^1 = 2$	

$\underline{2}012_3$

Number	$\underline{2}$	0	1	2
Place value	3^3	3^2	3^1	3^0
Digit value	$2 \times 3^3 = 54$			

$\underline{4}432_5$

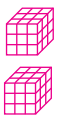
Number	$\underline{4}$	4	3	2
Place value	5^3	5^2	5^1	5^0
Digit value	$4 \times 5^3 = 500$			

Use of place value block

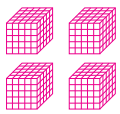
$10\underline{1}0_2$

Number	1	0	$\underline{1}$	0
Place value	2^3	2^2	2^1	2^0
Digit value			 2	

$\underline{2}012_3$

Number	$\underline{2}$	0	1	2
Place value	3^3	3^2	3^1	3^0
Digit value	 54			

$\underline{4}432_5$

Number	$\underline{4}$	4	3	2
Place value	5^3	5^2	5^1	5^0
Digit value	 500			

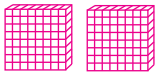
Multiplication of digit and place value

$\underline{2}71_8$

Number	$\underline{2}$	7	1
Place value	8^2	8^1	8^0
Digit value	$2 \times 8^2 = 128$		

Use of place value block

$\underline{2}71_8$

Number	$\underline{2}$	7	1
Place value	8^2	8^1	8^0
Digit value	 128		

Example 3

State the value of the underlined digit in each of the following numbers.

(a) $\underline{3}41_8$ (b) $\underline{5}037_9$ (c) $3\underline{5}01_6$ (d) $213\underline{4}_5$

Solution:

(a) $\underline{3}41_8$

8^2	8^1	8^0
$\underline{3}$	4	1

$3 \times 8^2 = 192$

(b) $\underline{5}037_9$

9^3	9^2	9^1	9^0
$\underline{5}$	0	3	7

$5 \times 9^3 = 3645$

(c) $3\underline{5}01_6$

6^3	6^2	6^1	6^0
3	$\underline{5}$	0	1

$5 \times 6^2 = 180$

(d) $213\underline{4}_5$

5^3	5^2	5^1	5^0
2	1	3	$\underline{4}$

$4 \times 5^0 = 4$

How do you state the numerical value of a number in various bases?

The numerical value of a number in various bases can be determined by calculating the sum of the digit values of the number.

Indicator

Numbers in base two have only digits 0 and 1.

(a) Determine the value of a number in base two.

Collection process

Number	1	1	0	0	1
Place value	2^4	2^3	2^2	2^1	2^0
Digit value	$1 \times 2^4 = 16$	$1 \times 2^3 = 8$	$0 \times 2^2 = 0$	$0 \times 2^1 = 0$	$1 \times 2^0 = 1$
Number value	$16 + 8 + 0 + 0 + 1 = 25_{10}$				

Adding digit values using blocks

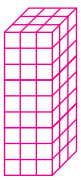
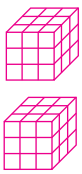
Number	1	1	0	0	1
Place value	2^4	2^3	2^2	2^1	2^0
Digit value	 16	 8			 1
Number value	$16 + 8 + 0 + 0 + 1 = 25_{10}$				

(b) Determine the value of a number in base three.

Collection process

Number	1	2	0	2	1
Place value	3^4	3^3	3^2	3^1	3^0
Digit value	$1 \times 3^4 = 81$	$2 \times 3^3 = 54$	$0 \times 3^2 = 0$	$2 \times 3^1 = 6$	$1 \times 3^0 = 1$
Number value	$81 + 54 + 0 + 6 + 1 = 142_{10}$				

Adding digit values using blocks

Number	1	2	0	2	1
Place value	3^4	3^3	3^2	3^1	3^0
Digit value	 81	 54		 6	 1
Number value	$81 + 54 + 0 + 6 + 1 = 142_{10}$				

(c) Determine the value of a number in base four.

Collection process

Number	3	0	2	1
Place value	4^3	4^2	4^1	4^0
Digit value	$3 \times 4^3 = 192$	$0 \times 4^2 = 0$	$2 \times 4^1 = 8$	$1 \times 4^0 = 1$
Number value	$192 + 0 + 8 + 1 = 201_{10}$			

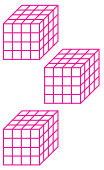
INTERACTIVE ZONE

Is the value of 243_8 equal to 243_5? Discuss.

INFO ZONE

Writing the base sign for a number in base 10 is optional, which can be written or left out.

Adding digit values using blocks

Number	3	0	2	1
Place value	4^3	4^2	4^1	4^0
Digit value	 192		 8	 1
Number value	$192 + 0 + 8 + 1 = 201_{10}$			

Smart Mind

State two numbers in different bases with the same value.

Example 4

Determine the values of the following numbers.

(a) 340_5 (b) 341_7 (c) 1506_8

Solution:

(a) 340_5

Number	3	4	0
Place value	5^2	5^1	5^0
Number value	$(3 \times 5^2) + (4 \times 5^1) + (0 \times 5^0)$ $= 75 + 20 + 0$ $= 95_{10}$		

(b) 341_7

Number	3	4	1
Place value	7^2	7^1	7^0
Number value	$(3 \times 7^2) + (4 \times 7^1) + (1 \times 7^0)$ $= 147 + 28 + 1$ $= 176_{10}$		

(c) 1506_8

Number	1	5	0	6
Place value	8^3	8^2	8^1	8^0
Number value	$(1 \times 8^3) + (5 \times 8^2) + (0 \times 8^1) + (6 \times 8^0)$ $= 512 + 320 + 0 + 6$ $= 838_{10}$			

Smart Mind

Convert your year of birth to a number base that you prefer.

INTERACTIVE ZONE

What will happen if a number in a base higher than 10 is used? Discuss.

Checking Answer

1. Press the MODE key 2 times until SD REG BASE 1 2 3 appears on the screen.
2. Press 3 to choose BASE.
3. Press OCT.
4. Press 1506 then press =.
5. Press DEC, the answer 838 is displayed.

Self Practice 2.1a

1. Write three numbers to represent numbers in base two up to base nine.

2. Circle three numbers which do not represent numbers in base six.

245	332	461	212	371	829	345	123

3.

234	673	336	281

Based on the four numbers above, identify and list all the numbers with the following bases.

(a) Base five (b) Base seven (c) Base eight (d) Base nine

4. Determine the place value of the underlined digit in each of the following numbers.

(a) $1\underline{1}0010_2$ (b) $\underline{2}14_5$ (c) $600\underline{1}_7$ (d) $5114\underline{0}_6$ (e) $1\underline{2}00_3$

(f) $6\underline{8}3_9$ (g) $2\underline{3}31_4$ (h) $\underline{7}321_8$ (i) $5\underline{2}41_6$ (j) $3221_{\underline{5}}$

5. Determine the value of the underlined digit in each of the following numbers.

(a) $1\underline{1}10_2$ (b) $\underline{3}24_5$ (c) $87\underline{3}_9$ (d) $\underline{2}35_6$ (e) $\underline{2}100_3$

(f) $1\underline{6}623_7$ (g) $\underline{1}101_2$ (h) $1\underline{7}76_8$ (i) $2\underline{3}1_4$ (j) $111\underline{1}01_2$

6. Determine the values of the following numbers in base ten.

(a) 23_6 (b) 425_8 (c) 110101_2 (d) 338_9 (e) 364_7

(f) 33_4 (g) 123_5 (h) 1217_8 (i) 515_6 (j) 1121_3

7. Determine the values of p and q.

(a) $1101_2 = (1 \times 2^p) + (1 \times q) + (1 \times 2^0)$

(b) $375_8 = (3 \times 8^p) + (q \times 8^1) + (5 \times 8^0)$

(c) $1321_4 = (1 \times p^q) + (3 \times 4^2) + (2 \times 4^1) + (1 \times 4^0)$

8. Calculate the sum of the values of digit 8 and digit 3 in 1823_9.

9. Rearrange the following numbers in ascending order.

(a) $110_2, 1101_2, 111_2, 1110_2$ (b) $1123_4, 132_4, 231_4, 112_4$ (c) $324_5, 124_5, 241_5, 231_5$

10. Rearrange the following numbers in descending order.

(a) $111101_2, 1213_4, 81_9$ (b) $123_4, 73_8, 313_5$ (c) $253_6, 161_7, 222_3$

11. Calculate the difference between the values of digit 5 in 1576_8 and 125_7.

How do you convert numbers from one base to another base using various methods?

Learning Standard

Convert numbers from one base to another using various methods.

A number can be converted to other bases by using various methods, such as the division using place value and the division using base value. These processes involve converting

(a) a number in base ten to another base.
(b) a number in a certain base to base ten and then to another base.
(c) a number in base two directly to base eight.
(d) a number in base eight directly to base two.

How do you convert a number in base ten to another base?

A number in base ten can be converted to another base by dividing the number using the place value or the base value required. The number 58_{10} can be converted to base two by

(a) dividing 58 using the place value in base two.
(b) dividing 58 by two.

Example 5

Rajang River which is the longest river in Malaysia is 563 kilometres.
Convert 563_{10} to a number in

(a) base five. (b) base eight.

Solution:

(a) Base five

Division using place value

Place value	625	125	25	5	1
Step	The value of 625 is greater than 563	4 125) 563 − 500 63	2 25) 63 − 50 13	2 5) 13 − 10 3	3 1) 3 − 3 0
Base 5	0	4	2	2	3
Answer	4223_5				

563 is divided by the place value of 125. Its remainder is transferred to the previous place value for the next division until a zero remainder is obtained.

Alternative Method

Division using base value

5	563	
5	112	− 3
5	22	− 2
5	4	− 2
	0	− 4

Remainder

The digits are read from the bottom upwards.

The division is continued until digit zero is obtained.

$563_{10} = 4223_5$

(b) Base eight

Division using place value

Place value	4096	512	64	8	1
Step	The value of 4096 is greater than 563	1 512) 563 − 512 51	The value of 64 is greater than 51	6 8) 51 − 48 3	3 1) 3 − 3 0
Base 8	0	1	0	6	3
Answer	1063_8				

563 is divided by the place value of 512. Its remainder is transferred to the previous place value for the next division until a zero remainder is obtained.

Alternative Method

Division using base value

Remainder

8 | 563
8 | 70 – 3
8 | 8 – 6
8 | 1 – 0
0 – 1

The digits are read from the bottom upwards.

The division is continued until digit zero is obtained.

$563_{10} = 1063_8$

How do you convert a number in a certain base to base ten and then to another base?

A number in base p can be converted to base ten and then to base q. In the process of converting a number in base two to base nine, the number in base two is converted to base ten and then to base nine.

Base p ⋯▸ **Base 10** ⋯▸ **Base q**

Example 6

Convert 253_6 to a number in base nine.

Solution:

Step 1

Convert the number in base six to base ten.

Place value	6^2	6^1	6^0
Number in base 6	2	5	3
Value of number in base 10	$(2 \times 6^2) + (5 \times 6^1) + (3 \times 6^0)$ $= 105_{10}$		

Step 2

Convert the number in base ten to base nine.

9 | 105
9 | 11 – 6
9 | 1 – 2
0 – 1

$253_6 = 105_{10} = 126_9$

Example 7

Convert 334_5 to a number in base two.

Solution:

Step 1

Convert the number in base five to base ten.

Place value	5^2	5^1	5^0
Number in base 5	3	3	4
Value of number in base 10	$(3 \times 5^2) + (3 \times 5^1) + (4 \times 5^0)$ $= 94_{10}$		

Step 2

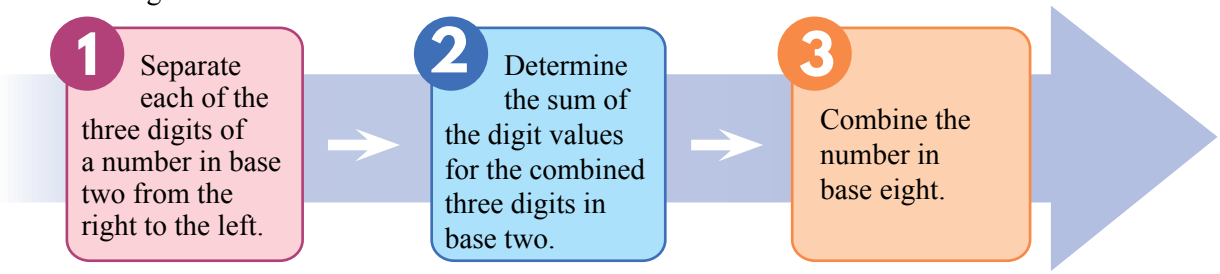
Convert the number in base ten to base two.

2	94	
2	47	– 0
2	23	– 1
2	11	– 1
2	5	– 1
2	2	– 1
2	1	– 0
	0	– 1

$334_5 = 94_{10} = 1011110_2$

How do you convert a number in base two to base eight?

A number in base two can be converted directly to base eight. Each digit in base eight is equivalent to three digits in base two.



Example 8

Convert the numbers in base two to numbers in base eight.

(a) 110111_2 (b) 1101101_2

Solution:

(a) 110111_2

Number in base 2	1	1	0	1	1	1
Place value	2^2	2^1	2^0	2^2	2^1	2^0
Digit value	4	2	0	4	2	1
Base 8	$4 + 2 + 0$ $= 6$			$4 + 2 + 1$ $= 7$		
	67_8					

$110111_2 = 67_8$

INFO ZONE

Base 2	Base 8
000	0
001	1
010	2
011	3
100	4
101	5
110	6
111	7

Alternative Method

$\underbrace{110}_{6}\ \underbrace{111}_{7}$

$110111_2 = 67_8$

By referring to the table in INFO ZONE, you can easily convert a number in base two to base eight.

(b) 1101101_2

Number in base 2			1	1	0	1	1	0	1
Place value	2^2	2^1	2^0	2^2	2^1	2^0	2^2	2^1	2^0
Digit value	0	0	1	4	0	1	4	0	1
Base 8	$0 + 0 + 1 = \mathbf{1}$			$4 + 0 + 1 = \mathbf{5}$			$4 + 0 + 1 = \mathbf{5}$		
	155_8								

$1101101_2 = 155_8$

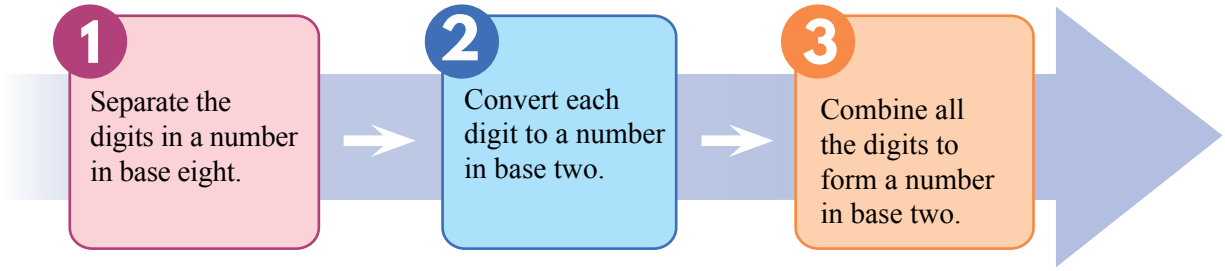
Alternative Method

$\underbrace{1}_{1}\ \underbrace{101}_{5}\ \underbrace{101}_{5}$

$1101101_2 = 155_8$

How do you convert a number in base eight to base two?

A number in base eight can be converted directly to base two. Each digit in base eight is equivalent to three digits in base two.



Example 9

Convert the numbers in base eight to numbers in base two.

(a) 517_8 (b) 725_8

Solution:

(a) 517_8

Base 8	5			1			7		
	4 + 1			1			4 + 2 + 1		
Place value	2^2	2^1	2^0	2^2	2^1	2^0	2^2	2^1	2^0
Base 2	1	0	1	0	0	1	1	1	1
	101001111_2								

$517_8 = 101001111_2$

(b) 725_8

Base 8	7			2			5		
	4 + 2 + 1			2			4 + 1		
Place value	2^2	2^1	2^0	2^2	2^1	2^0	2^2	2^1	2^0
Base 2	1	1	1	0	1	0	1	0	1
	111010101_2								

$725_8 = 111010101_2$

INFO ZONE

Base 2	Base 10	Base 16
0000_2	0	0
0001_2	1	1
0010_2	2	2
0011_2	3	3
0100_2	4	4
0101_2	5	5
0110_2	6	6
0111_2	7	7
1000_2	8	8
1001_2	9	9
1010_2	10	A
1011_2	11	B
1100_2	12	C
1101_2	13	D
1110_2	14	E
1111_2	15	F

Self Practice 2.1b

1. Convert 494_{10} to numbers in the following bases.

(a) Base two (b) Base four (c) Base five (d) Base eight (e) Base nine

2. Convert each of the following numbers to the number base given in brackets.

(a) 43_8 (base 3) (b) 112_3 (base 5) (c) 526_7 (base 2)
(d) 1213_4 (base 6) (e) 1134_5 (base 8) (f) 321_9 (base 4)

3. Convert the value of digit 5 in the number 154_6 to a number in base 3.

4. Convert each of the following numbers to a number in base eight.

(a) 111101_2 (b) 1110_2 (c) 11110111_2
(d) 101010_2 (e) 111000_2 (f) 111010101_2

5. Convert each of the following numbers to a number in base two.

(a) 43_8 (b) 112_8 (c) 57_8
(d) 1217_8 (e) 635_8 (f) 243_8

How do you perform calculations involving addition and subtraction of numbers in various bases?

Learning Standard

Perform calculations involving addition and subtraction of numbers in various bases.

Addition and subtraction in number bases can be carried out in the following two methods:

(a) Using vertical form, that is to write the numbers vertically when performing addition and subtraction.
(b) Conversion of numbers in a certain base to base ten.

Addition of numbers in various bases

Vertical Form

1. Add the given digits starting from the right to the left.
2. The sum of the digits in base ten is converted to the given base to be written in the answer space.
3. This process is repeated until all the digits in the number are added up.

Conversion of Base

1. Convert a number to base ten and perform addition.
2. Convert the answer in base ten to the required base.

Example 10

Calculate each of the following.

(a) $110_2 + 111_2$ (b) $673_8 + 175_8$ (c) $1837_9 + 765_9$

Solution:

Vertical Form

(a) $110_2 + 111_2$

$$\begin{array}{r} {\scriptstyle 1} \\ 1\ 1\ 0_2 \\ +\ 1\ 1\ 1_2 \\ \hline 1\ 1\ 0\ 1_2 \end{array}$$

TIPS

Perform addition as usual and convert the values in base ten to base two.

$0 + 1 = 1_{10} = 1_2$

$1 + 1 = 2_{10} = 10_2$ ← Write 0 in the answer space. 1 is carried forward to the next place value.

$1 + 1 + 1 = 3_{10} = 11_2$

$110_2 + 111_2 = 1101_2$

(b) $673_8 + 175_8$

$$\begin{array}{r} {\scriptstyle 1\ 1} \\ 6\ 7\ 3_8 \\ +\ 1\ 7\ 5_8 \\ \hline 1\ 0\ 7\ 0_8 \end{array}$$

TIPS

Perform addition as usual and convert the values in base ten to base eight.

Write 0 in the answer space. 1 is carried forward to the next place value.

$3 + 5 = 8_{10} = 10_8$

$1 + 7 + 7 = 15_{10} = 17_8$

$1 + 6 + 1 = 8_{10} = 10_8$ ← Write 7 in the answer space. 1 is carried forward to the next place value.

$673_8 + 175_8 = 1070_8$

(c) $1837_9 + 765_9$

$$\begin{array}{r} {\scriptstyle 1\ 1\ 1} \\ 1\ 8\ 3\ 7_9 \\ +\ \ 7\ 6\ 5_9 \\ \hline 2\ 7\ 1\ 3_9 \end{array}$$

TIPS

Perform addition as usual and convert the values in base ten to base nine.

Write 3 in the answer space. 1 is carried forward to the next place value.

$7 + 5 = 12_{10} = 13_9$

$1 + 3 + 6 = 10_{10} = 11_9$

$1 + 8 + 7 = 16_{10} = 17_9$ ← Write 1 in the answer space. The number 1 on the left is carried forward to the next place value.

$1 + 1 = 2_{10} = 2_9$ ← Write 7 in the answer space. 1 is carried forward to the next place value.

$1837_9 + 765_9 = 2713_9$

Conversion of Base

(a) $110_2 + 111_2$

$110_2 \rightarrow 6_{10}$

$111_2 \rightarrow +\ 7_{10}$

13_{10}

2	13	
2	6	− 1
2	3	− 0
2	1	− 1
	0	− 1

$110_2 + 111_2 = 1101_2$

(b) $673_8 + 175_8$

$673_8 \rightarrow 443_{10}$

$175_8 \rightarrow +\ 125_{10}$

568_{10}

8	568	
8	71	− 0
8	8	− 7
8	1	− 0
	0	− 1

$673_8 + 175_8 = 1070_8$

(c) $1837_9 + 765_9$

$1837_9 \rightarrow 1411_{10}$

$765_9 \rightarrow +\ 626_{10}$

2037_{10}

9	2037	
9	226	− 3
9	25	− 1
9	2	− 7
	0	− 2

$1837_9 + 765_9 = 2713_9$

Subtraction of numbers in various bases

Vertical Form

1. Subtract the given digits starting from the right to the left.
2. The difference is written in the answer space. The difference is always less than the given base and its value is equal to base ten.
3. This process is repeated until all the digits in the number are subtracted.

Conversion of Base

1. Convert a number to base ten and perform subtraction.
2. Convert the answer in base ten to the required base.

Example 11

Calculate each of the following.

(a) $4005_6 - 325_6$
(b) $6241_7 - 613_7$
(c) $372_8 - 77_8$
(d) $1827_9 - 65_9$

Solution:

Vertical Form

(a) $4005_6 - 325_6$

$$\begin{array}{r} {\scriptstyle 5} \\ 3\ \not{6}\ 6 \\ \not{4}\,0\,0\,5_6 \\ -\ \ 3\,2\,5_6 \\ \hline 3\,2\,4\,0_6 \end{array}$$

Transfer numbers according to base six. Use 6 to perform subtraction. Subtract as usual.

$5 - 5 = 0_6$
$6 - 2 = 4_6$
$5 - 3 = 2_6$

$4005_6 - 325_6 = 3240_6$

Conversion of Base

$4005_6 \rightarrow 869_{10}$
$325_6 \rightarrow -\ 125_{10}$
744_{10}

6	744	
6	124	– 0
6	20	– 4
6	3	– 2
	0	– 3

$4005_6 - 325_6 = 3240_6$

(b) $6241_7 - 613_7$

$$\begin{array}{r} {\scriptstyle 5\ 7\ 3\ 7} \\ 6\ 2\ 4\ 1_7 \\ -\ \ 6\ 1\ 3_7 \\ \hline 5\ 3\ 2\ 5_7 \end{array}$$

Transfer numbers according to base seven. Use 7 to perform subtraction. Subtract as usual.

$7 + 1 - 3 = 5_7$

$3 - 1 = 2_7$

$7 + 2 - 6 = 3_7$

$6241_7 - 613_7 = 5325_7$

$6241_7 \rightarrow 2\ 1\ 8\ 5_{10}$

$613_7 \rightarrow -\ 3\ 0\ 4_{10}$

$1\ 8\ 8\ 1_{10}$

7	1881
7	268 – 5
7	38 – 2
7	5 – 3
	0 – 5

$6241_7 - 613_7 = 5325_7$

(c) $372_8 - 77_8$

$$\begin{array}{r} {\scriptstyle 2\ 6\ 8} \\ 3\ 7\ 2_8 \\ -\ \ 7\ 7_8 \\ \hline 2\ 7\ 3_8 \end{array}$$

Transfer numbers according to base eight. Use 8 to perform subtraction. Subtract as usual.

$8 + 2 - 7 = 3_8$

$8 + 6 - 7 = 7_8$

$372_8 - 77_8 = 273_8$

$372_8 \rightarrow 2\ 5\ 0_{10}$

$77_8 \rightarrow -\ 6\ 3_{10}$

$1\ 8\ 7_{10}$

8	187
8	23 – 3
8	2 – 7
	0 – 2

$372_8 - 77_8 = 273_8$

(d) $1827_9 - 65_9$

$$\begin{array}{r} {\scriptstyle 7\ 9} \\ 1\ 8\ 2\ 7_9 \\ -\ \ \ 6\ 5_9 \\ \hline 1\ 7\ 5\ 2_9 \end{array}$$

Transfer numbers according to base nine. Use 9 to perform subtraction. Subtract as usual.

$7 - 5 = 2_9$

$9 + 2 - 6 = 5_9$

$1827_9 - 65_9 = 1752_9$

$1827_9 \rightarrow 1\ 4\ 0\ 2_{10}$

$65_9 \rightarrow -\ \ 5\ 9_{10}$

$1\ 3\ 4\ 3_{10}$

9	1343
9	149 – 2
9	16 – 5
9	1 – 7
	0 – 1

$1827_9 - 65_9 = 1752_9$

Self Practice 2.1c

1. Calculate the value of the following.

(a) $11_2 + 10_2$	(b) $11011_2 + 11110_2$	(c) $210_3 + 121_3$	(d) $1112_3 + 101_3$
(e) $13_4 + 10_4$	(f) $1330_4 + 1120_4$	(g) $423_5 + 130_5$	(h) $3244_5 + 203_5$
(i) $351_6 + 122_6$	(j) $123_6 + 50_6$	(k) $166_7 + 253_7$	(l) $633_7 + 150_7$
(m) $1713_8 + 105_8$	(n) $453_8 + 262_8$	(o) $183_9 + 17_9$	(p) $5703_9 + 750_9$

2. Calculate the value of the following.

(a) $1111_2 - 10_2$	(b) $1011_2 - 101_2$	(c) $2210_3 - 211_3$	(d) $1012_3 - 121_3$
(e) $131_4 - 121_4$	(f) $1030_4 - 122_4$	(g) $423_5 - 100_5$	(h) $3204_5 - 2013_5$
(i) $3531_6 - 114_6$	(j) $1253_6 - 150_6$	(k) $6026_7 - 243_7$	(l) $6503_7 - 160_7$
(m) $1753_8 - 1005_8$	(n) $4403_8 - 202_8$	(o) $1853_9 - 1207_9$	(p) $8703_9 - 7250_9$

How do you solve problems involving number bases?

Example 12

Learning Standard

Solve problems involving number bases.

Mode of transportation	Percentage (%)
Bus	25
Car	40
Walking	17
Bicycle	10
Motorcycle	8

The table above shows a study of various ways of 200 pupils going to school every day.

(a) State the number of pupils who go to school by bus and by car in base four.
(b) Calculate the total number of pupils who go to school by bus and by car in base four.
(c) Calculate the difference in number of pupils, in base seven, between those who walk to school and those who go to school by motorcycle.

Understanding the problem

Total number of pupils = 200

The given percentages need to be converted to number of pupils.

Planning a strategy

(a) Convert the number of pupils who go to school by bus and by car to a number in base four.
(b) Add the answer in (a).
(c) Subtract the number of pupils who walk by those who go to school by motorcycle. Then, convert the answer to a number in base seven.

Implementing the strategy

(a) Bus = 302_4

$\frac{25}{100} \times 200 = 50_{10}$

$4 \mid 50$
$4 \mid 12 - 2$
$4 \mid 3 - 0$
$0 - 3$

Car = 1100_4

$\frac{40}{100} \times 200 = 80_{10}$

$4 \mid 80$
$4 \mid 20 - 0$
$4 \mid 5 - 0$
$4 \mid 1 - 1$
$0 - 1$

(b)
$$\begin{array}{r} 1\,1\,0\,0_4 \\ +\quad 3\,0\,2_4 \\ \hline 2\,0\,0\,2_4 \end{array}$$

$1100_4 + 302_4 = 2002_4$

(c) $34 - 16 = 18_{10}$

$\frac{17}{100} \times 200 = 34_{10}$

$\frac{8}{100} \times 200 = 16_{10}$

$7 \mid 18$
$7 \mid 2 - 4$
$0 - 2$
$= 24_7$

Checking Answer

$80 + 50 = 130_{10}$

$4 \mid 130$
$4 \mid 32 - 2$
$4 \mid 8 - 0$
$4 \mid 2 - 0$
$0 - 2$
$= 2002_4$

Conclusion

(a) Bus = 302_4
Car = 1100_4
(b) $1100_4 + 302_4 = 2002_4$
(c) Difference in number of pupils between those who walk to school and those who go to school by motorcycle = 24_7

Self Practice 2.1d

1. A piece of rope is cut into 4 equal parts. Calculate the value of x.

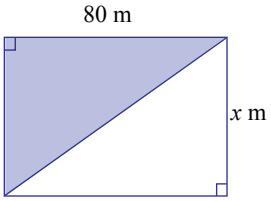
2. Alex Wong's marks in History, Mathematics and Bahasa Melayu are 82_9, 234_5 and 11001_2 respectively.

(a) What is the total mark obtained by Alex Wong in base ten?

(b) What is the difference in marks between Bahasa Melayu and Mathematics? Write the answer in base five.

3. A watch store gives discounts for several brands of ladies' wrist watches in conjunction with Mother's Day. After the discount, Madam Santhi buys a watch of brand M which costs $RM134_5$ and Puan Aminah buys a watch of brand N which costs $RM50_7$. Who gets a higher discount?

4. Ashri Satem, a residential developer, wants to build a rectangular playground. The playground is divided into two sections, the coloured section is the children's playground and the blank section is the exercise area for adults. If the perimeter of the playground is 3300_4 metres, calculate the area of the children's playground.



Comprehensive Practice

1. Complete the following number sequences in ascending order.

(a) 234_5, ☐, ☐, ☐.

(b) 101_2, ☐, ☐, ☐.

(c) 30_7, ☐, ☐, ☐.

2. State the value of digit 2 in $3\underline{2}40_4$.

3. (a) Convert 111001110_2 to base eight.

(b) Convert 367_8 to base two.

4. Convert 287_9 to numbers in the following bases.

(a) Base two (b) Base five

(c) Base seven (d) Base eight

5. Calculate the value of the following.

(a) $111_2 + 1110_2$ (b) $140_7 + 302_7$ (c) $275_9 - 218_9$

6. Determine whether each of the following statements is true or false.
(a) $123_4 > 23_8$ (b) $72_9 < 341_5$ (c) $452_6 > 334_8$

7. Calculate the value of $110_2 + 278_9 + 132_4$ in base ten.

8. Calculate the value of $231_4 - 110_2$ in base ten.

9. Given $1122_3 + y_7 = 118_9$, calculate the value of y.

10. Fill in the blanks with numbers in ascending order according to the given bases.
(a) 63_8, 110100_2, $\square\square_8$, $\square\square\square\square\square\square_2$, 67_8.
(b) 124_{10}, 148_9, $\square\square\square_8$, 241_7, $\square\square\square\square_5$.

11. The total number of visitors to Zoo Negara in January is 20202_5. What is the average number of visitors to Zoo Negara each day in January, if it opens every day? Give the answer in base five.

12. Azizan buys a bag filled with 100 small balls for his son. $\frac{1}{5}$ of the balls are green, $\frac{1}{4}$ are yellow, $\frac{1}{10}$ are pink and the remainder is blue. What is the number of blue balls in base eight?

13. Kishendran buys a batik shirt at a discount of 25%. The original price of the shirt is $RM220_4$. What is the price of the shirt after discount in base seven?

PROJECT

Do you know that besides numbers in base two up to base ten, there are other number bases such as numbers in base sixteen, base thirty-two and base sixty-four? By surfing the Internet, explain the usage of these bases in the field of science and information technology by giving suitable examples.

CONCEPT MAP

Number Bases

Conversion of base

Convert a number in base ten to another base and vice versa.

Place value	6^2	6^1	6^0
Number in base 6	2	5	3
Value of number in base 10	$(2 \times 6^2) + (5 \times 6^1) + (3 \times 6^0)$ $= 105_{10}$		

$$\begin{array}{r|l} 9 & 105 \\ 9 & 11 \quad -6 \\ 9 & 1 \quad -2 \\ & 0 \quad -1 \end{array} \uparrow$$

$253_6 = 105_{10} = 126_9$

Convert a number in base two to base eight and vice versa.

Number in base 2	1	1	0	1	1	1
Place value	2^2	2^1	2^0	2^2	2^1	2^0
Digit value	4	2	0	4	2	1
Base 8	$4 + 2 + 0$ $= 6$			$4 + 2 + 1$ $= 7$		
	67_8					

$110111_2 = 67_8$

Base 8	5			1			7		
	$4 + 1$			1			$4 + 2 + 1$		
Place value	2^2	2^1	2^0	2^2	2^1	2^0	2^2	2^1	2^0
Base 2	1	0	1	0	0	1	1	1	1
	101001111_2								

$517_8 = 101001111_2$

Computation involving addition and subtraction

Vertical Form

$$\begin{array}{r} {\scriptstyle 1} \\ 1\ 1\ 0_2 \\ +\ 1\ 1\ 1_2 \\ \hline 1\ 1\ 0\ 1_2 \end{array}$$

$0 + 1 = 1_{10} = 1_2$
$1 + 1 = 2_{10} = 10_2$
$1 + 1 + 1 = 3_{10} = 11_2$

Conversion of Base

$110_2 \rightarrow 6_{10}$
$111_2 \rightarrow +\ 7_{10}$
13_{10}

$$\begin{array}{r|l} 2 & 13 \\ 2 & 6 \quad -1 \\ 2 & 3 \quad -0 \\ 2 & 1 \quad -1 \\ & 0 \quad -1 \end{array} \uparrow$$

$110_2 + 111_2 = 1101_2$

Vertical Form

$$\begin{array}{r} {\scriptstyle 5\ 7\ 3\ 7} \\ 6\ 2\ 4\ 1_7 \\ -\ \ 6\ 1\ 3_7 \\ \hline 5\ 3\ 2\ 5_7 \end{array}$$

$7 + 1 - 3 = 5_7$
$3 - 1 = 2_7$
$7 + 2 - 6 = 3_7$

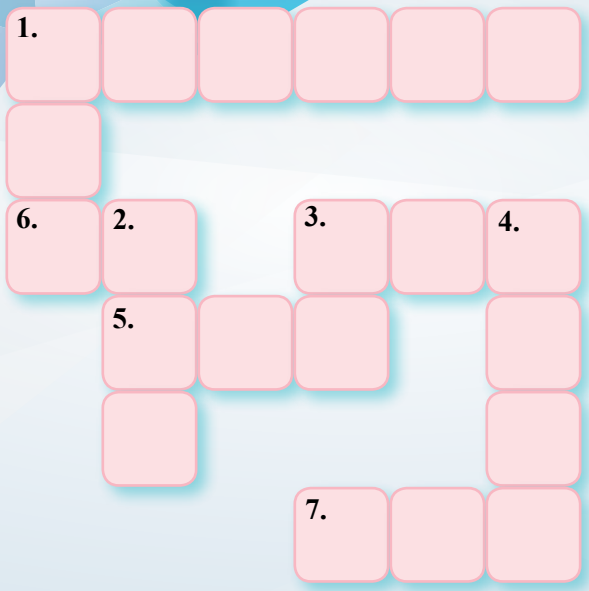
Conversion of Base

$6241_7 \rightarrow 2185_{10}$
$613_7 \rightarrow -\ 304_{10}$
1881_{10}

$$\begin{array}{r|l} 7 & 1881 \\ 7 & 268 \quad -5 \\ 7 & 38 \quad -2 \\ 7 & 5 \quad -3 \\ & 0 \quad -5 \end{array} \uparrow$$

$6241_7 - 613_7 = 5325_7$

Self Reflection



Across

1. Convert 65_8 to base two.
3. Convert 43_{10} to base four.
5. Convert 633_7 to base ten.
6. Calculate $132_4 + 110_2$ and give the answer in base ten.
7. Calculate $2220_3 + 211_3$ and give the answer in base ten.

Down

1. Calculate $461_7 - 141_5$ and give the answer in base ten.
2. Convert 110011110_2 to base eight.
3. Calculate $10101_2 + 111_2$ and give the answer in base ten.
4. Convert 400_{10} to base five.

Scan the QR Code to carry out this activity.
http://bt.sasbadi.com/m4053

Mathematics Exploration

Each number in base ten can be written as a number in another base. Prepare a chart of various number bases as shown below.
What is the relationship between the number bases given?

Base 10	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	...	50
Base 2																				
Base 4																				
Base 8																				
Base 16																				
Base 32																				

CHAPTER 3

Logical Reasoning

You will learn

- Statements
- Arguments

The Parliament is the legislative body of the Federal government. All acts of law must be debated and approved by the House of Representatives and the Senate before the approval of the Yang di-Pertuan Agong.

Do you know why every act of law must be debated in the Parliament?

Why Study This Chapter?

Apart from debate, logical reasoning is often applied in the electrical engineering field especially in electronic circuit. It also plays an important role in computer programming and computer hardware design. In our daily lives, logical reasoning is applied in the problem solving process because it enhances our critical and analytical thinking skills.

WORD BANK

• converse	• *akas*
• deductive	• *deduktif*
• argument	• *hujah*
• implication	• *implikasi*
• inductive	• *induktif*
• contrapositive	• *kontrapositif*
• negation	• *penafian*
• quantifier	• *pengkuantiti*
• statement	• *pernyataan*
• compound statement	• *pernyataan majmuk*
• inverse	• *songsangan*

Walking Through Time

Aristotle
(384 BC – 322 BC)

According to Aristotle, logic is not a part of Philosophy but it is just an instrument used by philosophers and scientists. Aristotle used logic as a technique for debating and linguistics knowledge.

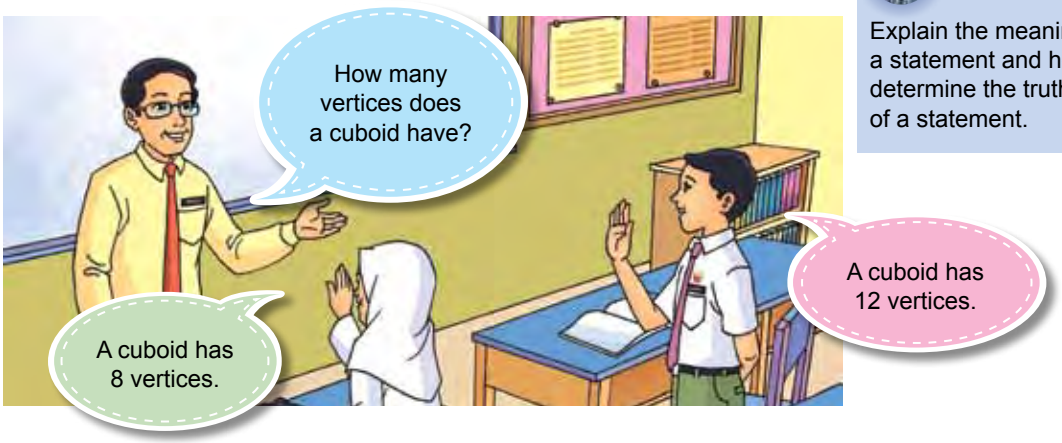
http://bt.sasbadi.com/m4055

3.1 Statements

What does a statement mean and how do you determine the truth value of a statement?

Learning Standard

Explain the meaning of a statement and hence determine the truth value of a statement.



Most of the mathematical results involve statements.

> A statement is a sentence of which the truth value can be determined, that is either true or false, but not both.

Question, exclamation and command sentences are not statements. The truth values of these sentences cannot be determined.

Both answers given in the above dialogue are statements. The statement of "A cuboid has 8 vertices" is a true statement while the statement of "A cuboid has 12 vertices" is a false statement. Statements can be divided into true statements and false statements.

Example 1

Determine whether each sentence below is a statement or not a statement. Justify your answers.

(a) Please send the workbook.
(b) Kuala Lumpur Tower is the tallest tower in Malaysia.
(c) How do you come to school?
(d) $x + 3 = 5$.
(e) $-6 < -8$.

Solution:

(a) Not a statement because the truth value cannot be determined.
(b) A statement because it is true.
(c) Not a statement because the truth value cannot be determined.
(d) Not a statement because the truth value cannot be determined.
(e) A statement because it is false.

Malaysiaku

The Kuala Lumpur Tower is located at the peak of Bukit Nanas. This tower with a height of 421 metres is the 7th tallest communication tower in the world and the tallest in Southeast Asia.

Mind Stimulation 1

Aim: To determine the truth values of the given statements.

Steps:

1. Divide the class into groups.
2. State whether the mathematical sentences provided in the Activity Sheet are true statements or false statements with Round Robin.
3. Discuss and draw conclusions from the findings of this activity.

Activity Sheet:

	Mathematical sentence	Truth value
(a)	$28 + 12 = 40$	
(b)	$3^2 + 4^2 = 7^2$	
(c)	$(2 + 3)(2 - 3) = 2^2 - 3^2$	
(d)	$\sqrt{729} = 81^{\frac{3}{2}}$	
(e)	$(x - y)^2 = x^2 - 2xy + y^2$	
(f)	{a, b} has 4 subsets.	
(g)	5 is a factor of 400.	
(h)	The lowest common multiple of the numbers 4 and 18 is 36.	

Discussion:

Are all the mathematical statements true? Discuss your reasons.

From the activity in Mind Stimulation 1, it is found that:

> Not all the mathematical statements are true. The truth values of the mathematical statements can be determined.

Example 2

Determine whether the following statements are true or false. If it is false, prove it.

(a) $(x + y)^2 = x^2 + 2xy - y^2, x \neq 0, y \neq 0$
(b) $(x + 5)^2 < 0, x \in \mathbf{R}$
(c) $2 + 6(4) > 4 + 6(2)$
(d) $2 \in$ {Factors of 8}
(e) {2, 5, 7} $\cup$ {Prime numbers} = {2, 5, 7}

INFO ZONE

$x \in \mathbf{R}$ means x is an element of the real numbers.

- Real numbers can be defined as any rational numbers or irrational numbers.

Solution:

(a) False Let $x = 2$ and $y = 3$.

$$(x + y)^2 = (2 + 3)^2 = 25$$
$$x^2 + 2xy - y^2 = 2^2 + 2(2)(3) - 3^2 = 7$$

Hence, $(x + y)^2 \neq x^2 + 2xy - y^2$

(b) False $(2 + 5)^2 = 49 > 0$

(c) True

(d) True

(e) False

{2, 5, 7} $\cup$ {2, 3, 5, 7, 11, 13, ...} = {Prime numbers}

Example 3

Determine whether the following mathematics statements are true or false. Explain if the statement is false.

(a) All polygons have diagonals.

(b) Some perfect squares are whole numbers.

(c) All even numbers have prime factors.

(d) Some straight lines intersect the y-axis.

Solution:

(a) False. Triangles do not have diagonals.

(b) False. All perfect squares are whole numbers.

(c) True

(d) True

Self Practice 3.1a

1. Determine whether each sentence below is a statement or not a statement. Justify your answers.

(a) Let's play in the field.

(b) Malaysia is located in Asian continent.

(c) Is $3 + 2 = 8$?

(d) $x + 3 > x - 8$

(e) $3x + 5 = -7$

2. Construct a true statement by using the given digits and symbols.

(a) 23, +, 9, 40, >

(b) {3, 6, 9}, {3}, $\subset$

(c) $\frac{5}{6}, \frac{1}{4}, \times, \frac{10}{3}, =$

(d) $x^2 + 3, \leqslant, (x + 3)^2$

(e) $\sqrt[3]{\ }$, 9, 27, 12, =, +

3. Determine whether the following statements are true or false.

(a) All quadrilaterals have right angles.

(b) Some rhombuses have four equal sides.

(c) All triangles have equal sides.

(d) Some polygons have five sides.

(e) All circles can be divided equally into eight sectors.

How do you negate a statement?

We use the word "no" or "not" to negate a statement. The negation of statement p is written as $\sim p$.

Learning Standard

Negate a statement.

Indicator

'$\sim p$' is read as 'tilde p'.

Example 4

Form a negation ($\sim p$) for each of the following statements (p) by using the word "no" or "not".

(a) 12 is a multiple of 5.

(b) 41 is a prime number.

(c) All multiples of 5 are multiples of 10.

(d) 0.4 m is equal to 400 mm.

Solution:

(a) 12 is not a multiple of 5.

(b) 41 is not a prime number.

(c) Not all multiples of 5 are multiples of 10.

(d) 0.4 m is not equal to 400 mm.

Mind Stimulation 2

Aim: To determine the truth value of a statement after the negation.

Steps:

1. Divide the class into groups.

2. Determine the truth values of the following statements.

(a) All even numbers are divisible by 2.
(b) All factors of 6 are factors of 8.
(c) 64 is a perfect cube number.
(d) 50% of 600 is 30.
(e) $\{a, b, c\}$ is a subset of $\{a, b\} \cap \{b, c\}$.

3. Form a negation for each statement in Step 2 by using the word "no" or "not".

4. Determine the truth values of the negation statements in Step 3.

Discussion:

What can you say about the truth values of the statements in Step 2 before and after the negation?

From the activity in Mind Stimulation 2, it is found that:

The truth values change from true to false or vice versa through the process of negation.

Self Practice 3.1b

Form a negation ($\sim p$) for each of the following statements (p) by using the word "no" or "not". Then, determine the truth values of the negations.

1. 819 is a multiple of 9.
2. A kite has two axes of symmetry.
3. A cone has one curved surface.
4. Two parallel lines have the same gradient.
5. All quadratic equations have two equal roots.

How do you determine the truth value of a compound statement?

A compound statement is a combination of two or more statements by using the word "and" or "or".

Learning Standard

Determine the truth value of a compound statement.

Example 5

Combine the following statements, p and q, by using the words

(i) and, (ii) or.

(a) p : A pentagon has two diagonals.
q : A heptagon has four diagonals.

(b) p : A pyramid has five planes.
q : A pyramid has five vertices.

(c) p : -4 is an integer.
q : 2 is an integer.

Solution:

(a) (i) A pentagon has two diagonals and a heptagon has four diagonals.
(ii) A pentagon has two diagonals or a heptagon has four diagonals.

(b) (i) A pyramid has five planes and five vertices.
(ii) A pyramid has five planes or five vertices.

(c) (i) -4 and 2 are integers.
(ii) -4 or 2 is an integer.

Example 6

Determine the two statements, p and q, in the following compound statements.

(a) $5 + 3 > 5$ and $5 - 3 < 5$.

(b) 9 and 91 are prime numbers.

(c) $2^2 = 4$ or $2^3 = 8$.

(d) $-9 < 10$ or $9 < 10$.

Solution:

(a) p : $5 + 3 > 5$.
q : $5 - 3 < 5$.

(b) p : 9 is a prime number.
q : 91 is a prime number.

(c) p : $2^2 = 4$.
q : $2^3 = 8$.

(d) p : $-9 < 10$.
q : $9 < 10$.

The word "and" in a mathematical statement means both while the word "or" means one of them or both.

Based on the picture on the right, three statements p, q and r are formed.

p : Rashid is running.

q : Kok Keong is running.

r : Melinda is running.

You can combine two statements by using the word "and" or "or".

p and q : Rashid and Kok Keong are running.

p and r : Rashid and Melinda are running.

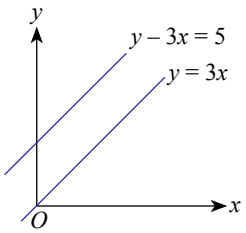
q or r : Kok Keong or Melinda is running.

From the three compound statements above, we notice that the statement "p and q" is true because both Rashid and Kok Keong are running but "p and r" is false because not both Rashid and Melinda are running. However, the compound statement "q or r" is true because part of the statement is true.

Hence, the truth value of a compound statement can be concluded as shown in the truth table below.

p	q	p and q	p or q
True	True	True	True
True	False	False	True
False	True	False	True
False	False	False	False

Example 7



Determine the truth values of the following compound statements.

(a) 2 and –5 are greater than 4.

(b) $x + 3 < x - 5$ and 99 is an odd number.

(c) 81 is a perfect square and 6 is a factor of 18.

(d) $y = 3x$ is parallel to $y - 3x = 5$ and the y-intercept of the straight line $y - 3x = 5$ is 3.

(e) $2 + 3 = 23$ or $2 \times 3 = 23$.

(f) $5 \times 5 \times 5 = 5^3$ or $\sqrt{125} = 5$.

(g) The sum of interior angles of a triangle or a quadrilateral is 360°.

(h) $4 - (-7) = 11$ or $4 + 7 = 11$.

Solution:

		Statement	Truth value
(a)	p	2 is greater than 4.	False
	q	–5 is greater than 4.	False
	p and q	2 and –5 are greater than 4.	False
(b)	p	$x + 3 < x - 5$.	False
	q	99 is an odd number.	True
	p and q	$x + 3 < x - 5$ and 99 is an odd number.	False
(c)	p	81 is a perfect square.	True
	q	6 is a factor of 18.	True
	p and q	81 is a perfect square and 6 is a factor of 18.	True
(d)	p	$y = 3x$ is parallel to $y - 3x = 5$.	True
	q	The y-intercept of the straight line $y - 3x = 5$ is 3.	False
	p and q	$y = 3x$ is parallel to $y - 3x = 5$ and the y-intercept of the straight line $y - 3x = 5$ is 3.	False
(e)	p	$2 + 3 = 23$.	False
	q	$2 \times 3 = 23$.	False
	p or q	$2 + 3 = 23$ or $2 \times 3 = 23$.	False
(f)	p	$5 \times 5 \times 5 = 5^3$.	True
	q	$\sqrt{125} = 5$.	False
	p or q	$5 \times 5 \times 5 = 5^3$ or $\sqrt{125} = 5$.	True
(g)	p	The sum of interior angles of a triangle is 360°.	False
	q	The sum of interior angles of a quadrilateral is 360°.	True
	p or q	The sum of interior angles of a triangle or a quadrilateral is 360°.	True
(h)	p	$4 - (-7) = 11$.	True
	q	$4 + 7 = 11$.	True
	p or q	$4 - (-7) = 11$ or $4 + 7 = 11$.	True

Self Practice 3.1c

1. Combine the following statements p and q by using the words given in the brackets to form compound statements.

(a) p : 2 is a prime factor of 6. (or)
q : 3 is a prime factor of 6.

(b) p : A cone has one vertex. (and)
q : A cone has one plane.

(c) p : A rhombus is a parallelogram. (and)
q : A trapezium is a parallelogram.

2. Determine the truth values of the following compound statements.

(a) 49 is a multiple of 7 and a perfect square.
(b) 3 hours = 120 minutes and 4 minutes = 240 seconds.
(c) The coefficient of $9x$ is 9 and $9^0 = 0$.
(d) $3 \in \{1, 2, 5\}$ and $\{8, 9\} \subset \{6, 7, 8\}$.
(e) $\frac{2}{9}$ can be expressed as a recurring decimal or less than 1.
(f) $\frac{4}{5}$ or $\frac{5}{4}$ is a proper fraction.
(g) 6 or 8 is an odd number.
(h) $\sqrt[4]{64} = 2$ or $2^3 = 8$.

How do you construct a statement in the form of an implication?

Learning Standard

Construct statement in the form of implication
(i) if p, then q
(ii) p if and only if q

Implication "If p, then q"

A statement "if p, then q" is known as an **implication** where

- p is denoted as the **antecedent**.
- q is denoted as the **consequent**.

Example 8

Form an implication "if p, then q" with the given antecedent and consequent.

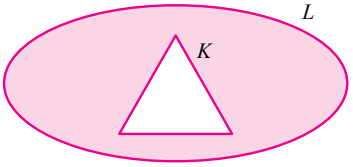
(a) Antecedent : k is divisible by 5.
Consequent : k is a multiple of 5.

(b) Antecedent : Set K is a subset of set L.
Consequent : $n(K) \leq n(L)$.

Solution:

(a) If k is divisible by 5, then k is a multiple of 5.

(b) If set K is a subset of set L, then $n(K) \leq n(L)$.



Example 9

Determine the antecedent and consequent for the following implications "if p, then q".

(a) If x is a factor of 16, then x is a factor of 64.

(b) If $x - y > 0$, then $x > y$.

Solution:

(a) Antecedent : x is a factor of 16.
Consequent : x is a factor of 64.

(b) Antecedent : $x - y > 0$.
Consequent : $x > y$.

Implication "p if and only if q"

Apart from the implication "if p, then q", the implication "p if and only if q" is also used frequently in logical reasoning.

An implication "p if and only if q" consists of the following two implications:

- if p, then q
- if q, then p

Example 10

Form an implication "p if and only if q" with the following implications.

(a) If k is a prime number, then k has only two factors.
If k has only two factors, then k is a prime number.

(b) If $y = ax^n + b$ is a linear equation, then $n = 1$.
If $n = 1$, then $y = ax^n + b$ is a linear equation.

Solution:

(a) k is a prime number if and only if k has only two factors.

(b) $y = ax^n + b$ is a linear equation if and only if $n = 1$.

Example 11

Write two implications based on the implication "p if and only if q" given below.

(a) $\sqrt{r} = 15$ if and only if $r = 225$.

(b) $x < 6$ if and only if $x + 10 < 16$.

Solution:

(a) Implication 1: If $\sqrt{r} = 15$, then $r = 225$.
Implication 2: If $r = 225$, then $\sqrt{r} = 15$.

(b) Implication 1: If $x < 6$, then $x + 10 < 16$.
Implication 2: If $x + 10 < 16$, then $x < 6$.

Self Practice 3.1d

1. Form an implication "if p, then q" with the given antecedent and consequent.

(a) Antecedent : $x = 3$
Consequent : $x^4 = 81$

(b) Antecedent : $ax^3 + bx^2 + cx + d = 0$ is a cubic equation.
Consequent : $a \neq 0$

(c) Antecedent : $n - 5 > 2n$
Consequent : $n < -5$

(d) Antecedent : $\frac{m}{n} > 1$
Consequent : $m^2 > n^2$

2. Determine the antecedent and consequent for the following implications "if p, then q".

(a) If x is an even number, then x^2 is an even number.

(b) If set $K = \phi$, then $n(K) = 0$.

(c) If x is a whole number, then $2x$ is an even number.

(d) If a straight line AB is a tangent to a circle P, then the straight line AB touches the circle P at one point only.

3. Form an implication "p if and only if q" with the following implications.

(a) If k is a perfect square, then $\sqrt{k}$ is a whole number.
If $\sqrt{k}$ is a whole number, then k is a perfect square.

(b) If $P \cap Q = P$, then $P \subset Q$.
If $P \subset Q$, then $P \cap Q = P$.

(c) If $pq = 1$, then $p = q^{-1}$ and $q = p^{-1}$.
If $p = q^{-1}$ and $q = p^{-1}$, then $pq = 1$.

(d) If $k^2 = 4$, then $(k + 2)(k - 2) = 0$.
If $(k + 2)(k - 2) = 0$, then $k^2 = 4$.

4. Write two implications based on the implication "p if and only if q" given below.

(a) PQR is a regular polygon if and only if $PQ = QR = PR$.

(b) $\frac{m}{n}$ is an improper fraction if and only if $m > n$.

(c) 9 is the y-intercept of a straight line $y = mx + c$ if and only if $c = 9$.

(d) $f(x) = ax^2 + bx + c$ has a maximum point if and only if $a < 0$.

How do you construct and compare the truth value of converse, inverse and contrapositive of an implication?

Learning Standard

Construct and compare the truth value of converse, inverse and contrapositive of an implication.

Before comparing the truth value of converse, inverse and contrapositive of an implication, observe and state the differences between four statements in the following example.

Statement	: If Ali is a prefect, then Ali is a disciplined person.
Converse	: If Ali is a disciplined person, then Ali is a prefect.
Inverse	: If Ali is not a prefect, then Ali is not a disciplined person.
Contrapositive	: If Ali is not a disciplined person, then Ali is not a prefect.

In general,

Statement	: If p, then q.
Converse	: If q, then p.
Inverse	: If $\sim p$, then $\sim q$.
Contrapositive	: If $\sim q$, then $\sim p$.

Example 12

Write the converse, inverse and contrapositive of the following implications.

(a) If x is a positive number, then x is greater than 0.

(b) If $p^2 - q^2 > 0$, then $(p + q)(p - q) > 0$.

(c) If $x = 5$, then $x + 1 = 6$.

Solution:

(a) Statement : If x is a positive number, then x is greater than 0.
Converse : If x is greater than 0, then x is a positive number.
Inverse : If x is not a positive number, then x is not greater than 0.
Contrapositive : If x is not greater than 0, then x is not a positive number.

(b) Statement : If $p^2 - q^2 > 0$, then $(p + q)(p - q) > 0$.
Converse : If $(p + q)(p - q) > 0$, then $p^2 - q^2 > 0$.
Inverse : If $p^2 - q^2 \leqslant 0$, then $(p + q)(p - q) \leqslant 0$.
Contrapositive : If $(p + q)(p - q) \leqslant 0$, then $p^2 - q^2 \leqslant 0$.

(c) Statement : If $x = 5$, then $x + 1 = 6$.
Converse : If $x + 1 = 6$, then $x = 5$.
Inverse : If $x \neq 5$, then $x + 1 \neq 6$.
Contrapositive : If $x + 1 \neq 6$, then $x \neq 5$.

TIPS

$\sim p$ is a complement of p. Then, the complement of $p^2 - q^2 > 0$ is $p^2 - q^2 \leqslant 0$.

After identifying the converse, inverse and contrapositive of an implication, you will evaluate the truth of the statements mentioned earlier.

Mind Stimulation 3

Aim: To compare the truth values of converse, inverse and contrapositive of an implication.

Steps:

1. Determine the truth values for the statements p and q of each implication in the Activity Sheet.
2. Write the converse, inverse and contrapositive of an implication "if p, then q". Then, determine the truth values of the statements.

Activity Sheet

			Truth value
(a)	p:	12 is an even number.	True
	q:	12 is divisible by 2.	True
	Statement:	If 12 is an even number, then 12 is divisible by 2.	True
	Converse:		
	Inverse:		
	Contrapositive:		
(b)	p:	50 is a multiple of 10.	
	q:	50 is a multiple of 20.	
	Statement:		
	Converse:	If 50 is a multiple of 20, then 50 is a multiple of 10.	True
	Inverse:		
	Contrapositive:		
(c)	p:	The sum of interior angles in pentagon $PQRST$ is 360°.	
	q:	Pentagon $PQRST$ is a quadrilateral.	
	Statement:		
	Converse:		
	Inverse:	If the sum of interior angles in pentagon $PQRST$ is not 360°, then pentagon $PQRST$ is not a quadrilateral.	True
	Contrapositive:		
(d)	p:	$x^2 < 0$.	
	q:	$x^2 + 3 > 0$.	
	Statement:		
	Converse:		
	Inverse:		
	Contrapositive:	If $x^2 + 3 \leqslant 0$, then $x^2 \geqslant 0$.	True

Discussion:

1. Compare the truth value of a contrapositive and the truth value of an implication "if p, then q".
2. What is the relationship between the converse and inverse of an implication "if p, then q"?
3. What is the difference of the truth values between the converse and inverse of an implication "if p, then q"?

From the activity in Mind Stimulation 3, it is found that:

1. The truth value of contrapositive is the same as the truth value of an implication "if p, then q".
2. The converse and inverse are contrapositive to each other.
3. The converse and inverse also have the same truth value.

Hence, you can list the truth values of an implication "if p, then q", and its corresponding converse, inverse and contrapositive in the table below.

p	q	Statement	Converse	Inverse	Contrapositive
		If p, then q.	**If q, then p.**	**If $\sim p$, then $\sim q$.**	**If $\sim q$, then $\sim p$.**
True	True	True	True	True	True
True	False	False	True	True	False
False	True	True	False	False	True
False	False	True	True	True	True

In conclusion,

The truth value of an implication "if p, then q" is always true except when p is true and q is false happen at the same time. If an antecedent is false, then the implication "if p, then q" is always true without depending on the truth value of the consequent.

Example 13

Determine the truth values of statement, converse, inverse and contrapositive of the implication

"If $2 \times 3 = 6$, then $8 - 2 \times 3 = 18$."

Solution:

		Antecedent	Consequent	Truth value
Statement:	If $2 \times 3 = 6$, then $8 - 2 \times 3 = 18$.	True	False	False
Converse:	If $8 - 2 \times 3 = 18$, then $2 \times 3 = 6$.	False	True	True
Inverse:	If $2 \times 3 \neq 6$, then $8 - 2 \times 3 \neq 18$.	False	True	True
Contrapositive:	If $8 - 2 \times 3 \neq 18$, then $2 \times 3 \neq 6$.	True	False	False

Self Practice 3.1e

1. Write the converse, inverse and contrapositive of the following implications.

(a) If $x + 3 > 2$, then $x > -1$.

(b) If $(k - 3)(k + 4) = 0$, then $k = 3$ or $k = -4$.

(c) If $ABCD$ is a parallelogram, then AB is parallel to CD.

2. Determine the truth values of implication, converse, inverse and contrapositive for each of the following statements.

(a) If 2 and 5 are factors of 10, then 2×5 is 10.

(b) If 4 is a root of $x^2 - 16 = 0$, then 4 is not a root of $(x + 4)(x - 4) = 0$.

(c) If a rectangle has four axes of symmetry, then the rectangle has four sides.

(d) If $55 + 55 = 4 \times 5$, then $666 + 666 = 6 \times 6$.

How do you determine a counter-example to negate the truth of a particular statement?

Learning Standard

Determine a counter-example to negate the truth of a particular statement.

For a false statement, at least one counter-example can be given to negate the truth of that statement. For example, the statement "All polygons have two or more diagonals." is false as a triangle does not have a diagonal. The triangle is a counter-example to support the false value.

Example 14

Determine the truth value of the following mathematical statements. If it is false, give one counter-example to support your answer.

(a) The sum of interior angles of all polygons is 180°.

(b) Some prime numbers are even numbers.

(c) 4 and 8 are the factors of 20.

(d) 6 or 36 is a multiple of 9.

Solution:

(a) False because the sum of interior angles of a pentagon is 540°.

(b) True.

(c) False because 8 is not a factor of 20.

(d) True.

Example 15

Write the mathematical statement requested in the brackets for each of the following. Then, determine the truth value of each statement written. If it is false, give one counter-example to support your answer.

(a) $6 \in \{3, 6, 9\}$. (Negation)

(b) All multiples of 10 are multiples of 2. (Negation)

(c) If $x > 5$, then $x > 3$. (Converse)

(d) If x is a root of $x^3 - 1 = 0$, then $x = 1$. (Inverse)

(e) If $k^2 > 0$, then $k > 0$. (Contrapositive)

Solution:

(a) Negation: $6 \notin \{3, 6, 9\}$. False because 6 is an element of $\{3, 6, 9\}$.

(b) Negation: Not all multiples of 10 are multiples of 2. False because all the multiples of 10 are divisible by 2.

(c) Converse: If $x > 3$, then $x > 5$. False because $4 > 3$ but $4 < 5$.

(d) Inverse: If x is not a root of $x^3 - 1 = 0$, then $x \neq 1$. True.

(e) Contrapositive: If $k \leqslant 0$, then $k^2 \leqslant 0$. False because $-2 < 0$ but $(-2)^2 = 4 > 0$.

Self Practice 3.1f

1. Determine the truth values of the following mathematical statements. If it is false, give one counter-example to support your answer.

(a) All rectangles are squares.

(b) Some perfect squares are divisible by 5.

(c) 5 or 9 have two factors.

(d) 36 is a multiple of 4 and 14.

2. Write the mathematical statement requested in the brackets for each of the following. Then, determine the truth value of each statement written. If it is false, give one counter-example to support your answer.

(a) $100_8 - 77_8 = 1_8$ (Negation)

(b) A cuboid has four uniform cross sections. (Negation)

(c) If $y = 2x$ is parallel to $y = 2x - 1$, then $y = 2x$ and $y = 2x - 1$ have the same gradient. (Converse)

(d) If a triangle ABC has a right angle at C, then $c^2 = a^2 + b^2$. (Inverse)

(e) If $w < 7$, then $w < 5$. (Contrapositive)

3.2 Arguments

What does an argument mean? What is the difference between deductive and inductive arguments?

Learning Standard

Explain the meaning of an argument and differentiate between deductive and inductive arguments.

From the above conversation, what is the conclusion that you can make? Has Suhaimi finished all his mathematics homework?

The process of making a conclusion based on statements is known as argumentation. An argument can consist of several premises and one conclusion. A premise is a statement that gives information before making a conclusion and a conclusion is an outcome or a decision. Normally, a simple argument consists of at least two premises and one conclusion.

Mind Stimulation 4

Aim: To differentiate between deductive and inductive arguments.

Steps:

1. Divide the class into groups.
2. Differentiate whether the statements below are specific statements or general statements. Circle your answer.

	Statement	Type of statement	
(a)	The area of triangle ABC is 8 cm^2.	Specific	General
(b)	All prisms have uniform cross sections.	Specific	General
(c)	All multiples of 2 end with even digits.	Specific	General
(d)	15 is divisible by 3.	Specific	General
(e)	$9^3 = 729$	Specific	General
(f)	The volume of a cube $= x^3$, where x is the edge of the cube.	Specific	General
(g)	The height of cylinder P is 80 cm.	Specific	General
(h)	All numbers in base 5 consist digits which are smaller than 5.	Specific	General
(i)	$(x - 2)(x + 3) = 0$ has two roots.	Specific	General
(j)	The volume of a sphere is $\frac{4}{3}\pi r^3$, where r is the radius of the sphere.	Specific	General

Discussion:
Justify your answer.

From the activity in Mind Stimulation 4, it is found that:

The specific statements are statements that refer to a particular case, while the general statements are statements that describe a concept.

There are two types of arguments, that are deductive argument and inductive argument. Try to justify the inductive argument and deductive argument through Mind Stimulation 5.

Mind Stimulation 5

Aim: To justify deductive argument and inductive argument.

Steps:

1. Divide the class into groups.
2. Study the arguments in Activity Sheet A.
3. Complete Activity Sheet B by writing premise 1 and the conclusion of each argument from Activity Sheet A. Hence, determine the type of statement for the premise and conclusion and circle the specific or general words below.

Activity Sheet A

Argument			
(a)	The area of a circle is πr^2. Circle A has a radius of 7 cm. In conclusion, the area of circle A is 154 cm^2.	(b)	All multiples of 6 are multiples of 2 and multiples of 3. 72 is a multiple of 6. Hence, 72 is a multiple of 2 and 3.
(c)	$1^0 = 1$ $2^0 = 1$ $3^0 = 1$ $4^0 = 1$ ⋮ In conclusion $n^0 = 1$, where $n = 1, 2, 3, 4, \ldots$	(d)	$2(1)^3 - 4 = -2$ $2(2)^3 - 4 = 12$ $2(3)^3 - 4 = 50$ $2(4)^3 - 4 = 124$ ⋮ Therefore, the number sequence –2, 12, 50, 124, … can be formulated as $2n^3 - 4$, where $n = 1, 2, 3, 4, \ldots$
(e)	All Class 4 Celik pupils scored A in the Mathematics test. Camelia is a pupil in Class 4 Celik. In conclusion, Camelia scored A in the Mathematics test.	(f)	Tigers are carnivores. Lions are carnivores. Crocodiles are carnivores. Penguins are carnivores. Therefore, all the animals above are carnivores.

Activity Sheet B

	Premise 1	Conclusion	Type of argument
(a)	The area of a circle is πr^2. General / Specific	In conclusion, the area of circle A is 154 cm^2. General / Specific	Deductive argument
(b)	General / Specific	General / Specific	Deductive argument
(c)	General / Specific	General / Specific	Inductive argument
(d)	General / Specific	General / Specific	Inductive argument
(e)	General / Specific	General / Specific	Deductive argument
(f)	General / Specific	General / Specific	Inductive argument

Discussion:
Based on the types of arguments given, justify the deductive argument and inductive argument.

From the activity in Mind Stimulation 5, it is found that:

- Deductive argument is a process of making a specific conclusion based on general premises.
- Inductive argument is a process of making a general conclusion based on specific premises.

Example 16

Determine whether each argument below is a deductive argument or an inductive argument.

(a) All acute angles are less than 90°. Angle PQR is an acute angle. Thus, angle PQR is less than 90°.

(b) All sudoku competition representatives are members of the Mathematics Club. Jamal is a sudoku competition representative. Thus, Jamal is a member of the Mathematics Club.

(c) The sum of exterior angles of a triangle is 360°. The sum of exterior angles of a quadrilateral is 360°. The sum of exterior angles of a pentagon is 360°. Thus, the sum of exterior angles of each polygon is 360°.

(d) The sum of the digits in 18 is divisible by 9. The sum of the digits in 27 is divisible by 9. The sum of the digits in 36 is divisible by 9. Thus, the sum of the digits in multiples of 9 is divisible by 9.

Solution:

(a)

Premise 1:	All acute angles are less than 90°. (General)
Conclusion:	Then, angle PQR is less than 90°. (Specific)

Deductive argument

(b)

Premise 1:	All sudoku competition representatives are members of the Mathematics Club. (General)
Conclusion:	Then, Jamal is a member of the Mathematics Club. (Specific)

Deductive argument

(c)

Premise 1:	The sum of exterior angles of a triangle is 360°. (Specific)
Conclusion:	Then, the sum of exterior angles of each polygon is 360°. (General)

Inductive argument

(d)

Premise 1:	The sum of the digits in 18 is divisible by 9. (Specific)
Conclusion:	Then, the sum of the digits in multiples of 9 is divisible by 9. (General)

Inductive argument

Self Practice 3.2a

Determine whether each argument below is a deductive argument or an inductive argument.

1. All factors of 6 are factors of 12. 1, 2, 3 and 6 are factors of 6. Thus, 1, 2, 3 and 6 are factors of 12.

2. $5^2 \times 5^3 = 5^5$, $5^3 \times 5^4 = 5^7$, $5^4 \times 5^5 = 5^9$. Thus, $5^m \times 5^n = 5^{m+n}$.

3. $2(1) = 2$, $2(2) = 4$, $2(3) = 6$, ... Thus, the number pattern 2, 4, 6, … can be expressed as $2n$; $n = 1, 2, 3, \ldots$.

4. All regular polygons have sides of equal length. $ABCDEFG$ is a regular polygon. Thus, $ABCDEFG$ has sides of equal length.

5. All multiples of 10 end with the digit 0. 50 is a multiple of 10. Thus, 50 ends with the digit 0.

6. $(1)^2 + 2 = 3$, $(2)^2 + 2 = 6$, $(3)^2 + 2 = 11$, …. Thus, the number sequence 3, 6, 11, … can be expressed as $n^2 + 2$; $n = 1, 2, 3, \ldots$.

7. $(1 + 1)^2 = 4$, $(1 + 2)^2 = 9$, $(1 + 3)^2 = 16$, …. Thus, the number sequence 4, 9, 16, … can be expressed as $(1 + n)^2$; $n = 1, 2, 3, \ldots$.

8. All multiples of 9 are multiples of 3. 72 is a multiple of 9. Thus, 72 is a multiple of 3.

9. All rational numbers can be written in the fraction form. 1.5 is a rational number. Thus, 1.5 can be written in the fraction form.

10. The supplementary angle of 60° is 120°. The supplementary angle of 45° is 135°. Thus, the supplementary angle of θ is $180° - \theta$.

How do you determine and justify the validity of a deductive argument and hence determine whether the valid argument is sound?

Learning Standard

Determine and justify the validity of a deductive argument and hence determine whether the valid argument is sound.

A valid deductive argument can be categorised into three forms.

	Form I	Form II	Form III
Premise 1:	All A are B	If p, then q	If p, then q
Premise 2:	C is A	p is true	Not q is true
Conclusion:	C is B	q is true	Not p is true

A deductive argument is said to be valid if all the premises and the conclusion are true.

Premise 1 : All actresses are good at dancing.
Premise 2 : Jasmine is an actress.
Conclusion : Jasmine is good at dancing.

The above argument is a valid argument. Although we know that Premise 1 is false (not all actresses are good at dancing) but this argument is still valid because the argument fulfils Form I as shown in the table above. But the argument above is unsound because Premise 1 is false.

Mind Stimulation 6

Aim: To determine and justify the validity of an argument.
Steps:
1. Divide the class into groups.
2. Observe the following argument.

Premise 1 : All actresses are good at dancing.
Premise 2 : Jasmine is good at dancing.
Conclusion : Jasmine is an actress.

Discussion:
(a) Why is the above argument not valid?
(b) Do all true premises in an argument guarantee the validity of the argument?

From the activity in Mind Stimulation 6, it is found that:

The argument above is not valid because it does not comply with any of the three forms of valid deductive argument. Therefore, a true premise does not guarantee the validity of an argument.

In general,

The validity of an argument is determined based on the forms of argument, and not based on the truth of the premises or the conclusion.

Example 17

Are the following arguments valid and sound? If it is not, justify your answer.

(a) Premise 1 : All multiples of 16 are even numbers.
Premise 2 : 64 is a multiple of 16.
Conclusion : 64 is an even number.

(b) Premise 1 : All basketballs are in spherical shape.
Premise 2 : The Earth is in spherical shape.
Conclusion : The Earth is a basketball.

(c) Premise 1 : If $w < 9$, then $w < 19$.
Premise 2 : $4 < 9$.
Conclusion : $4 < 19$.

(d) Premise 1 : If $a \neq 0$, then $ax^n + bx + c$ is a quadratic expression.
Premise 2 : $a \neq 0$.
Conclusion : $ax^n + bx + c$ is a quadratic expression.

(e) Premise 1 : If k is divisible by 8, then k is divisible by 4.
Premise 2 : 12 is not divisible by 8.
Conclusion : 12 is not divisible by 4.

(f) Premise 1 : If k is an even number, then $k + 1$ is an odd number.
Premise 2 : $8 + 1$ is an odd number.
Conclusion : 8 is an even number.

Solution:

(a) Valid and sound.

(b) Not valid because it does not comply with a valid form of deductive argument. Not sound because the conclusion is false.

(c) Valid and sound.

(d) Valid but not sound because Premise 1 and conclusion are not true.

(e) Not valid because it does not comply with a valid form of deductive argument. Not sound because the conclusion is false.

(f) Not valid but sound because it does not comply with a valid form of deductive argument.

Self Practice 3.2b

Are the following arguments valid and sound? If it is not, justify your answer.

1. Premise 1 : All multiples of 5 are multiples of 10.
Premise 2 : 35 is a multiple of 5.
Conclusion : 35 is a multiple of 10.

2. Premise 1 : All squares have right angles.
Premise 2 : $PQRS$ is a square.
Conclusion : $PQRS$ has right angles.

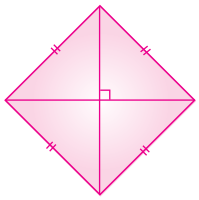
3. Premise 1 : If $\sqrt{x} < 3$, then $x < 9$.
Premise 2 : $\sqrt{4} < 3$.
Conclusion : $4 < 9$.

4. Premise 1 : If $k - 5 < 9$, then $k > 9$.
Premise 2 : $10 - 5 < 9$.
Conclusion : $10 > 9$.

5. Premise 1 : If x is a factor of 6, then 6 is divisible by x.
Premise 2 : 6 is divisible by 3.
Conclusion : 3 is a factor of 6.

6. Premise 1 : If l_1 is parallel to l_2, then gradient of l_1 = gradient of l_2.
Premise 2 : Gradient of $l_1 \neq$ Gradient of l_2.
Conclusion : l_1 is not parallel to l_2.

7. Premise 1 : All rhombuses have perpendicular diagonals.
Premise 2 : $PQRS$ has perpendicular diagonals.
Conclusion : $PQRS$ is a rhombus.



8. Premise 1 : If x is an even number, then $3x$ is an even number.
Premise 2 : $3x$ is not an even number.
Conclusion : x is not an even number.

9. Premise 1 : If $k > 5$, then $k^2 > 25$.
Premise 2 : $k \leqslant 5$.
Conclusion : $k^2 \leqslant 25$.

10. Premise 1 : All cubes are cuboids.
Premise 2 : Object P is a cube.
Conclusion : Object P is a cuboid.

How do you form a valid deductive argument for a situation?

Learning Standard

Form a valid deductive argument for a situation.

Example 18

Form a valid deductive argument for each of the following situations.

(a) All mammals are warm blooded. Cats are mammals. Cats are warm blooded.

(b) If x is greater than 0, then x has a positive value. 6 is greater than 0. 6 has a positive value.

(c) If x is an odd number, then $x + 1$ is divisible by 2. $18 + 1$ is not divisible by 2. 18 is not an odd number.

Solution:

(a) Premise 1 : All mammals are warm blooded.
Premise 2 : Cats are mammals.
Conclusion : Cats are warm blooded.

(b) Premise 1 : If x is greater than 0, then x has a positive value.
Premise 2 : 6 is greater than 0.
Conclusion : 6 has a positive value.

(c) Premise 1 : If x is an odd number, then $x + 1$ is divisible by 2.
Premise 2 : $18 + 1$ is not divisible by 2.
Conclusion : 18 is not an odd number.

Example 19

Write a conclusion for each of the following deductive arguments to form a valid and sound deductive argument.

(a) Premise 1 : All whole numbers are real numbers.
Premise 2 : 38 is a whole number.
Conclusion :

(b) Premise 1 : If $ax^2 + bx + c = 0$ has real roots, then $b^2 - 4ac \geqslant 0$.
Premise 2 : $2x^2 + px - 2 = 0$ has real roots.
Conclusion :

(c) Premise 1 : If a straight line $y = mx + c$ is parallel to the x-axis, then $m = 0$.
Premise 2 : $m \neq 0$.
Conclusion :

Solution:

(a) Premise 1 : All whole numbers are real numbers.
Premise 2 : 38 is a whole number.
Conclusion : 38 is a real number.

(b) Premise 1 : If $ax^2 + bx + c = 0$ has real roots, then $b^2 - 4ac \geq 0$.
Premise 2 : $2x^2 + px - 2 = 0$ has real roots.
Conclusion : $p^2 - 4(2)(-2) \geq 0$.

(c) Premise 1 : If a straight line $y = mx + c$ is parallel to the x-axis, then $m = 0$.
Premise 2 : $m \neq 0$.
Conclusion : The straight line $y = mx + c$ is not parallel to the x-axis.

MY MEMORY

For $2x^2 + px - 2 = 0$
$a = 2$, $b = p$ and $c = -2$

Example 20

Write the premise for each of the following deductive arguments to form a valid and sound deductive argument.

(a) Premise 1 :
Premise 2 : 37 is a prime number.
Conclusion : 37 has only two factors.

(b) Premise 1 : If the annual sales of ANC company exceeds three millions, then the employees get a bonus of three months' salary.
Premise 2 :
Conclusion : The annual sales of ANC company do not exceed three millions.

(c) Premise 1 : If $x = k$, then k is a root of the equation $3x^2 - 5 = 12$.
Premise 2 :
Conclusion : $x \neq 9$.

Solution:

(a) Premise 1 : All prime numbers have only two factors.
Premise 2 : 37 is a prime number.
Conclusion : 37 has only two factors.

(b) Premise 1 : If the annual sales of ANC company exceeds three millions, then the employees get a bonus of three months' salary.
Premise 2 : The employees of ANC company do not get a bonus of three months' salary.
Conclusion : The annual sales of ANC company do not exceed three millions.

(c) Premise 1 : If $x = k$, then k is a root of $3x^2 - 5 = 12$.
Premise 2 : 9 is not a root of $3x^2 - 5 = 12$.
Conclusion : $x \neq 9$.

Self Practice 3.2c

1. Write the conclusion for each of the following deductive arguments to form a valid and sound deductive argument.

(a) Premise 1 : All the pupils in 4 Amanah use digital textbooks.
Premise 2 : Preevena is a pupil of 4 Amanah.
Conclusion :

(b) Premise 1 : If Kai Meng is the champion of the state level chess competition, then he will get a cash prize of RM200.

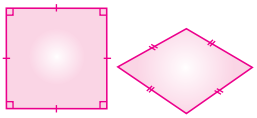
Premise 2 : Kai Meng is the champion of the state level chess competition.

Conclusion :

(c) Premise 1 : If quadrilateral $PQRS$ is a regular polygon, then quadrilateral $PQRS$ is a square.

Premise 2 : Quadrilateral $PQRS$ is not a square.

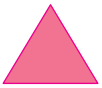
Conclusion :



(d) Premise 1 : All isosceles triangles have one axis of symmetry.

Premise 2 : ΔABC is an isosceles triangle.

Conclusion :



(e) Premise 1 : If $3m = 2n$, then $m : n = 2 : 3$.

Premise 2 : $3m = 2n$.

Conclusion :

(f) Premise 1 : If $m + 3 \leqslant 2m - 9$, then $m \geqslant 12$.

Premise 2 : $m < 12$.

Conclusion :

2. Write the conclusion for each of the following deductive arguments to form a valid deductive argument.

(a) Premise 1 : All straight lines with zero gradient are parallel to the x-axis.

Premise 2 :

Conclusion : Straight line AB is parallel to the x-axis.

(b) Premise 1 :

Premise 2 : 891 is a multiple of 9.

Conclusion : 891 is divisible by 3.

(c) Premise 1 : If polygon P is a nonagon, then polygon P has nine vertices.

Premise 2 :

Conclusion : Polygon P has nine vertices.

(d) Premise 1 :

Premise 2 : $x > 6$.

Conclusion : $x > 4$.

(e) Premise 1 : If it is raining today, then the room temperature is lower than 19°C.

Premise 2 :

Conclusion : It is not raining today.

(f) Premise 1 :

Premise 2 : $x \neq 8$.

Conclusion : $3x - 8 \neq 16$.

How do you determine and justify the strength of an inductive argument and hence determine whether the strong argument is cogent?

Learning Standard

Determine and justify the strength of an inductive argument and hence determine whether the strong argument is cogent.

A deductive argument emphasises the validity of the argument while an inductive argument emphasises the strength of the argument. The strength of an inductive argument is determined based on the probability that the conclusion is true, assuming that all premises are true. To determine an argument that is cogent or not cogent, it needs to be discussed based on the truth of the premises and its conclusion.

Example 21

Determine whether the given arguments are strong or weak. Hence, determine whether the strong argument is cogent or not cogent and justify your answer.

(a) Premise 1 : The chairs in the living room are red.
Premise 2 : The chairs in the dining room are red.
Premise 3 : The chairs in the study room are red.
Premise 4 : The chairs in the bedroom are red.
Conclusion : All chairs in the house are red.

(b) Premise 1 : 27 is a multiple of 3.
Premise 2 : 81 is a multiple of 3.
Conclusion : All multiples of 9 are multiples of 3.

(c) Premise 1 : A mackerel breathes through its gills.
Premise 2 : A shark breathes through its gills.
Conclusion : All fishes breathe through their gills.

(d) Premise 1 : $11 \times 5 = 55$
Premise 2 : $12 \times 5 = 60$
Conclusion : All multiples of 5 end with digit 0 or 5.

(e) Premise 1 : 1 is a prime number.
Premise 2 : 2 is a prime number.
Premise 3 : 3 is a prime number.
Premise 4 : 5 is a prime number.
Conclusion : All prime numbers are divisible by 1 and itself.

Solution:

(a) This argument is weak and not cogent because although the premises are true, the conclusion is probably false.

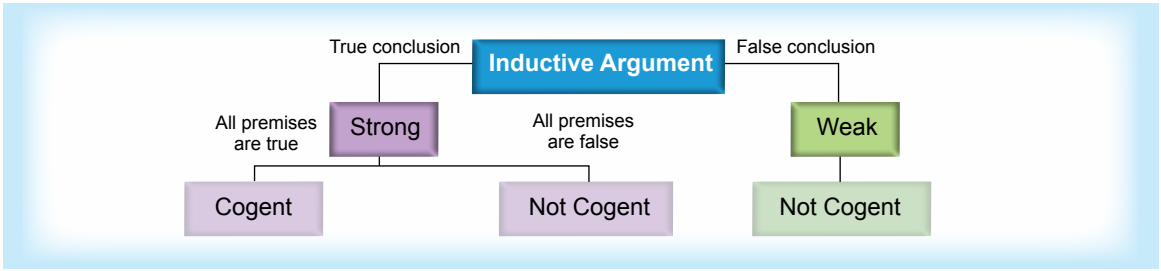
(b) This argument is strong and cogent because all the premises and conclusion are true.

(c) This argument is weak and not cogent because although the premises are true, the conclusion is false.

(d) This argument is strong and cogent because all the premises and conclusion are true.

(e) This argument is strong but not cogent because premise 1 is false.

From Example 21, it is found that the number of premises does not guarantee a strong argument because the strength of an argument depends on the truth value of the conclusion. A weak argument is not cogent while a strong argument will only be cogent if all its premises are true.



Self Practice 3.2d

Determine whether the given arguments are strong or weak, and cogent or not cogent. Justify your answers.

1. Premise 1 : The table is made of wood.
Premise 2 : The chair is made of wood.
Premise 3 : The cupboard is made of wood.
Conclusion : All furniture are made of wood.

2. Premise 1 : $(k^5)^2 = k^{10}$
Premise 2 : $(k^8)^2 = k^{16}$
Conclusion : $(k^m)^n = k^{mn}$

3. Premise 1 : 2^3 is divisible by 4.
Premise 2 : 2^5 is divisible by 4.
Conclusion : 2^n is divisible by 4.

4. Premise 1 : $2 \times 5 = 10$
Premise 2 : $4 \times 5 = 20$
Premise 3 : $6 \times 5 = 30$
Conclusion : The product of multiples of 2 and 5 end with digit 0.

5. Premise 1 : 24 is a multiple of 6.
Premise 2 : 36 is a multiple of 6.
Premise 3 : 40 is a multiple of 6.
Conclusion : All the multiples of 6 are even numbers.

6. Premise 1 : Rats have 4 legs.
Premise 2 : Cats have 4 legs.
Premise 3 : Horses have 4 legs.
Conclusion : All animals have 4 legs.

How do you form a strong inductive argument of a certain situation?

Learning Standard

Form a strong inductive argument of a certain situation.

A strong and cogent inductive argument depends on the true premises and conclusion. The given premises are the evidence or support to the conclusion made. Inductive reasoning can be carried out in the following steps.

Observe some examples or specific situations. → Observe the common features. → Make a general conclusion.

Example 22

Form a strong inductive conclusion for each of the following number sequences.

(a) Sequence $\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$

$$\frac{1}{1} = 1^{-1}$$
$$\frac{1}{2} = 2^{-1}$$
$$\frac{1}{3} = 3^{-1}$$
$$\frac{1}{4} = 4^{-1}$$
$$\vdots$$

(b) Sequence $0.5, 0.25, 0.125, 0.0625, \ldots$

$$0.5 = 0.5^1$$
$$0.25 = (0.5)^2$$
$$0.125 = (0.5)^3$$
$$0.0625 = (0.5)^4$$
$$\vdots$$

(c) Sequence $1, 3, 5, 7, \ldots$

$$1 = 2(0) + 1$$
$$3 = 2(1) + 1$$
$$5 = 2(2) + 1$$
$$7 = 2(3) + 1$$
$$\vdots$$

(d) Sequence $0, 9, 24, 45, \ldots$

$$0 = 3(1)^2 - 3$$
$$9 = 3(2)^2 - 3$$
$$24 = 3(3)^2 - 3$$
$$45 = 3(4)^2 - 3$$
$$\vdots$$

Solution:

(a) n^{-1}; $n = 1, 2, 3, 4, \ldots$

(b) $(0.5)^n$; $n = 1, 2, 3, 4, \ldots$

(c) $2n + 1$; $n = 0, 1, 2, 3, \ldots$

(d) $3n^2 - 3$; $n = 1, 2, 3, 4, \ldots$

Self Practice 3.2e

Form a strong inductive conclusion for each of the following number sequences.

1. $\frac{1}{3}, \frac{1}{6}, \frac{1}{9}, \frac{1}{12}, \ldots$

$$\frac{1}{3} = (3 \times 1)^{-1}$$
$$\frac{1}{6} = (3 \times 2)^{-1}$$
$$\frac{1}{9} = (3 \times 3)^{-1}$$
$$\frac{1}{12} = (3 \times 4)^{-1}$$
$$\vdots$$

2. $0.2, 0.4, 0.6, 0.8, \ldots$

$$0.2 = \frac{1}{5}$$
$$0.4 = \frac{2}{5}$$
$$0.6 = \frac{3}{5}$$
$$0.8 = \frac{4}{5}$$
$$\vdots$$

3. $0, 3, 18, 57, \ldots$

$$0 = 2(0)$$
$$3 = 2(1) + 1$$
$$18 = 2(8) + 2$$
$$57 = 2(27) + 3$$
$$\vdots$$

4. $19, 16, 4, -44, \ldots$

$$19 = 20 - 4^0$$
$$16 = 20 - 4^1$$
$$4 = 20 - 4^2$$
$$-44 = 20 - 4^3$$
$$\vdots$$

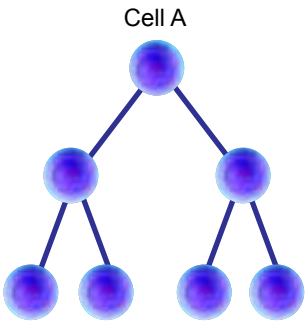
How do you solve problems involving logical reasoning?

Learning Standard

Solve problems involving logical reasoning.

Example 23

The diagram on the right shows the growth of a cell which begins with cell A. On the first day, two new cells are produced. Every cell will produce two other cells on subsequent days. The number of cells growth is $P(t) = 2^t$, where t is the number of days.



(a) How many new cells will be produced on the 8th day?

(b) On which day will the number of new cells become 2 048?

Solution:

(a)

Understanding the problem

(i) Make a conclusion by deduction.
(ii) Calculate the number of new cells on the 8th day.
(iii) $t = 8$
(iv) Calculate $P(8)$.

Planning a strategy

Substitute t with 8 into $P(t) = 2t$.

Implementing the strategy

$P(8) = 2^8$
$= 256$

Checking Answer

2, 4, 8, 16, 32, 64, 128, 256

Conclusion

$P(t) = 256$
256 new cells will be produced on the 8th day.

(b)

Understanding the problem

(i) Calculate on which day the number of new cells is 2 048.
(ii) Calculate the value of t when $P(t) = 2\,048$.

Planning a strategy

Solve $2^t = 2\,048$.

Implementing the strategy

$2^t = 2\,048$
$2^t = 2^{11}$

Conclusion

$t = 11$
The number of new cells is 2 048 on the 11th day.

Example 24

The table below shows the number of toy cars produced by the TOY Factory in a certain morning.

Time	Number of toy cars
8:00 a.m.	270
9:00 a.m.	520
10:00 a.m.	770
11:00 a.m.	1 020

(a) Construct a general formula for the number of toy cars produced by the TOY Factory based on the above table.

(b) The TOY Factory operates from 7 o'clock in the morning until 10 o'clock at night everyday for 5 days in a week.

(i) How many toy cars can be produced by the TOY Factory in a day?

(ii) The TOY Factory receives an order of 25 000 toy cars. This order needs to be completed in a week. Will the TOY Factory be able to deliver the toy cars on time? If it is not, suggest a solution so that the TOY Factory can deliver this order.

Solution:

(a)

Understanding the problem

(i) Make a conclusion by induction.
(ii) Construct a general formula for the number of toy cars produced.

Planning a strategy

Observe the pattern forms by the number of toy cars produced.

Conclusion

The general formula for the number of toy cars produced by TOY Factory is $250n + 20$; $n = 1, 2, 3, 4, ...$

Implementing the strategy

$270 = 250 + 20$
$520 = 2(250) + 20$
$770 = 3(250) + 20$
$1\,020 = 4(250) + 20$
$\vdots$

Hence, the number of toy cars produced can be formulated as
$250n + 20$; $n = 1, 2, 3, 4, ...$

(b) (i)

Understanding the problem

- The TOY Factory operates 15 hours a day.
- $n = 15$

Planning a strategy

Using the general formula constructed in part (a), substitute n with 15.

Conclusion

The TOY Factory produces 3 770 toy cars in a day.

Implementing the strategy

$250(15) + 20 = 3\,770$

(b) (ii)

Understanding the problem

- The TOY Factory needs to produce at least 25 000 toy cars in 5 days.
- The TOY Factory produces 3 770 toy cars in a day.

Planning a strategy

- Multiply 3 770 by 5.
- Compare the results with 25 000 and make a conclusion.

Conclusion

Therefore, the TOY Factory is not able to deliver the toy cars on time. Suggestion: TOY Factory can extend the operation hours to 20 hours a day so that on each day they can produce $250(20) + 20 = 5\,020$ toy cars.

Checking Answer

$5\,020 \times 5 = 25\,100$
$(> 25\,000)$

Implementing the strategy

$3\,770 \times 5 = 18\,850$ $(< 25\,000)$

Self Practice 3.2f

1. The parking charges at the Cahaya Hotel are calculated based on the rates below.

Time	Charge
The first hour or part of it	RM6.00
Each hour thereafter until the 6th hour	RM5.00
Each hour thereafter	RM3.00

Zamuddin parked his car from 0750 hours to attend a course at Cahaya Hotel. After the course, Zamuddin took his car at 1725 hours. Calculate by deduction the total charges that Zamuddin has to pay prior to exiting from the car park.

2. The number of residents in Taman Gembira follows the formula of $g(t) = 250(t^2 + t + 100)$, where t is the number of years.
It is given that the number of residents in Taman Gembira on 1 January 2012 was 25 000.

(a) Make a deductive conclusion about the total number of residents in Taman Gembira on 31 December 2016.

(b) In which year will the number of residents in Taman Gembira reach 77 500?

3. The number of new born babies in a certain country in the year 2014 was 536 100. The number of new born babies in that country from the years 2015 to 2017 form a number pattern as follows.

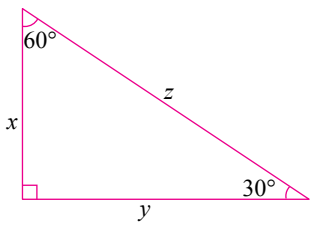
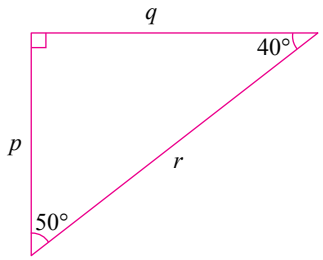
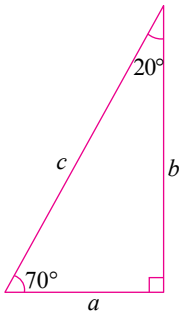
Year	New born babies
2014	536 100
2015	521 100
2016	506 100
2017	491 100

(a) Construct a formula based on the number pattern of new born babies.

(b) If the number of new born babies in that country follows the above number pattern for the next 5 years, estimate the number of babies born in the year 2021.

4. The following diagram shows three right-angled triangles.

(a) Complete the table below.

		
sin 60° = cos 30° =	sin 40° = cos 50° =	sin 20° = cos 70° =

(b) Observe and state the relationship between the angles and values of sine and cosine functions for each pair of angles above. Make a conclusion by induction about the relationship between sin θ and cos $(90° - \theta)$.

(c) Given sin 80° = 0.9848 and based on the inductive conclusion from part (b) above, state the value of cos 10°.

Comprehensive Practice

1. Determine whether each sentence below is a statement or not a statement. Justify your answer.

(a) A cuboid has 6 faces.
(b) Solve the equation $x^3 = 3x^2 + 3x - 1$.
(c) Each cylinder has two curved faces.
(d) Don't forget to bring workbook tomorrow.
(e) $3x + 5 = 6$.
(f) $(a + b)(a - b) = a^2 - b^2$.
(g) Wow, this flower is beautiful!
(h) The members of PDRM are government officials.
(i) $3 + 5 > 8$.

2. Determine whether the following statements are true or false.
If it is false, give one counter-example.

(a) $(x - y)^2 = x^2 - 2xy + y^2$.
(b) All integers have positive values.
(c) A fraction is smaller than one.
(d) All diagonals are perpendicular bisectors.

3. Determine whether the following compound statements are true or false.

(a) $26 = 64$ and $2 \times 6 = 26$.

(b) $9^{-1} = \frac{1}{9}$ and 9 is a factor of 72.

(c) $\{2, 5\} \subset \{2, 3, 6\} \cup \{5, 7\}$ or $n(\phi) = 0$.

(d) $90 \times 80\% = 70$ or $8 \times 8 \times 8 = 324$.

4. Write a true statement by using the quantifier "all" or "some" for the objects and characteristics given below.

	Object	Characteristic
(a)	Hexagon	Has 6 vertices.
(b)	Circle	Has a radius of 18 cm.
(c)	Triangle	Has three axes of symmetry.

5. (a) Determine the antecedents and consequents of the following statements:

(i) If $p < q$, then $q - p > 0$.

(ii) If the perimeter of rectangle A is $2(x + y)$, then the area of rectangle A is xy.

(b) Make an appropriate implication based on each of the following pairs of implications:

(i) If x is a multiple of 10, then x is a multiple of 5.
If x is a multiple of 5, then x is a multiple of 10.

(ii) If 6 is a factor of 12, then 6 is a factor of 24.
If 6 is a factor of 24, then 6 is a factor of 12.

(c) Construct two appropriate implications for each of the following implications:

(i) 20% of 30 is 6 if and only if $0.2 \times 30 = 6$.

(ii) M is divisible by 20 if and only if M is divisible by 2 and 10.

6. Write the requested statement in brackets for each of the following and determine the truth value of the written statement. Justify your answer if it is false.

(a) If α and β are two complementary angles, then $\alpha + \beta = 90°$. (Converse)

(b) If $w > 20$, then $w > 30$. (Contrapositive)

(c) If $p > 0$, then $p^2 > 0$. (Inverse)

(d) The sum of exterior angles of a polygon is 360°. (Negation)

7. Complete the following arguments to form valid and sound deductive arguments.

(a) Premise 1 : All factors of 4 are factors of 8.
Premise 2 : 2 is a factor of 4.
Conclusion :

(b) Premise 1 : If $x = 5$, then $2x + 8 = 18$.
Premise 2 :
Conclusion : $2x + 8 = 18$.

(c) Premise 1 :
Premise 2 : $\sin^2 \alpha + \cos^2 \beta \neq 1$.
Conclusion : $\alpha \neq \beta$.

(d) Premise 1 : If p is divisible by 18, then p is a multiple of 18.
Premise 2 : 54 is divisible by 18.
Conclusion :

(e) Premise 1 : If $-4m < 0$, then $m > 0$.
Premise 2 :
Conclusion : $-4m \geqslant 0$.

(f) Premise 1 : All quadratic functions have a turning point.
Premise 2 :
Conclusion : The function $g(x)$ has a turning point.

8. (a) Given the surface area of a cone $= \pi r(r + s)$, make a conclusion by deduction for the surface area of the five similar cones such that $r = 7$ cm and $s = 13$ cm.

(b) Given the equation of a straight line is $y = mx + c$, make a conclusion by deduction for the equation of the straight line PQ such that $m = 3$ and $c = 5$.

9. Make a conclusion by induction for the following number sequences.

(a) $-4, -1, 4, 11, \ldots$
$-4 = 1^2 - 5$
$-1 = 2^2 - 5$
$4 = 3^2 - 5$
$11 = 4^2 - 5$
$\vdots$

(b) $4, 5, 7, 11, \ldots$
$4 = 2^0 + 3$
$5 = 2^1 + 3$
$7 = 2^2 + 3$
$11 = 2^3 + 3$
$\vdots$

(c) $5, 12, 21, 32, \ldots$
$5 = 4(1) + 1$
$12 = 4(2) + 4$
$21 = 4(3) + 9$
$32 = 4(4) + 16$
$\vdots$

(d) $3, 8, 17, 30, \ldots$
$3 = 3(1) + 2(0)^2$
$8 = 3(2) + 2(1)^2$
$17 = 3(3) + 2(2)^2$
$30 = 3(4) + 2(3)^2$
$\vdots$

10. Determine whether the following arguments are inductive arguments or deductive arguments.

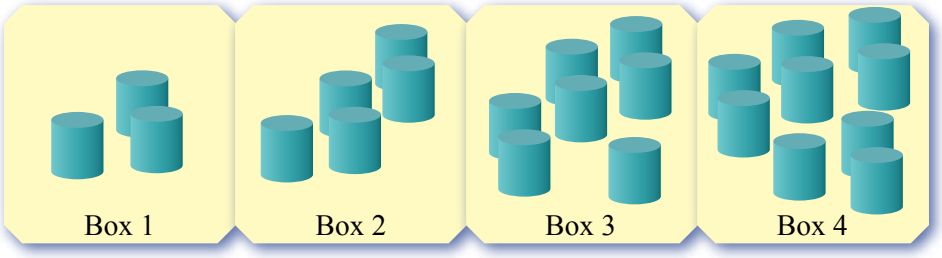
(a)

All the pupils from Class 4 Bahagia perform on Teachers' Day. Jayanthi is a pupil from Class 4 Bahagia. Then, Jayanthi performs on Teachers' Day.

(b)

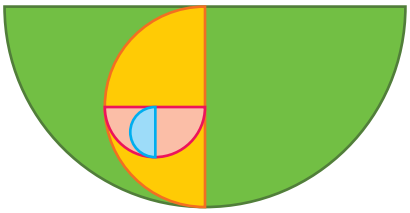
The sum of 1 and 3 is an even number.
The sum of 3 and 5 is an even number.
The sum of 5 and 7 is an even number.
The sum of 7 and 9 is an even number.
In conclusion, the sum of two odd numbers is an even number.

11. The diagram below shows the number of cylinders of equal size arranged in the boxes according to a number pattern 3, 5, 7, 9, …



(a) Construct a conclusion by induction for the pattern of the number of cylinders above.
(b) If the radius and height of each cylinder are 14 cm and 10 cm respectively, calculate the total volume of cylinders in the box 8.

12. The diagram on the right shows the first four semicircles arranged according to a certain pattern. The radius of the largest semicircle is 32 cm.



(a) Calculate and list the perimeters of the four semicircles, in terms of π.
(b) Based on the answer from (a), show that the generalisation for the perimeters of the four semicircles is $2^{6-n}(\pi + 2)$; $n = 0, 1, 2, 3, 4, \ldots$.
(c) Calculate the perimeter, in cm, of the 8th semicircle.

PROJECT

The rise in sea level is a critical issue for the whole world nowadays. The rise in sea level is closely related to the changes in temperature on the earth. Given the equilibrium of sea level and temperatures are connected by the formula, that is

$$L = T(0.54T^2 + 0.39T + 7.7)$$

L is the change in sea level and T is the changes in temperature.

Prepare a folio about the rise in sea level and changes in temperature for the last five years. Your folio must contain

1. Front page
2. Content page
   (a) An introduction to the issue of the rise in sea level.
   (b) A conclusion by deduction about the changes in sea level for the last five years.
   (c) Reasons causing the rise in sea level.
   (d) Impact of human activity on the rise in sea level.
   (e) Steps to control the rise in sea level.
3. Conclusion

CONCEPT MAP

Logical Reasoning

Statements True or False

Negation
"No" or "Not"

Example:
12 is a multiple of 5.
Negation: 12 is not a multiple of 5.

Compound Statement
"or" or "and"

Example:
p: 12 is a multiple of 5.
q: 12 is a multiple of 6.
- 12 is a multiple of 5 or a multiple of 6.
- 12 is a multiple of 5 and a multiple of 6.

Implication
"If p, then q"
"p if and only if q"

Example:
p: 12 is a multiple of 3.
q: 12 is a multiple of 6.
- If 12 is a multiple of 3, then 12 is a multiple of 6.
- 12 is a multiple of 3 if and only if 12 is a multiple of 6.

Converse : "If q, then p"
Inverse : "If $\sim p$, then $\sim q$"
Contrapositive : "If $\sim q$, then $\sim p$"

Example:
If 12 is a multiple of 3, then 12 is a multiple of 6.
Converse : If 12 is a multiple of 6, then 12 is a multiple of 3.
Inverse : If 12 is not a multiple of 3, then 12 is not a multiple of 6.
Contrapositive : If 12 is not a multiple of 6, then 12 is not a multiple of 3.

Arguments

Inductive Argument
Strong and Cogent

Make general conclusion based on specific premises

Example:
Premise 1: $2(1) - 1 = 1$
Premise 2: $2(2) - 1 = 3$
Premise 3: $2(3) - 1 = 5$
Premise 4: $2(4) - 1 = 7$
Conclusion:
$2n - 1$; $n = 1, 2, 3, 4, \ldots$

Deductive Argument
Valid and Sound

Make specific conclusion based on general premises

Premise 1 : All A is B.
Premise 2 : C is A.
Conclusion : C is B.

Example:
Premise 1 : All multiples of 6 are multiples of 3.
Premise 2 : 18 is a multiple of 6.
Conclusion : 18 is a multiple of 3.

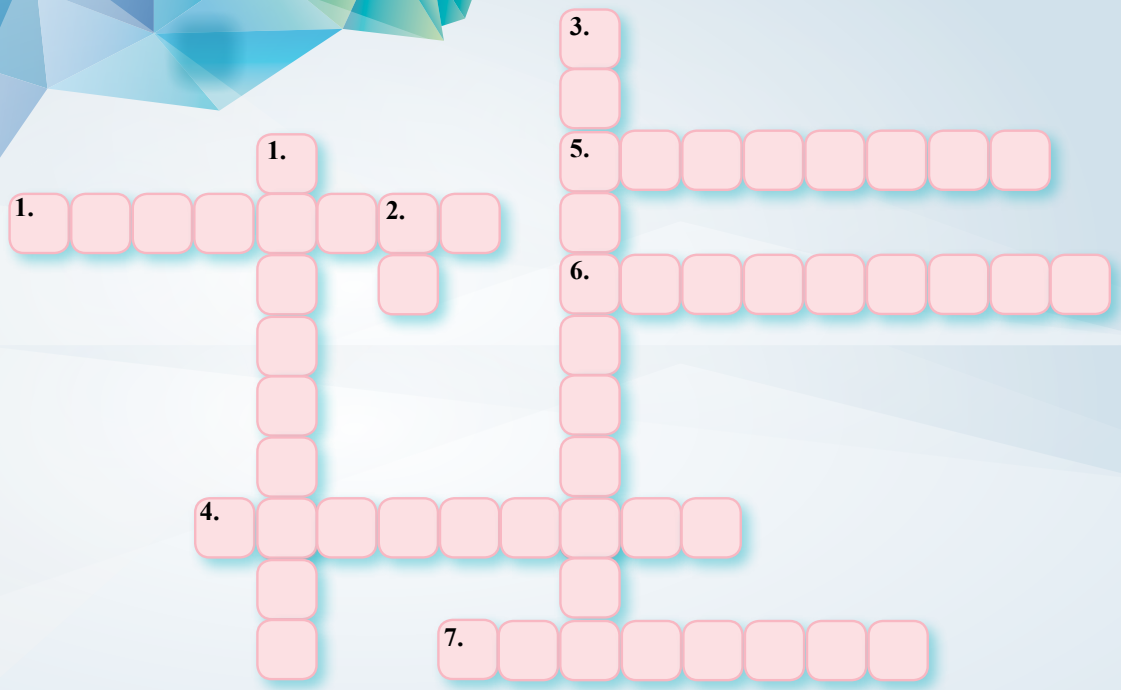
Premise 1 : If p, then q.
Premise 2 : p is true.
Conclusion : q is true.

Example:
Premise 1 : If x is a multiple of 6, then x is a multiple of 3.
Premise 2 : 18 is a multiple of 6.
Conclusion : 18 is a multiple of 3.

Premise 1 : If p, then q.
Premise 2 : Not q is true.
Conclusion : Not p is true.

Example:
Premise 1 : If x is a multiple of 6, then x is a multiple of 3.
Premise 2 : 17 is not a multiple of 3.
Conclusion : 17 is not a multiple of 6.

Self Reflection



Across

1. The _____ of statement p is written as $\sim p$.
4. Premise 1: All multiples of 6 are multiples of 2 and 3.
   Premise 2: 18 is a multiple of 6.
   Conclusion: 18 is a multiple of 2 and 3.
   The _____ argument above is valid and sound.
5. A simple argument consists of two or more _____ and one conclusion.
6. The _____ argument is a general conclusion made based on specific premises.
7. The _____ and inverse have the same truth value.

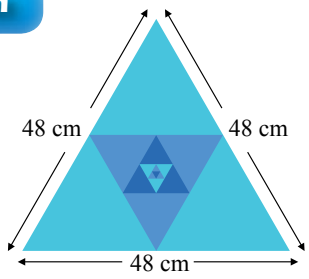
Down

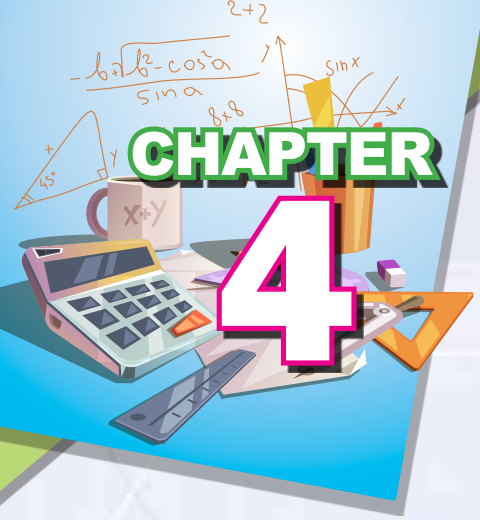
1. A sentence by which the truth value can be determined is known as a _____ .
2. Make a true compound statement from the statement below:
   91 is a multiple of 9 _____ 81 is a multiple of 9.
3. The inverse of _____ "if p, then q" is "if $\sim p$, then $\sim q$."

Mathematics Exploration

The diagram on the right shows an equilateral triangle with the sides of 48 cm. The midpoint of each side is connected to form another equilateral triangle. This process is repeated infinitely.

Calculate the total perimeter of all the infinite equilateral triangles.





CHAPTER 4

Operations on Sets

You will learn

- Intersection of Sets
- Union of Sets
- Combined Operations on Sets

The agriculture sector is one of the sectors contributing to the income of Malaysia. The use of modern and high technology in agriculture not only enables this sector to increase its output, it even attracts the new generation to this sector in the future.

Do you know what other economy sectors that are the main contributors to the income of our nation?

Why Study This Chapter?

Operations on sets are used in representing, grouping and analysing similar data in our daily life. This knowledge is the basis to various fields such as programming, statistics, economics and stock markets, which need analytical skills.

WORD BANK

• union	• *kesatuan*
• intersection	• *persilangan*
• complement	• *pelengkap*
• set	• *set*
• empty set	• *set kosong*
• subset	• *subset*
• set notation	• *tatatanda set*
• element	• *unsur*

Walking Through Time

Georg Ferdinand Ludwig Philipp Cantor
(1845 – 1918)

Georg Cantor is a German mathematician who pioneered the theory of set. He also introduced the ideas of ordinal numbers, cardinal numbers and the arithmetic of infinite sets.

http://bt.sasbadi.com/m4095

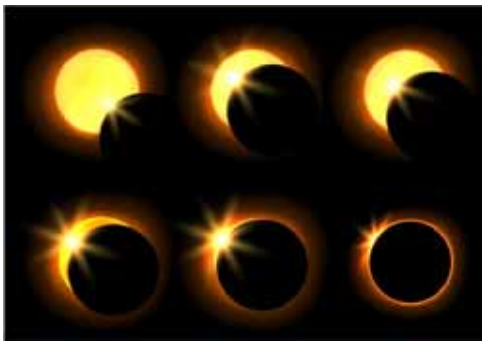
4.1 Intersection of Sets

How do you determine and describe the intersection of sets using various representations?

Learning Standard

Determine and describe the intersection of sets using various representations.

An intersection of sets exists when there are more than one set. The intersection of sets P and Q is written using the symbol $\cap$. Set $P \cap Q$ contains the common elements of both sets P and Q.



Eclipse of the sun occurs when the moon is in between the sun and the earth, in a straight line. The intersection between the moon and the sun can be seen when parts of the moon and the sun are at the same position.

Mind Stimulation 1

Aim: To determine and describe the intersection of sets using various representations.

The table below shows a group of pupils who prefer to eat local fruits.

Nabil prefers to eat durian.	Afiq prefers to eat durian and rambutan.
Hani prefers to eat durian.	Amirul prefers to eat rambutan.
Navin prefers to eat rambutan.	Meena prefers to eat durian.
Yan Kit prefers to eat durian.	Benjamin prefers to eat rambutan and durian.
Raj prefers to eat durian and rambutan.	Jenny prefers to eat durian.
Mei Yee prefers to eat durian.	Khairi prefers to eat rambutan.

Steps:

1. Based on the above table, write the names of the pupils using set notations.

A ={pupils who prefer to eat durian}

A = {Nabil, Hani, Yan Kit, Raj, Mei Yee, Afiq, Meena, Benjamin, Jenny}

B ={pupils who prefer to eat rambutan}

B = { , , , , , }

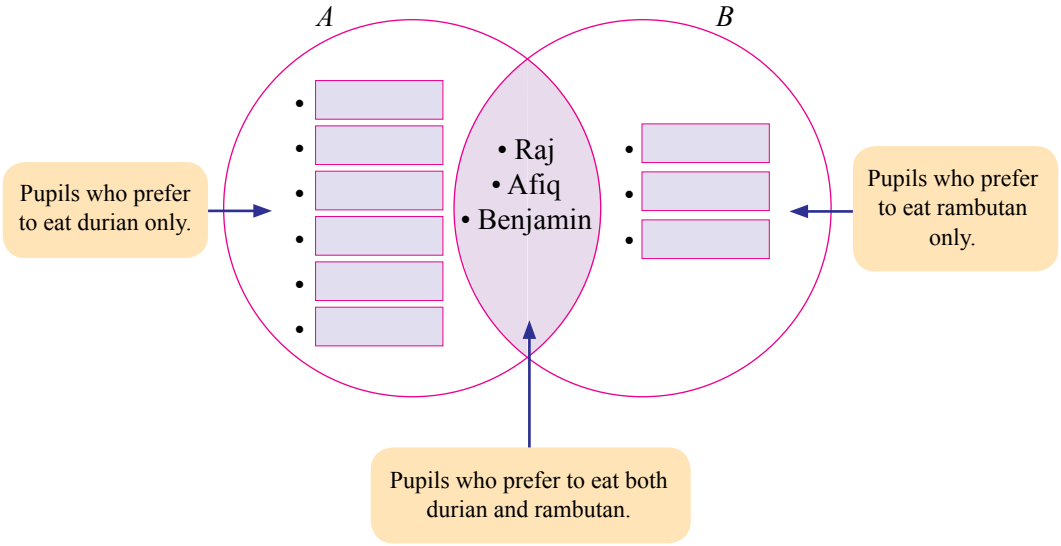
$A \cap B$ = {pupils who prefer to eat both durian and rambutan}

$A \cap B$ = { , , }

2. Complete the following table with names of pupils who prefer to eat durian only, rambutan only and both durian and rambutan.

Names of pupils who prefer to eat durian only	Names of pupils who prefer to eat rambutan only	Names of pupils who prefer to eat both durian and rambutan
 • Nabil • Hani • Yan Kit • Mei Yee • Meena • Jenny	 • Navin • ☐ • ☐	• Raj • Afiq • Benjamin  

3. Complete the following Venn diagram based on the information above.



Discussion:

How can you determine the intersection of sets based on the above activity?

From the activity in Mind Stimulation 1, it is found that:

The intersection of set A and set B contains common elements, that are Raj, Afiq and Benjamin who prefer to eat both durian and rambutan.

Example 1

It is given that the universal set, $\xi = \{x : x \text{ is an integer}, 1 \leqslant x \leqslant 10\}$, set $P = \{x : x \text{ is an odd number}\}$, set $Q = \{x : x \text{ is a prime number}\}$ and set $R = \{x : x \text{ is a multiple of } 3\}$.

(a) List all the elements of the following intersections of sets.

(i) $P \cap Q$ (ii) $P \cap R$ (iii) $Q \cap R$ (iv) $P \cap Q \cap R$

(b) State the number of elements of the following sets.

(i) $n(P \cap Q)$ (ii) $n(P \cap R)$ (iii) $n(Q \cap R)$ (iv) $n(P \cap Q \cap R)$

Solution:

(a) (i) $P \cap Q$

$P = \{1, 3, 5, 7, 9\}$

$Q = \{2, 3, 5, 7\}$

$P \cap Q = \{3, 5, 7\}$

(ii) $P \cap R$

$P = \{1, 3, 5, 7, 9\}$

$R = \{3, 6, 9\}$

$P \cap R = \{3, 9\}$

(iii) $Q \cap R$

$Q = \{2, 3, 5, 7\}$

$R = \{3, 6, 9\}$

$Q \cap R = \{3\}$

(iv) $P \cap Q \cap R$

$P = \{1, 3, 5, 7, 9\}$

$Q = \{2, 3, 5, 7\}$

$R = \{3, 6, 9\}$

$P \cap Q \cap R = \{3\}$

(b) (i) $P \cap Q = \{3, 5, 7\}$

$n(P \cap Q) = 3$

(ii) $P \cap R = \{3, 9\}$

$n(P \cap R) = 2$

(iii) $Q \cap R = \{3\}$

$n(Q \cap R) = 1$

(iv) $P \cap Q \cap R = \{3\}$

$n(P \cap Q \cap R) = 1$

MY MEMORY

$B \subset A$

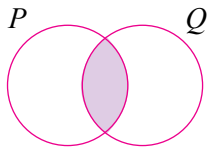
Set B is a subset of set A when all the elements of set B are found in set A.

INFO ZONE

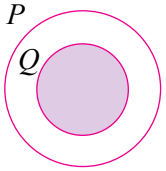
The symbol $\subseteq$ can also be used to denote subset.

How do you determine the intersections of two or more sets using Venn diagrams?

The intersections of two or more sets are represented by the shaded regions, as shown in the following Venn diagrams.

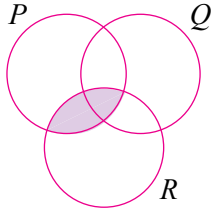


$P \cap Q$

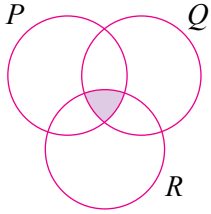


$P \cap Q = Q$

$(P \cap Q) \subset P$



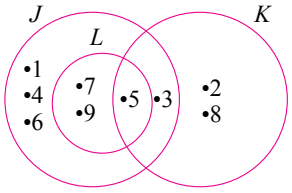
$P \cap R$



$P \cap Q \cap R$

Example 2

The Venn diagram shows set J, set K and set L such that the universal set, $\xi = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$.



List all the elements of the following intersections of sets.

(a) $J \cap K$ (b) $J \cap L$ (c) $K \cap L$ (d) $J \cap K \cap L$

Solution:

(a) $J \cap K = \{3, 5\}$

(b) $J \cap L = \{5, 7, 9\}$

(c) $K \cap L = \{5\}$

(d) $J \cap K \cap L = \{5\}$

MY MEMORY

An empty set is a set that has no element and is represented by the symbol ϕ or $\{\ \}$.

Example 3

It is given that set A = {numbers on a dice}, set B = {even numbers on a dice} and set $C = \{7, 8, 9\}$.

(a) List all the elements of the following intersections of sets.

(i) $A \cap B$ (ii) $B \cap C$ (iii) $A \cap C$

(b) Draw a Venn diagram to represent sets A, B and C, and shade the region that represents each of the following intersections of sets.

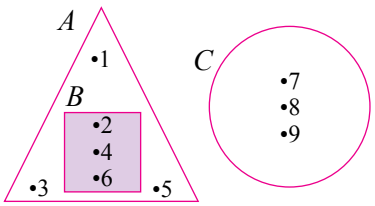
(i) $A \cap B$ (ii) $B \cap C$

Solution:

(a) $A = \{1, 2, 3, 4, 5, 6\}$
$B = \{2, 4, 6\}$
$C = \{7, 8, 9\}$

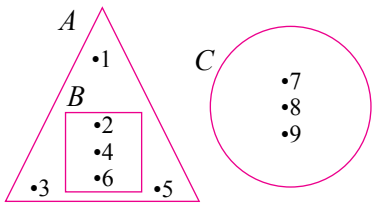
(i) $A \cap B = \{2, 4, 6\}$
(ii) $B \cap C = \{\ \}$
(iii) $A \cap C = \phi$

(b) (i)



All the elements of set B are in set A.
$A \cap B = B$

(ii)



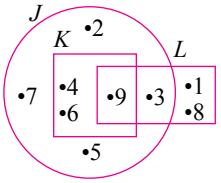
Set B and set C do not have common elements.

Self Practice 4.1a

1. Given $\xi = \{x : x \text{ is an integer, } 1 \leqslant x \leqslant 10\}$, set $M = \{x : x \text{ is an odd number}\}$ and set $N = \{x : x \text{ is a multiple of } 3\}$, list all the elements of the following sets.

(a) set M (b) set N (c) $M \cap N$

2. The Venn diagram shows sets J, K and L such that the universal set, $\xi = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. List all the elements of the following intersections of sets.



(a) $J \cap K$ (b) $J \cap L$ (c) $K \cap L$ (d) $J \cap K \cap L$

3. It is given that the universal set, $\xi = \{x : x \text{ is an integer}, 1 \leqslant x \leqslant 20\}$, set $P = \{x : x \text{ is a prime number}\}$, set $Q = \{x : x \text{ is a multiple of } 5\}$ and set $R = \{x : x \text{ is a factor of } 10\}$.

(a) Draw a Venn diagram to represent all the given sets.

(b) Based on the diagram in (a), shade the region that represents the set $P \cap Q \cap R$.

4. Given set $A = \{x : x \text{ is a letter in the word "GIGIH"}\}$, set $B = \{x : x \text{ is a letter in the word "DEDIKASI"}\}$ and set $C = \{x : x \text{ is a letter in the word "JUJUR"}\}$. State the number of elements by listing all the elements of the following intersections of sets.

(a) $n(A \cap B)$ (b) $n(A \cap C)$ (c) $n(B \cap C)$ (d) $n(A \cap B \cap C)$

CHAPTER 4

How do you determine the complement of an intersection of sets?

Learning Standard

Determine the complement of the intersection of sets.

The complement of an intersection of sets is written using the symbol " $'$ ". $(A \cap B)'$ is read as "the complement of the intersection of sets A and B". $(A \cap B)'$ refers to all the elements not in the intersection of sets A and B.

Example 4

Given the universal set, $\xi = \{x : x \text{ is an integer}, 1 \leqslant x \leqslant 8\}$, set $A = \{1, 2, 3, 4, 5, 6\}$, set $B = \{2, 4, 6\}$ and set $C = \{1, 2, 3, 4\}$, list all the elements and state the number of elements of the following sets.

(a) $(A \cap B)'$ (b) $(A \cap C)'$ (c) $(A \cap B \cap C)'$

Solution:

$\xi = \{1, 2, 3, 4, 5, 6, 7, 8\}$

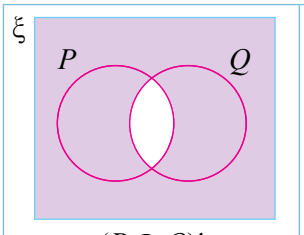
(a) $A \cap B = \{2, 4, 6\}$
$(A \cap B)' = \{1, 3, 5, 7, 8\}$
$n(A \cap B)' = 5$

(b) $A \cap C = \{1, 2, 3, 4\}$
$(A \cap C)' = \{5, 6, 7, 8\}$
$n(A \cap C)' = 4$

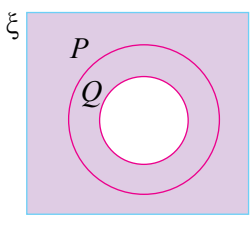
(c) $A \cap B \cap C = \{2, 4\}$
$(A \cap B \cap C)' = \{1, 3, 5, 6, 7, 8\}$
$n(A \cap B \cap C)' = 6$

How do you determine the complements of the intersections of two or more sets on Venn diagrams?

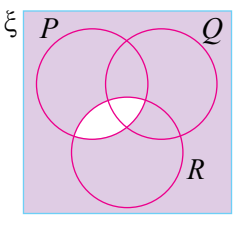
The complements of the intersections of two or more sets are represented by the shaded regions, as shown in the following Venn diagrams.



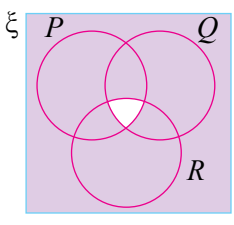
$(P \cap Q)'$



$(P \cap Q)'$



$(P \cap R)'$



$(P \cap Q \cap R)'$

Example 5

The co-curricular activities participated by three pupils are given in set P, set Q and set R such that the universal set, ξ = {Scouts, Mathematics, Hockey, Football, History, Badminton, Police Cadet}.
P = {Scouts, Mathematics, Hockey}
Q = {Police Cadet, History, Badminton}
R = {Scouts, History, Football}

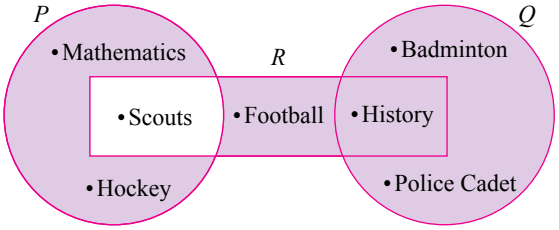
(a) List all the elements of the following sets.
(i) $(P \cap R)'$ (ii) $(R \cap Q)'$ (iii) $(P \cap Q \cap R)'$

(b) Draw a Venn diagram to represent sets P, Q and R, and shade the region that represents each of the following complements of intersections of sets.
(i) $(P \cap R)'$ (ii) $(R \cap Q)'$ (iii) $(P \cap Q \cap R)'$

Solution:
ξ = {Scouts, Mathematics, Hockey, Football, History, Badminton, Police Cadet}

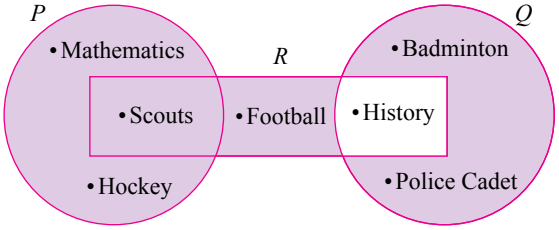
(a) (i) $P \cap R$ = {Scouts}
$(P \cap R)'$ = {Mathematics, Hockey, Football, History, Badminton, Police Cadet}

(b) (i)



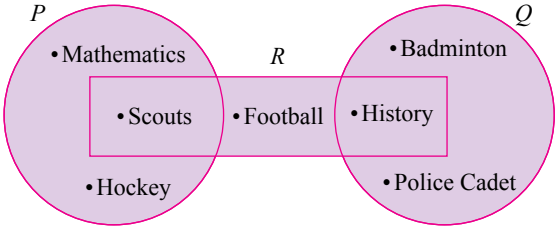
(a) (ii) $R \cap Q$ = {History}
$(R \cap Q)'$ = {Scouts, Mathematics, Hockey, Football, Badminton, Police Cadet}

(b) (ii)



(a) (iii) $P \cap Q \cap R$ = { }
$(P \cap Q \cap R)'$ = {Scouts, Mathematics, Hockey, Football, History, Badminton, Police Cadet}

(b) (iii)

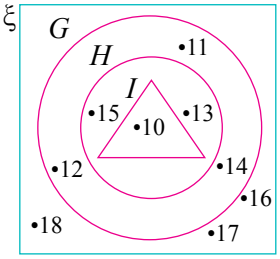


Self Practice 4.1b

1. Given the universal set, $\xi = \{2, 3, 4, 5, 6, 7, 8, 9, 10\}$, set $P = \{3, 5, 7, 9\}$, set $Q = \{2, 3, 5, 7\}$ and set $R = \{2, 4, 6, 8, 10\}$, list all the elements of the following sets.

(a) $(P \cap Q)'$ (b) $(Q \cap R)'$ (c) $(P \cap Q \cap R)'$

2. The Venn diagram shows set G, set H, set I and the universal set, ξ. List all the elements of the following sets.



(a) $(G \cap H)'$

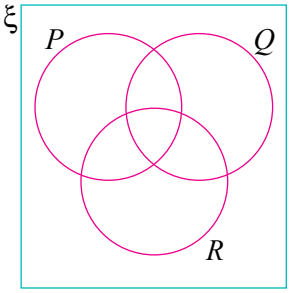
(b) $(G \cap I)'$

(c) $(H \cap I)'$

(d) $(G \cap H \cap I)'$

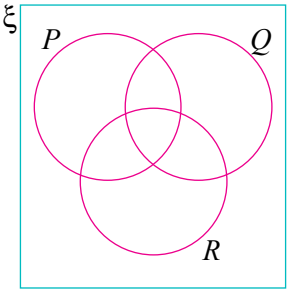
3. Shade the regions that represent the given sets.

(a)



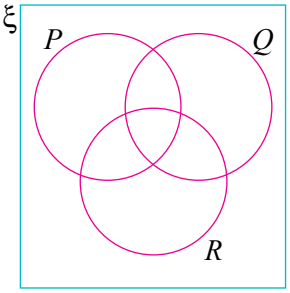
$(P \cap Q)'$

(b)



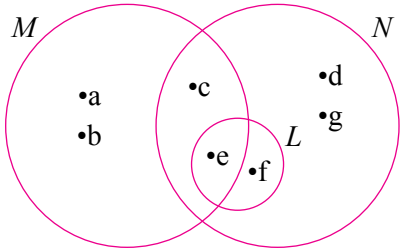
$(P \cap R)'$

(c)



$(P \cap Q \cap R)'$

4. The Venn diagram shows set L, set M, set N and the universal set, $\xi = \{a, b, c, d, e, f, g\}$.



List all the elements of the following sets.

(a) $(M \cap L)'$

(b) $(N \cap L)'$

(c) $(M \cap N)'$

(d) $(L \cap M \cap N)'$

How do you solve problems involving the intersection of sets?

Learning Standard

Solve problems involving the intersection of sets.

Example 6

A total of 140 Form 5 pupils are given the opportunity to attend the intensive classes for History and Bahasa Melayu subjects. 65 pupils choose Bahasa Melayu, 70 pupils choose History while 50 pupils choose both Bahasa Melayu and History. Calculate

(a) the total number of pupils who attend the intensive classes.
(b) the total number of pupils who do not attend any intensive classes.

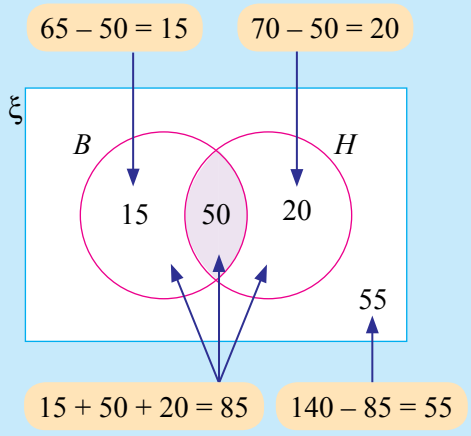
Understanding the problem

Total number of pupils = 140
Bahasa Melayu = 65
History = 70
Bahasa Melayu and History = 50
Calculate
(a) the total number of pupils who attend the intensive classes.
(b) the total number of pupils who do not attend any intensive classes.

Planning a strategy

ξ = {total number of pupils}
B = {pupils who attend Bahasa Melayu class}
H = {pupils who attend History class}
Draw a Venn diagram that represents all the given information.

Implementing the strategy



Steps:

1. Fill in $n(B \cap H) = 50$.
2. Fill in the number of pupils who attend Bahasa Melayu class only.
   $65 - 50 = 15$
3. Fill in the number of pupils who attend History class only.
   $70 - 50 = 20$
4. Pupils who attend the intensive classes
   $15 + 50 + 20 = 85$
5. Pupils who do not attend any intensive classes
   $140 - 85 = 55$

Conclusion

(a) 85 pupils attend the intensive classes.
(b) 55 pupils do not attend any intensive classes.

Example 7

A total of 200 university students take part in a survey on the use of technological devices. The result of the survey shows that 155 students have mobile phones, 90 students have laptops, 37 students have tablets, 4 students have both laptops and tablets only, 50 students have both mobile phones and laptops only, 5 students have both mobile phones and tablets only, and 83 students have mobile phones only. Calculate

(a) the total number of students who have all three technological devices.
(b) the total number of students who do not have any of the technological devices.

Understanding the problem

Total number of students = 200 Mobile phones = 155 Laptops = 90 Tablets = 37
Mobile phones and laptops only = 50 Laptops and tablets only = 4
Mobile phones only = 83 Mobile phones and tablets only = 5

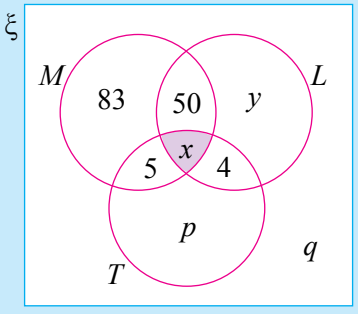
Calculate
(a) the total number of students who have all three technological devices.
(b) the total number of students who do not have any of the technological devices.

Planning a strategy

ξ = {200 students}
M = {students who have mobile phones}
L = {students who have laptops}
T = {students who have tablets}

Draw a Venn diagram to represent all the given information.

Implementing the strategy



Steps:

1. Draw the Venn diagram as shown on the left based on the given information.
2. Calculate the values of x, y, p and q.

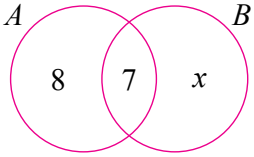
$x = 155 - 83 - 50 - 5$
$= 17$
$y = 90 - 50 - 4 - 17$
$= 19$
$p = 37 - 5 - 17 - 4$
$= 11$
$q = 200 - 83 - 50 - 5 - 4 - x - y - p$
$= 200 - 83 - 50 - 5 - 4 - 17 - 19 - 11$
$= 11$

Conclusion

(a) 17 students have all three technological devices.
(b) 11 students do not have any of the technological devices.

Self Practice 4.1c

1. The Venn diagram shows set A = {members of Art Club} and set B = {members of Science Club}. If $n(A) = 15$ and $n(B) = 22$, calculate



(a) the value of x.

(b) the total number of members of Art Club and Science Club.

2. A total of 150 pupils take part in a diagnostic test for two subjects, Science and Mathematics. The results show that 40% of the pupils passed Science and 30% of the pupils passed both subjects. If 8% of the pupils failed both subjects, calculate

(a) the number of pupils who passed Mathematics.

(b) the number of pupils who passed Science only.

(c) the number of pupils who passed Mathematics only.

3. A textile shop is holding a clearance sale. The sales shows that a total of 210 customers buy batik clothes. 70 customers buy green batik clothes only, 13 customers buy both green and blue batik clothes, 50 customers buy blue batik clothes only, 15 customers buy both green and red batik clothes and no customer buys all three colours. How many customers buy red batik clothes only?

4. It is given that ξ = {Form 4 pupils}, set K = {pupils who like to play piano} and set L = {pupils who like to play violin}. If $n(\xi) = 35$, $n(K) = 15$, $n(L) = 9$ and $n(K \cap L) = 5$, calculate the number of pupils who do not like to play both musical instruments.

5. A Badminton Club organises a competition among its members. A total of 38 members are involved in this competition. 20 members play in the double event and 26 members play in the single event. Calculate the number of members who play in both events.

4.2 Union of Sets

How do you determine and describe the union of sets using various representations?

Learning Standard

Determine and describe the union of sets using various representations.

The union of sets P and Q is written using the symbol $\cup$. $P \cup Q$ represents all the elements in set P or set Q or in both sets P and Q.

Mind Stimulation 2

Aim: To determine and describe the union of sets using various representations.

A group of pupils choose their favorite leisure activities from reading, surfing the Internet and playing sport.

Amirah likes reading.	Sofie likes surfing the Internet.
Kiran likes surfing the Internet.	Habibah likes reading.
Adeline likes surfing the Internet.	Ranjit likes playing sport.
Karim likes playing sport and surfing the Internet.	Kamal likes reading and playing sport.
Mee Yee likes reading.	Farhan likes playing sport.

Steps:

1. Divide the class into groups.
2. Each group is given A3 paper, sticky note, traffic light card and marker pens.
3. Each group reads the given situation above and complete the task.
4. Each group presents their work through Gallery Walk activity.

Task:

(a) Prepare a table as follows.

Names of pupils who like reading	Names of pupils who like surfing the Internet	Names of pupils who like playing sport
• Amirah • Mei Yee • Habibah • Kamal		

(b) Write the names of pupils representing each of the given sets using set notations.

A = {pupils who like reading}

A = {Amirah, Mei Yee, Habibah, Kamal}

B = {pupils who like surfing the Internet}

B = { , , , }

C = {pupils who like playing sport}

C = { , , , }

$A \cup B$ = {all pupils who like reading or surfing the Internet}

$A \cup B$ = {Amirah, Mei Yee, Habibah, Kamal, Kiran, Adeline, Karim, Sofie}

$B \cup C$ = {all pupils who like surfing the Internet or playing sport}

$B \cup C$ = { , , , , , , }

$A \cup C$ = {all pupils who like reading or playing sport}

$A \cup C$ = { , , , , , , }

$A \cup B \cup C$ = {all pupils who like reading, surfing the Internet or playing sport}

$A \cup B \cup C$ = { , , , , , , , , , }

(c) Draw the following unions of sets using the Venn diagram.

(i) set $A \cup B$

(ii) set $B \cup C$

(iii) set $A \cup B \cup C$

Discussion:

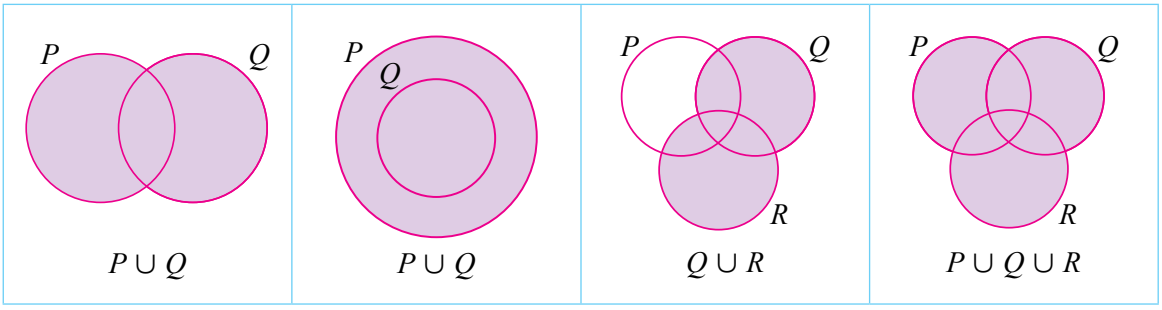
How can you determine the union of sets based on the above activity?

From the activity in Mind Stimulation 2, it is found that:

> $A \cup B \cup C$ consists of all pupils in set A, set B or set C, who are Amirah, Mei Yee, Habibah, Kamal, Karim, Kiran, Adeline, Sofie, Ranjit and Farhan.

How do you determine the union of two or more sets using the Venn diagram?

The union of two or more sets can be represented by the shaded regions in the Venn diagrams below.



Example 8

It is given that set P = {factors of 24}, set Q = {multiples of 3 which are less than 20} and set R = {multiples of 4 which are less than 20}.

(a) List all the elements of the following unions of sets.

(i) $P \cup Q$ (ii) $P \cup R$ (iii) $Q \cup R$ (iv) $P \cup Q \cup R$

(b) Draw a Venn diagram to represent sets P, Q and R, and shade the regions that represent the following unions of sets.

(i) $P \cup Q$ (ii) $P \cup Q \cup R$

Solution:

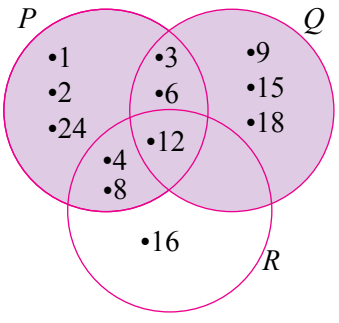
(a) (i) P = {1, 2, 3, 4, 6, 8, 12, 24}
Q = {3, 6, 9, 12, 15, 18}
$P \cup Q$ = {1, 2, 3, 4, 6, 8, 9, 12, 15, 18, 24}

(ii) P = {1, 2, 3, 4, 6, 8, 12, 24}
R = {4, 8, 12, 16}
$P \cup R$ = {1, 2, 3, 4, 6, 8, 12, 16, 24}

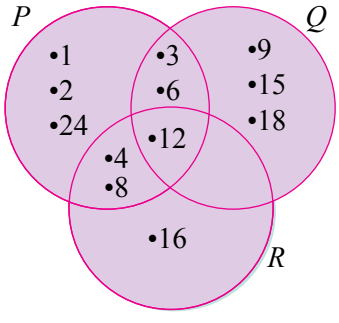
(iii) Q = {3, 6, 9, 12, 15, 18}
R = {4, 8, 12, 16}
$Q \cup R$ = {3, 4, 6, 8, 9, 12, 15, 16, 18}

(iv) P = {1, 2, 3, 4, 6, 8, 12, 24}
Q = {3, 6, 9, 12, 15, 18}
R = {4, 8, 12, 16}
$P \cup Q \cup R$ = {1, 2, 3, 4, 6, 8, 9, 12, 15, 16, 18, 24}

(b) (i)

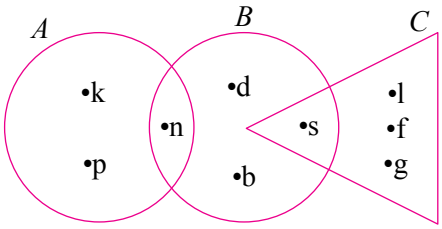


(ii)



Self Practice 4.2a

1. The Venn diagram shows set A, set B and set C such that the universal set, $\xi = A \cup B \cup C$.



List all the elements of the following sets.

(a) $A \cup B$ (b) $A \cup C$ (c) $B \cup C$ (d) $A \cup B \cup C$

2. It is given that $\xi = \{x : x \text{ is an integer, } 50 \leqslant x \leqslant 60\}$, set $P = \{x : x \text{ is a multiple of } 3\}$, set $Q = \{x : x \text{ is an odd number}\}$ and set $R = \{x : x \text{ is a prime number}\}$.

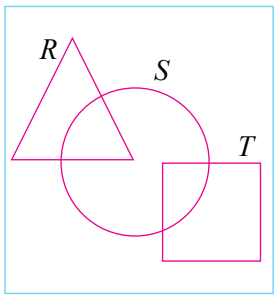
(a) Draw a Venn diagram to represent the universal set ξ, set P, set Q and set R.

(b) List all the elements of the following sets.

(i) $P \cup Q$ (ii) $P \cup R$ (iii) $Q \cup R$ (iv) $P \cup Q \cup R$

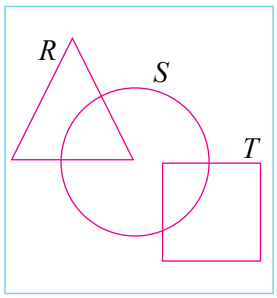
3. Shade the regions that represent the given sets.

(a) ξ



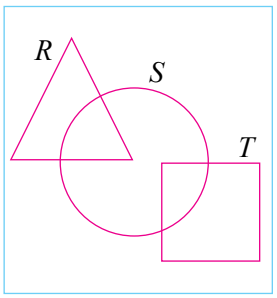
$R \cup T$

(b) ξ



$R \cup S$

(c) ξ



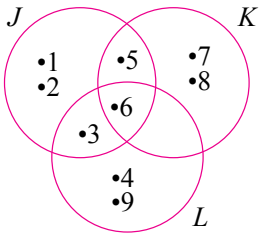
$R \cup S \cup T$

4. The Venn diagram shows set J, set K and set L such that the universal set, $\xi = J \cup K \cup L$. List all the elements of the following sets.

(a) $J \cup K$

(b) $J \cup L$

(c) $J \cup K \cup L$



How do you determine the complement of the union of sets?

The complement of the union of sets is written as $(A \cup B)'$, and is read as "the complement of the union of sets A and B".
The complement of the union of sets A and B refers to all the elements not in set A and set B.

Learning Standard
Determine the complement of the union of sets.

Example 9

Given the universal set, $\xi = \{x : x \text{ is an integer, } 50 \leqslant x \leqslant 60\}$, set $G = \{x : x \text{ is a prime number}\}$, set $H = \{x : x \text{ is a multiple of 4}\}$ and set $I = \{x : x \text{ is a multiple of 5}\}$, list all the elements and state the number of elements of the following sets.
(a) $(G \cup H)'$ (b) $(G \cup I)'$ (c) $(H \cup I)'$ (d) $(G \cup H \cup I)'$

Solution:

$\xi = \{50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60\}$
$G = \{53, 59\}$
$H = \{52, 56, 60\}$
$I = \{50, 55, 60\}$

(a) $G \cup H = \{52, 53, 56, 59, 60\}$
$(G \cup H)' = \{50, 51, 54, 55, 57, 58\}$
$n(G \cup H)' = 6$

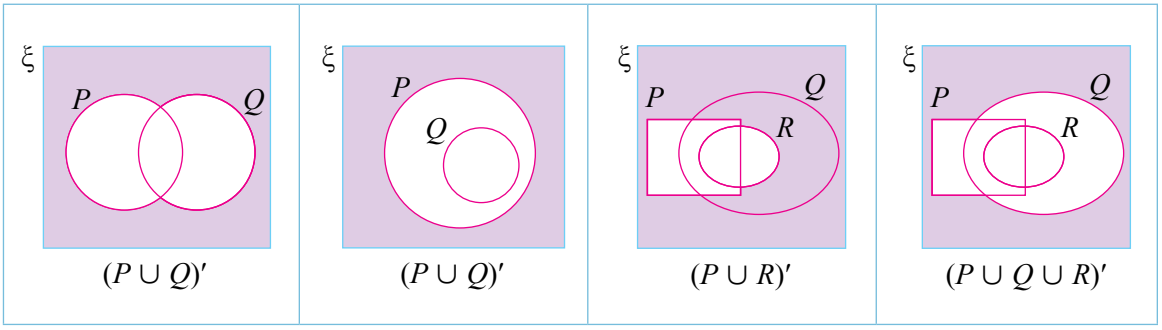
(b) $G \cup I = \{50, 53, 55, 59, 60\}$
$(G \cup I)' = \{51, 52, 54, 56, 57, 58\}$
$n(G \cup I)' = 6$

(c) $H \cup I = \{50, 52, 55, 56, 60\}$
$(H \cup I)' = \{51, 53, 54, 57, 58, 59\}$
$n(H \cup I)' = 6$

(d) $G \cup H \cup I = \{50, 52, 53, 55, 56, 59, 60\}$
$(G \cup H \cup I)' = \{51, 54, 57, 58\}$
$n(G \cup H \cup I)' = 4$

How do you determine the complements of the unions of two or more sets using Venn diagrams?

The complements of the unions of two or more sets can be represented by the shaded regions in the Venn diagrams below.



Example 10

Three private travel agencies, A, B and C, are chosen to organise the tourism exhibitions 2020 in Sarawak. Several divisions in Sarawak are chosen to hold the exhibitions as follows.

ξ = {Kapit, Miri, Bintulu, Sibu, Limbang, Mukah, Kuching, Betong}
A = {Miri, Sibu, Kuching, Betong}
B = {Miri, Sibu, Kapit, Limbang}
C = {Miri, Betong, Kapit, Mukah}

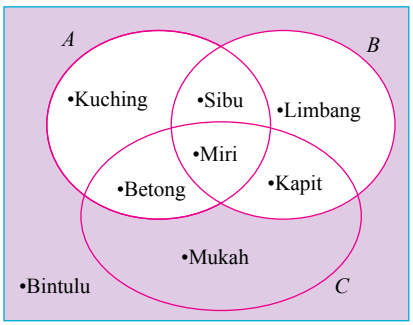
(a) List all the elements of the following sets.
(i) $(A \cup B)'$ (ii) $(B \cup C)'$ (iii) $(A \cup B \cup C)'$

(b) Draw a Venn diagram to represent sets A, B and C, and shade the region that represents each of the following complements of unions of sets.
(i) $(A \cup B)'$ (ii) $(B \cup C)'$ (iii) $(A \cup B \cup C)'$

Solution:

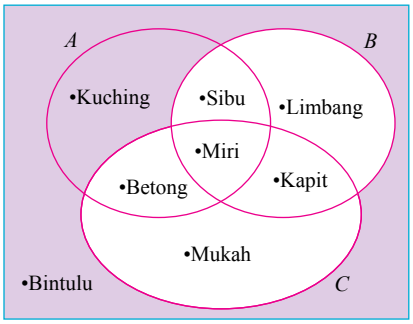
(a) (i) $A \cup B$ = {Kapit, Miri, Sibu, Limbang, Kuching, Betong}
$(A \cup B)'$ = {Mukah, Bintulu}

(b) (i)



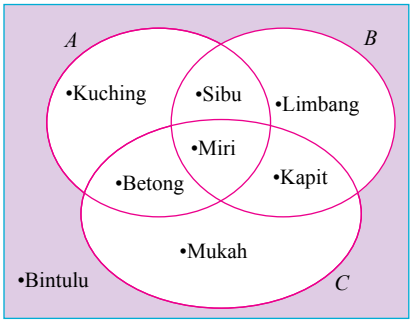
(a) (ii) $B \cup C$ = {Kapit, Miri, Sibu, Limbang, Betong, Mukah}
$(B \cup C)'$ = {Kuching, Bintulu}

(b) (ii)



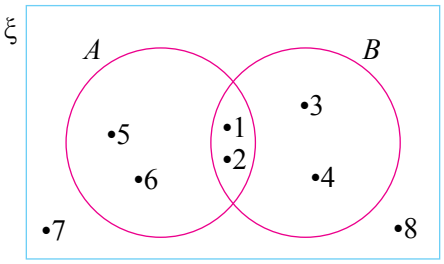
(a) (iii) $A \cup B \cup C$ = {Kapit, Miri, Sibu, Limbang, Mukah, Betong, Kuching}
$(A \cup B \cup C)'$ = {Bintulu}

(b) (iii)



Self Practice 4.2b

1. The Venn diagram shows the universal set ξ, set A and set B.



List all the elements of the following sets.

(a) A' (b) B' (c) $(A \cup B)'$

2. It is given that $\xi = \{x : x \text{ is an integer, } 10 \leqslant x \leqslant 30\}$, set $G = \{x : x \text{ is a prime number}\}$, set $H = \{x : x \text{ is a number such that the sum of its two digits is odd}\}$ and set $I = \{x : x \text{ is a multiple of } 6\}$.

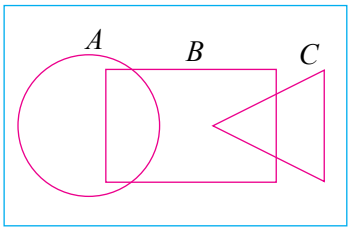
(a) Draw a Venn diagram to represent the universal set ξ , set G, set H and set I.

(b) List all the elements of the following sets.

(i) $(G \cup H)'$ (ii) $(H \cup I)'$ (iii) $(G \cup H \cup I)'$

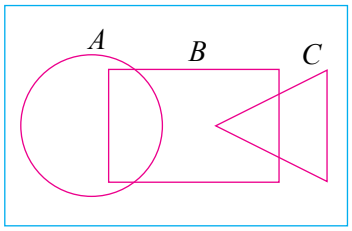
3. Shade the regions that represent the given sets.

(a) ξ



$(A \cup B)'$

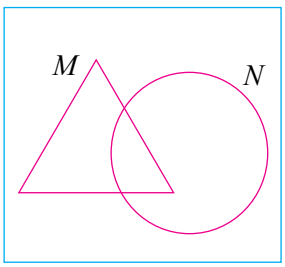
(b) ξ



$(A \cup B \cup C)'$

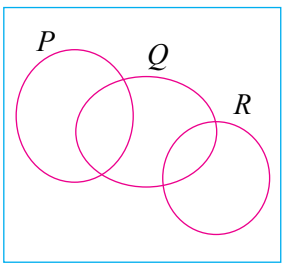
4. Shade the regions that represent the given sets.

(a) ξ



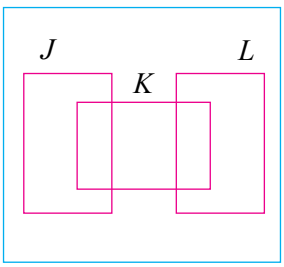
$(M \cup N)'$

(b) ξ



$(Q \cup R)'$

(c) ξ



$(J \cup K \cup L)'$

How do you solve problems involving the union of sets?

Learning Standard

Solve problems involving the union of sets.

Example 11

A total of 26 pupils participate in a scouting programme at the river bank. The activities of the programme are kayaking and fishing. 18 pupils participate in kayaking and 15 pupils participate in fishing while 9 pupils participate in both kayaking and fishing. What is the total number of pupils who participate in the activities of the programme?

Understanding the problem

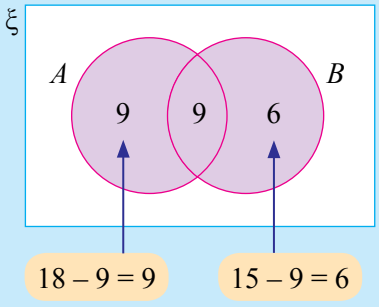
Total number of pupils = 26
Kayaking = 18
Fishing = 15
Kayaking and fishing = 9
Calculate the total number of pupils who participate in the activities of the programme.

Planning a strategy

ξ = {total number of pupils}
A = {pupils who participate in kayaking}
B = {pupils who participate in fishing}

Draw a Venn diagram to represent all the given information. Calculate
(a) the total number of pupils who participate in kayaking only.
(b) the total number of pupils who participate in fishing only.
(c) the total number of pupils who do not participate in the activities of the programme.
(d) the total number of pupils who participate in the activities of the programme, $n(A \cup B)$.

Implementing the strategy



(a) Kayaking only = 18 – 9
= 9

(b) Fishing only = 15 – 9
= 6

(c) Total number of pupils who do not participate in the activities of the programme
= 26 – 9 – 9 – 6
= 2

(d) Total number of pupils who participate in the activities of the programme, $n(A \cup B)$ (Shaded region)
= 26 – 2 = 24

Checking Answer

9 + 9 + 6 = 24

Conclusion

24 pupils participate in the activities of the programme, $n(A \cup B) = 24$

Example 12

A total of 100 adults are involved in a survey on their top choices of reading materials. 40 people choose newspapers, 25 people choose magazines, 18 people choose storybooks, 8 people choose both newspapers and magazines, 7 people choose both magazines and storybooks, 5 people choose both newspapers and storybooks, and 3 people choose all three types of reading materials. How many people do not choose any of the reading materials?

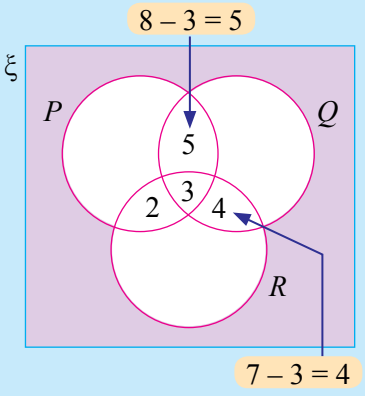
Understanding the problem

Total number of adults = 100 Newspapers = 40 Magazines = 25 Storybooks = 18
Newspapers and magazines = 8
Magazines and storybooks = 7
Newspapers and storybooks = 5
Newspapers, magazines and storybooks = 3
Calculate the number of people who do not choose any of the reading materials.

Planning a strategy

ξ = {total number of adults}
P = {newspapers} Q = {magazines} R = {storybooks}
Draw a Venn diagram to represent all the given information and calculate
(a) the total number of people who choose newspapers only.
(b) the total number of people who choose magazines only.
(c) the total number of people who choose storybooks only.
(d) the total number of people who do not choose any of the reading materials, $n(A \cup B \cup C)'$

Implementing the strategy



(a) Newspapers only
$= 40 - 5 - 3 - 2$
$= 30$

(b) Magazines only
$= 25 - 5 - 3 - 4$
$= 13$

(c) Storybooks only
$= 18 - 3 - 4 - 2$
$= 9$

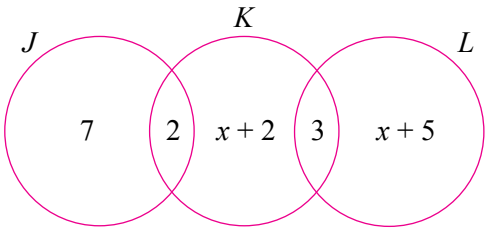
(d) Total number of people who do not choose any of the reading materials,
$n(A \cup B \cup C)' = 100 - 5 - 3 - 4 - 2 - 30 - 13 - 9$
$= 34$

Conclusion

34 people do not choose any of the reading materials.
$n(A \cup B \cup C)' = 34$

Self Practice 4.2c

1. The Venn diagram shows the number of elements in set J, set K and set L. Given $\xi = J \cup K \cup L$ and $n(\xi) = 25$, calculate the value of x.



2. It is given that set B = {pupils from Sabah} and set T = {pupils from Sarawak}. If there are 40 pupils in a class and $n(B \cup T) = 32$, calculate the number of pupils who are not from any of the states.

3. During the Independence Month celebration, the school organises a drama activity, a patriotic song singing activity and a History quiz. 40 pupils are involved in the activities. $\frac{1}{2}$ of the pupils participate in drama, $\frac{1}{4}$ of the pupils participate in singing, 6 pupils participate in drama and quiz and one pupil participates in all three activities. There is no pupil who participates in drama and singing only, and also no pupil participates in singing and quiz only. How many pupils participate in the History quiz only?

4. A bookstore conducts a survey for 200 customers on whether they buy fiction or non-fiction books. The survey shows that 114 customers buy non-fiction books, 52 customers buy fiction books and 27 customers buy both fiction and non-fiction books. Calculate

(a) the number of customers who buy fiction books only.

(b) the number of customers who buy non-fiction books only.

(c) the number of customers who do not buy any types of books.

5. The Form 4 pupils who are involved in Recycling Programme manage to collect old newspapers, plastic bottles and tins. 72 pupils collect plastic bottles, 36 pupils collect old newspapers, 25 pupils collect tins, 20 pupils collect old newspapers and plastic bottles, 8 pupils collect old newspapers and tins, 18 pupils collect plastic bottles and tins, and 7 pupils collect all the three types of materials. Calculate the total number of pupils who are involved in the programme.

4.3 Combined Operations on Sets

How do you determine and describe combined operations on sets using various representations?

Learning Standard

Determine and describe the combined operations on sets using various representations.

Combined operations on sets involve both intersection of sets ($\cap$) and union of sets ($\cup$) at the same time. Combined operations of sets are solved from left to right. However, if there are operations in brackets, the operations in brackets must be carried out first.

Example 13

The table below shows the hobbies of a group of pupils.

Arif likes singing.	Iris likes drawing, singing and dancing.
Zarif likes drawing.	Alan likes singing and dancing.
Lily likes dancing.	May likes dancing and drawing.
Emy likes dancing and singing.	Jay likes singing and drawing.
Getha likes drawing.	Nani likes dancing.

P = {pupils who like singing}, Q = {pupils who like dancing} and R = {pupils who like drawing}.

Based on the above table,

(a) list all the elements of the following sets.

(i) $(P \cup Q) \cap R$ (ii) $Q \cup (P \cap R)$

(b) draw a Venn diagram and shade the region that represents each of the following sets.

(i) $(P \cup Q) \cap R$ (ii) $Q \cup (P \cap R)$

Solution:

P = {Arif, Emy, Iris, Alan, Jay}

Q = {Lily, Emy, Iris, Alan, May, Nani}

R = {Zarif, Getha, Iris, May, Jay}

(a) (i) $(P \cup Q)$ = {Arif, Emy, Iris, Alan, Jay, Lily, May, Nani}

R = {Zarif, Getha, Iris, May, Jay}

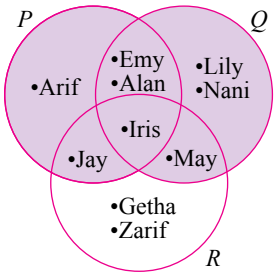
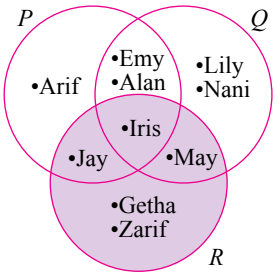
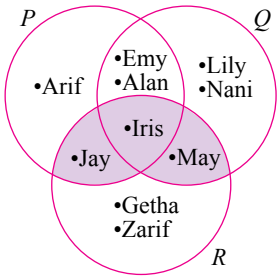
$(P \cup Q) \cap R$ = {Jay, Iris, May}

(ii) Q = {Lily, Emy, Iris, Alan, May, Nani}

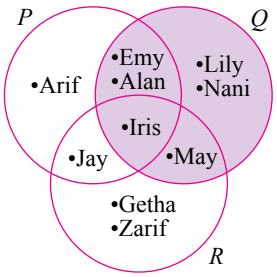
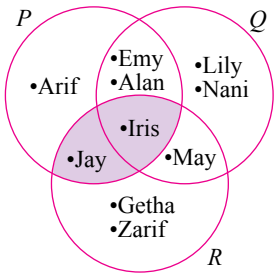
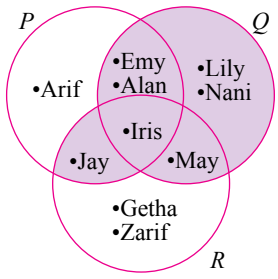
$(P \cap R)$ = {Iris, Jay}

$Q \cup (P \cap R)$ = {Lily, Emy, Iris, Alan, May, Nani, Jay}

(b) (i) Shade the region for set $(P \cup Q)$.
Shade the region for set R.
Set $(P \cup Q) \cap R$ refers to the common region for both sets $(P \cup Q)$ and R.

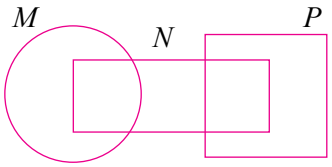
$(P \cup Q)$	R	$(P \cup Q) \cap R$
		

(ii) Shade the region for set Q.
Shade the region for set $(P \cap R)$.
Set $Q \cup (P \cap R)$ refers to all the regions covered by both sets Q and $(P \cap R)$.

Q	$(P \cap R)$	$Q \cup (P \cap R)$
		

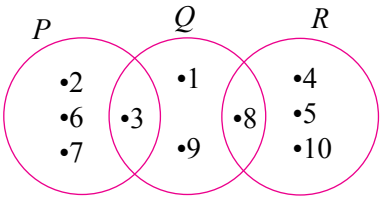
Self Practice 4.3a

1. The Venn diagram shows sets M, N and P such that the universal set, $\xi = M \cup N \cup P$. Shade set $(M \cup P) \cap N$.



2. Given the universal set, $\xi = \{x : x \text{ is an integer}, 1 \leqslant x \leqslant 15\}$, set $S = \{x : x \text{ is an odd number}\}$, set $R = \{x : x \text{ is a prime number}\}$ and set $T = \{1, 4, 7, 10, 13\}$, list all the elements of set $(S \cup T) \cap R$.

3. The Venn diagram shows sets P, Q and R such that the universal set, $\xi = P \cup Q \cup R$. List all the elements of the following sets.



(a) $P \cap (Q \cup R)$

(b) $Q \cap (P \cup R)$

(c) $(Q \cap R) \cup P$

How do you determine the complement of combined operations on sets?

The complement of combined operations on sets involves the complement of either intersection of sets ($\cap$) or union of sets ($\cup$) or both intersections. The complement of combined operations on sets is solved from left to right. However, if there are operations in brackets, the operations in brackets must be carried out first.

Learning Standard

Determine the complement of combined operations on sets.

Example 14

It is given that $\xi = \{x : x \text{ is an integer, } 30 \leqslant x \leqslant 40\}$, set $A = \{x : x \text{ is a multiple of } 3\}$, set $B = \{x : x \text{ is a number such that the sum of its two digits is odd}\}$ and set $C = \{30, 32, 35, 39, 40\}$.

(a) List all the elements of the following sets.

(i) $(A \cup B)' \cap C$ (ii) $A' \cap (B \cup C)$ (iii) $(A \cap C)' \cup (B \cap C)$

(b) Draw a Venn diagram and shade the region that represents each of the following sets.

(i) $(A \cup B)' \cap C$ (ii) $A' \cap (B \cup C)$ (iii) $(A \cap C)' \cup (B \cap C)$

Solution:

$\xi = \{30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40\}$
$A = \{30, 33, 36, 39\}$
$B = \{30, 32, 34, 36, 38\}$
$C = \{30, 32, 35, 39, 40\}$

MY MEMORY

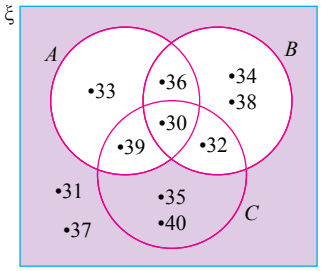
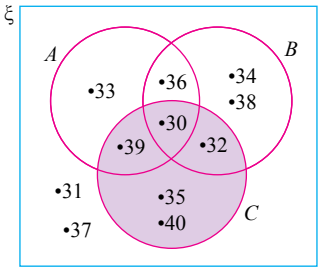
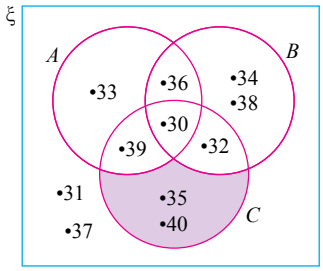
For a set A in the universal set, the complement of set A, is written as A', meaning all the elements that are not in set A.

(a) (i) $(A \cup B)' = \{31, 35, 37, 40\}$
$C = \{30, 32, 35, 39\ 40\}$
$(A \cup B)' \cap C = \{35, 40\}$

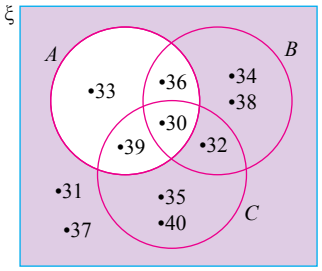
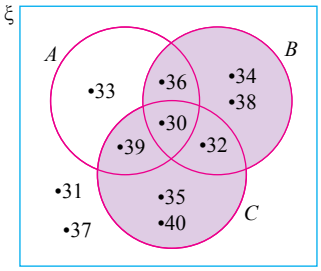
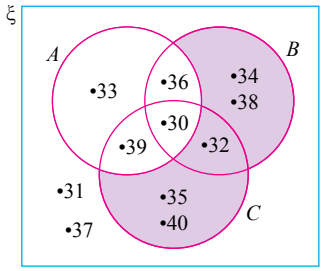
(ii) $A' = \{31, 32, 34, 35, 37, 38, 40\}$
$(B \cup C) = \{30, 32, 34, 35, 36, 38, 39, 40\}$
$A' \cap (B \cup C) = \{32, 34, 35, 38, 40\}$

(iii) $(A \cap C)' = \{31, 32, 33, 34, 35, 36, 37, 38, 40\}$
$(B \cap C) = \{30, 32\}$
$(A \cap C)' \cup (B \cap C) = \{30, 31, 32, 33, 34, 35, 36, 37, 38, 40\}$

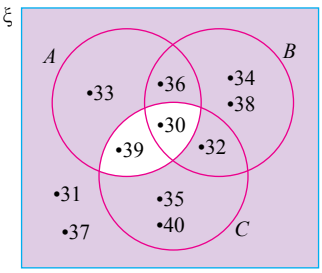
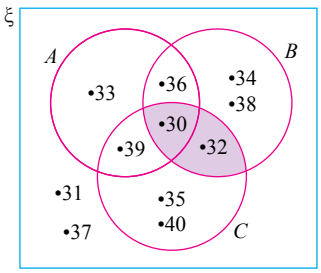
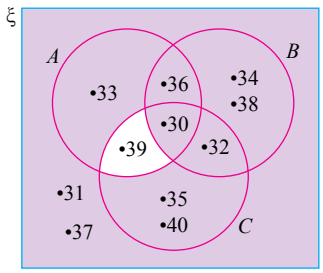
(b) (i) Shade the region for set $(A \cup B)'$.
Shade the region for set C.
Set $(A \cup B)' \cap C$ refers to the common region for both sets $(A \cup B)'$ and C.

$(A \cup B)'$	C	$(A \cup B)' \cap C$
		

(ii) Shade the region for set A'.
Shade the region for set $(B \cup C)$.
Set $A' \cap (B \cup C)$ refers to the common region for both sets A' and $(B \cup C)$.

A'	$B \cup C$	$A' \cap (B \cup C)$
		

(iii) Shade the region for set $(A \cap C)'$.
Shade the region for set $(B \cap C)$.
Set $(A \cap C)' \cup (B \cap C)$ refers to all the regions covered by both sets $(A \cap C)'$ and $(B \cap C)$.

$(A \cap C)'$	$B \cap C$	$(A \cap C)' \cup (B \cap C)$
		

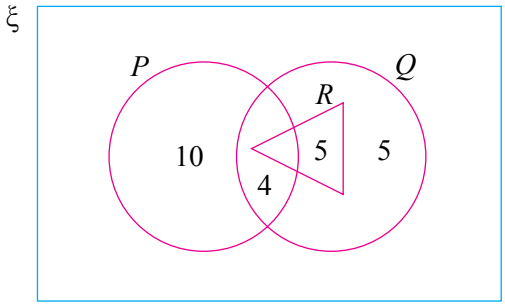
Self Practice 4.3b

1. Given $\xi = \{x : x \text{ is an integer, } 10 \leqslant x \leqslant 20\}$, set $L = \{x : x \text{ is a multiple of } 2\}$, set $M = \{13, 16, 19\}$ and set $N = \{x : x \text{ is a multiple of } 5\}$, list all the elements of the following sets.

(a) $L' \cap (M \cup N)$

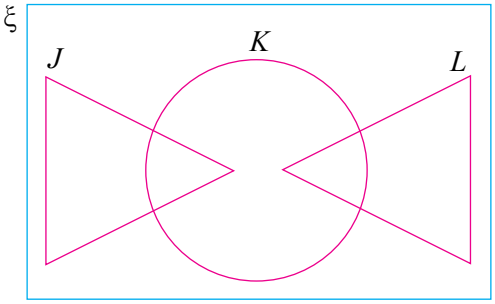
(b) $(M \cup N)' \cap L$

2. The incomplete Venn diagram shows the number of elements in sets P, Q and R. It is given that $n(P \cap Q) = n(P \cup Q)'$ and $n(\xi) = 50$. Determine $n(P)$.

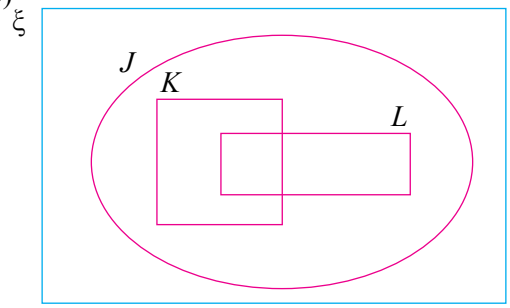


3. Shade the region that represents set $(J \cap K)' \cap (K \cup L)$ on each Venn diagram below.

(a)



(b)

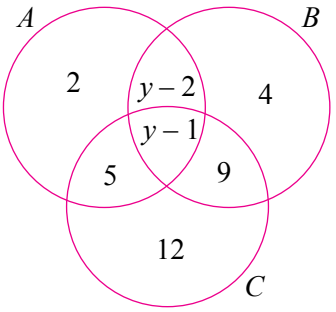


4. The Venn diagram shows the universal set, $\xi = A \cup B \cup C$ and $n(B') = n(B \cap C)$.

Determine

(a) the value of y.

(b) $n(A \cup B \cup C)$.



How do you solve problems involving combined operations on sets?

Learning Standard

Solve problems involving combined operations on sets.

Example 15

The Residents' Association of Happy Garden organises various sports competitions to instil health awareness among residents. A total of 35 participants join the football competition, 24 participants join the table tennis competition and 13 participants join the badminton competition. There are 4 participants who join both the football and table tennis competitions, 8 participants who join both the table tennis and badminton competitions, and 2 participants join all three competitions. There is no participant joining the badminton and football competitions only. Calculate the total number of participants who join one competition only.

Understanding the problem

Total number of participants = ?
Football = 35 Table tennis = 24 Badminton = 13
Football and table tennis only = 4 Badminton and football only = 0
Table tennis and badminton only = 8 Football, table tennis and badminton = 2
Calculate the total number of participants who join one competition only.

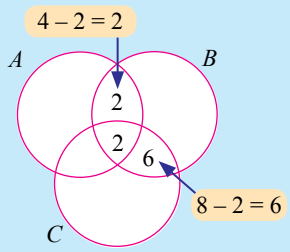
Planning a strategy

ξ = {all participants}
A = {participants who join the football competition}
B = {participants who join the table tennis competition}
C = {participants who join the badminton competition}

Draw a Venn diagram to represent all the given information.

Calculate
(a) the total number of participants who join the football competition only.
(b) the total number of participants who join the badminton competition only.
(c) the total number of participants who join the table tennis competition only.
(d) the total number of participants who join one competition only.

Implementing the strategy



(a) Football only
$= 35 - 4$
$= 31$

(b) Badminton only
$= 13 - 6 - 2$
$= 5$

(c) Table tennis only
$= 24 - 6 - 2 - 2$
$= 14$

(d) Total number of participants who join one competition only
$= 31 + 14 + 5$
$= 50$

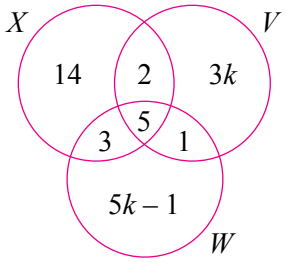
Conclusion

50 participants join one competition only.

Self Practice 4.3c

1. A supermarket initiates a campaign "No Plastic Bag Day". Customers need to buy plastic bags or recycle bags to pack the items bought. Among a total of 90 customers, 51 customers buy plastic bags, 48 customers buy recycle bags, and 9 customers buy both plastic and recycle bags. Calculate the number of customers who do not use plastic bags.

2. The Venn diagram shows the number of elements in sets X, V and W. It is given that $\xi = X \cup V \cup W$ and $n(\xi) = 56$. Determine the value of k.



3. A total of 100 children need to choose their favourite food from burger, salad and *nasi lemak*. 50 children choose burger, 60 children choose *nasi lemak*, 5 children choose burger and salad, 3 children choose *nasi lemak* and salad, and 22 children choose burger and *nasi lemak*. If only one child chooses all the three types of food, calculate the number of children who choose salad only.

4. Based on a survey among 100 adults, 68 of them own national cars while 52 of them own imported cars. If 27 adults own both the imported and national cars, calculate the number of adults who

(a) own national cars only.

(b) own imported cars only.

(c) do not own any cars.

Comprehensive Practice

1. It is given that the universal set, $\xi = \{2, 3, 5, 6\}$, set $P = \{3, 5\}$, set $Q = \{2, 3, 5\}$ and set $R = \{3, 6\}$. List all the elements of the following sets.

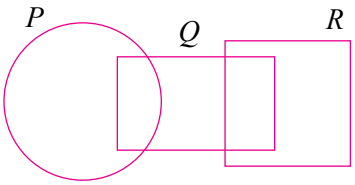
(a) $P \cap Q$ (b) $P \cap R$ (c) $P \cap Q \cap R$ (d) $(P \cap Q \cap R)'$

2. Given set $M = \{b, a, i, k\}$, set $N = \{b, u, d, i\}$ and set $P = \{b, e, r, a, n, i\}$, list all the elements of the following sets.

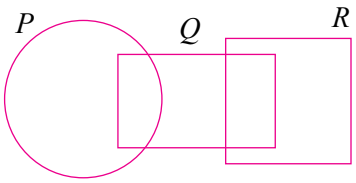
(a) $M \cup N$ (b) $M \cup P$ (c) $M \cup N \cup P$

3. Shade the regions that represent the given sets, such that the universal set, $\xi = P \cup Q \cup R$.

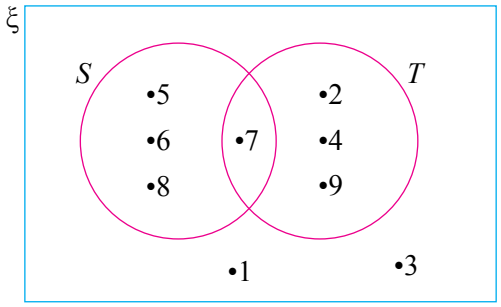
(a) $P \cap Q$



(b) $P \cup R$



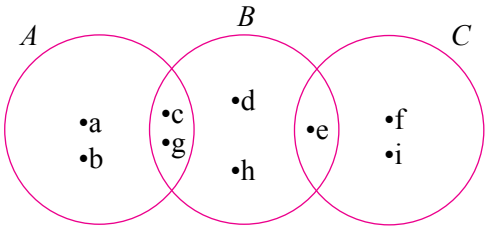
4. The Venn diagram shows the universal set ξ, set S and set T.



List all the elements of the following sets.

(a) T' (b) $S \cup T$ (c) $S' \cap T$ (d) $(S \cap T)'$

5. The Venn diagram shows the elements of sets A, B and C. Given the universal set, $\xi = A \cup B \cup C$, list all the elements of set A'.

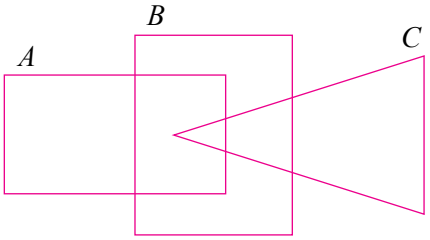


6. It is given that $\xi = \{x : x \text{ is an integer, } 10 \leqslant x \leqslant 30\}$, set $P = \{x : x \text{ is a prime number}\}$, set $Q = \{x : x \text{ is a multiple of } 5\}$ and set $R = \{x : x \text{ is a factor of } 24\}$. List all the elements of the following sets.

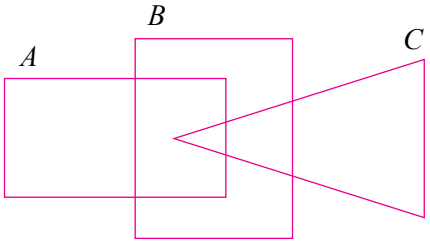
(a) Q' (b) $P \cup R'$ (c) $(P \cup R)' \cap Q$

7. Shade the regions to represent the following sets, such that the universal set, $\xi = A \cup B \cup C$.

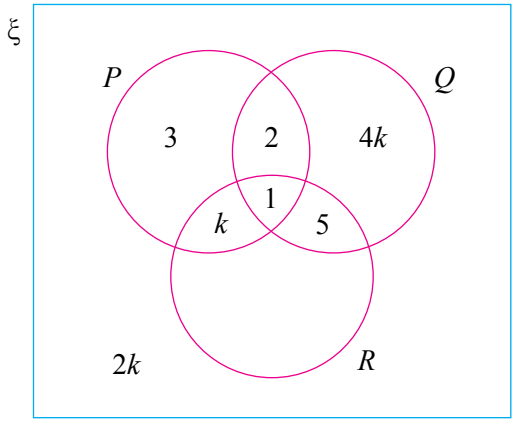
(a) $A \cap (B \cup C)$



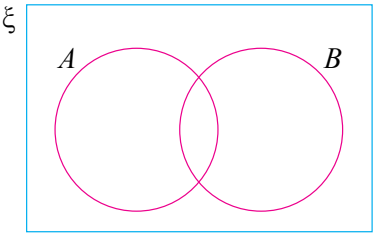
(b) $C \cup (A \cap B)'$



8. The Venn diagram shows the universal set ξ, set P, set Q and set R. Given $n(Q) = n(P \cup R)'$, determine $n(\xi)$.



9. The Venn diagram on the right shows the universal set, $\xi = \{\text{Form 4 pupils}\}$, set $A = \{\text{members of Music Club}\}$ and set $B = \{\text{members of Robotic Club}\}$. Given $n(\xi) = 58$, $n(A) = 20$, $n(B) = 16$ and $n(A \cap B) = 9$, determine the number of pupils who are not members of both clubs.



10. A total of 55 pupils are required to choose between two activities during school holidays. 28 pupils choose sports activities while 21 pupils choose community service activities. If 12 pupils do not choose any of the activities, how many pupils choose both activities?

11. The thrifty habit among teenagers is influenced by their families, society and mass media. 80 teenagers participate in a survey. It is found that 30 teenagers are influenced by their families, 15 teenagers are influenced by their families and society, 9 teenagers are influenced by society and mass media, 7 teenagers are only influenced by their families, and 3 teenagers are influenced by all the three factors. The ratio of the teenagers who are influenced by society only to the teenagers influenced by mass media only is 3 : 1. Calculate the number of teenagers who are influenced by

(a) two factors, which are their families and mass media only.

(b) mass media only.

(c) society.

12. The History Society introduces traditional games to its 40 members. 17 members play *ceper*, 25 members play *batu seremban*, 18 members play *congkak*, 8 members play *ceper* and *batu seremban*, 12 members play *batu seremban* and *congkak*, and 3 members play *ceper*, *batu seremban* and *congkak*.

If the number of members who play *batu seremban* only is twice the number of members who play *congkak* only, calculate the number of members who

(a) play *batu seremban* only.

(b) play both *ceper* and *congkak*.

(c) play *ceper* only.

(d) are not involved in these traditional games.

13. A study of 80 pupils was conducted on the modes of transportation when they travel back to their hometowns. 25 pupils travel by train and 48 pupils travel by train or car. If 7 pupils travel by train and car, 5 pupils travel by bus and train, and 2 pupils travel by all three modes of transportation, how many pupils do travel by bus or train but not by car?

PROJECT

Discuss three fields of careers that will be your choice in the future.

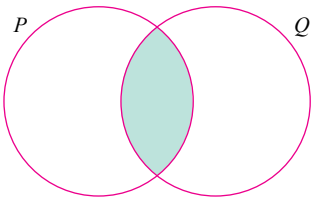
1. List the three fields of careers.
2. Give examples of jobs in the fields of careers.
3. What are the subjects that are important in the above fields of careers? Explain your answers with the help of Venn diagrams.

CONCEPT MAP

Operations on Sets

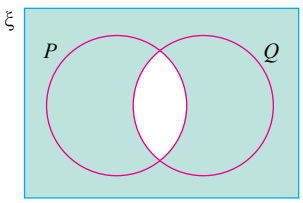
Intersection of Sets

The intersection of set P and set Q is written using the symbol $\cap$. $P \cap Q$ refers to both sets P and Q have common elements.



$P \cap Q$

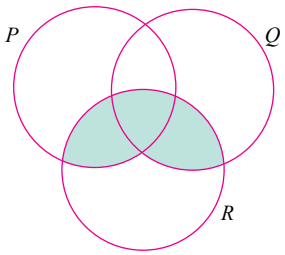
The complement of the intersection of sets P and Q refers to all the elements which are not in the intersection of sets P and Q.



$(P \cap Q)'$

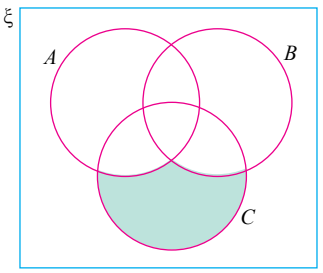
Combined Operations on Sets

Combined operations on sets involve both the intersection of sets ($\cap$) and union of sets ($\cup$) at the same time.



$(P \cup Q) \cap R$

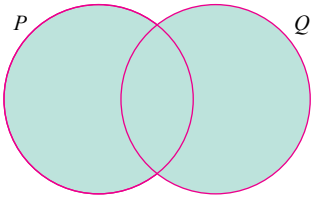
The complement of combined operations on sets involves the complements of either intersection of sets ($\cap$) or union of sets ($\cup$) or both intersection of sets ($\cap$) and union of sets ($\cup$) at the same time.



$(A \cup B)' \cap C$

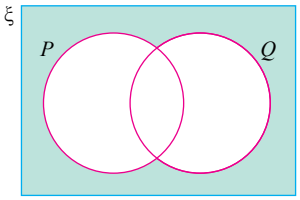
Union of Sets

$P \cup Q$ represents all the elements in set P, set Q or both sets P and Q.



$P \cup Q$

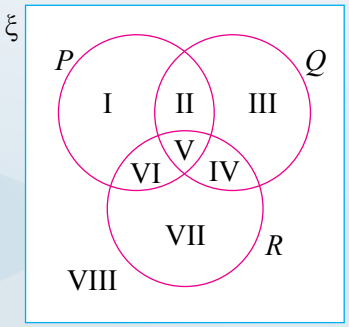
The complement of the union of set P or set Q refers to all the elements which are not in set P or set Q.



$(P \cup Q)'$

Self Reflection

The Venn diagram shows the universal set ξ, set P, set Q and set R with labelled regions. Answer all the questions by stating the regions that represent the following sets.



1. P: Regions I, II, V, VI
2. Q'
3. R
4. $P \cap Q$
5. $Q \cap R$
6. $P \cap Q \cap R$
7. $(P \cap Q \cap R)'$
8. $P \cap (Q \cup R)$
9. $Q \cap (P \cup R)$
10. $P \cup (Q \cap R)$
11. $P \cup (Q \cap R)'$
12. $Q \cup (P \cap R)'$
13. $P \cup Q \cup R$
14. $(P \cup Q \cup R)'$
15. $P' \cap (Q \cup R)$
16. $Q' \cap (P \cap R)'$
17. $R' \cup (P \cap Q)'$
18. $(P \cap Q) \cup (Q \cap R)$
19. $(P \cap Q)' \cup (Q \cap R)$
20. $(P \cap Q)' \cup (Q \cup R)'$

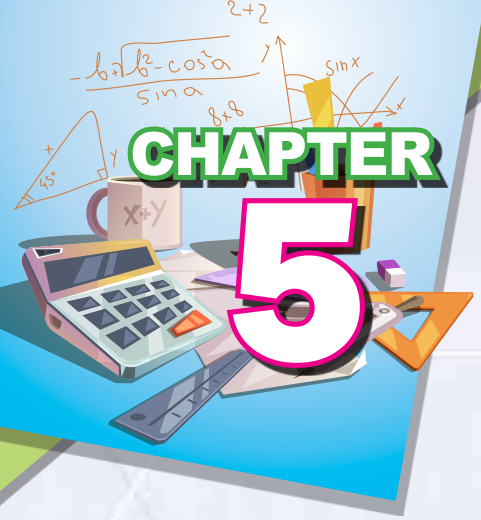
Mathematics Exploration

Based on the given food pyramid, draw two Venn diagrams to show the balanced diet for breakfast, lunch and dinner.

Steps

1. Divide the class into groups.
2. Choose the food needed for breakfast, lunch and dinner.
3. Draw Venn diagrams based on the food chosen from the food pyramid. It is given that
   K = {breakfast},
   L = {lunch} and
   M = {dinner}.
4. Present your group's findings using Three Stray, One Stay activity.
5. Give your opinions on the appropriateness of the other groups' food options for breakfast, lunch and dinner.

Food Pyramid



CHAPTER 5

Network in Graph Theory

You will learn

- Network

The transportation system in Malaysia which comprises land, water and air transportation are developed in line with the country's progress. The land transportation system, particularly public transport is developed at a fast pace based on the increase in the number of commuters in big cities. City rail services such as Light Rail Transit (LRT), Mass Rapid Transit (MRT), monorails and commuter trains in the cities are amongst the types of public transportation that are increasingly well accepted by the public.

Do you know that the transportation system is a type of network?

Why Study This Chapter?

A network is used to link objects in the same field based on its needs. Networks are widely used in transportation, computer, social, business, investigation, medicine, science, neuroscience, social science and gaming fields.

WORD BANK

• weighted	• *berpemberat*
• vertex	• *bucu*
• degree	• *darjah*
• discrete	• *diskret*
• loop	• *gelung*
• graph	• *graf*
• simple	• *mudah*
• tree	• *pokok*
• network	• *rangkaian*
• subgraph	• *subgraf*
• edge	• *tepi*
• directed	• *terarah*

Walking Through Time

Leonhard Euler
(1707 – 1783)

Leonhard Euler, a mathematician in the 18th century, was born in Basel, Switzerland. In the year 1735, Euler solved a mathematics and logic problem known as the Seven Bridges of Königsberg and developed a mathematical structure called graph – a diagram consisting of dots (vertices) which are linked by lines or arcs (edges).

http://bt.sasbadi.com/m4129

5.1 Network

What is the relationship between a network and a graph?

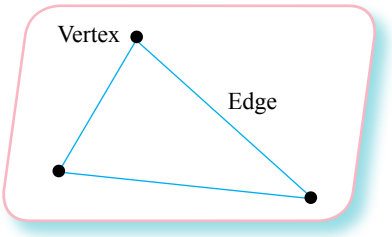
Learning Standard

To identify and explain a network as a graph.

In Year 5, you were introduced to systems of computer networks and the world of Internet through the subject of Information and Communication Technology.

The link between a group of computers and the associated devices, that is a computer network, enables information to be searched, used and shared easily. Do you know the relationship between a network and a graph?

A graph is used to represent data consisting of discrete objects and to show the relationship between these objects in a simple graphical manner. In the field of mathematics, graph theory in particular, a **graph** is interpreted as a series of dots which are either linked or not linked to one another by lines. Each dot is known as a **vertex** and the line joining two vertices is known as an **edge**.



A graph is usually used to represent a certain network. A **network** is part of a graph with the vertices and edges having their own characteristics. The structure of network data has a many-to-many relation. Examples of graphs that involve networks are as follows.

Land transport network

Vertex
Regions, towns, cities or certain buildings that are linked

Edge
Roads, highways or railway lines

Social network

Vertex
Individuals, groups or organisations

Edge
Types of relationships such as friends, colleagues or families

INFO ZONE

A Cold Chain System is a system used in the world of medicine. The function of this system is to transport, distribute and store vaccine and blood in a fixed temperature range from the source to the place they are used.

A graph is denoted by a set of ordered pairs $G = (V, E)$, where

- V is the set of dots or vertices.
  $V = \{v_1, v_2, v_3, \ldots v_n\}$
- E is the set of edges or lines linking each pair of vertices.
  $E = \{e_1, e_2, e_3, \ldots e_n\}$
  $E = \{(a_1, b_1), (a_2, b_2), \ldots (a_n, b_n)\}$; a and b are pairs of vertices.

INFO ZONE

G = Graph
v = Vertex or dot
e = Edge or line or arc
d = Degree
Σ = Sum

The **degree**, ***d***, is the number of edges that connect two vertices. The sum of degrees of a graph is twice the number of edges, that is

$$\Sigma d(v) = 2E;\ v \in V$$

MY MEMORY

$\in$: an element of

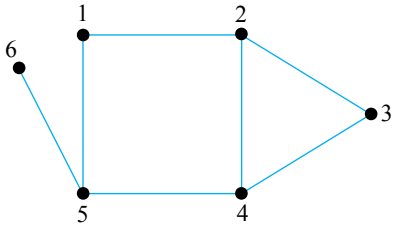
What do you understand about a simple graph?

A simple graph has no loops and no multiple edges. The sum of degrees of the graph is twice the number of edges.

Example 1

Based on the simple graph given, determine

(a) V and $n(V)$
(b) E and $n(E)$
(c) sum of degrees.



Solution:

(a) $V = \{1, 2, 3, 4, 5, 6\}$ ← Set of vertices
$n(V) = 6$ ← Number of vertices

(b) $E = \{(1, 2), (1, 5), (2, 3), (2, 4), (3, 4), (4, 5), (5, 6)\}$ ← Set of vertex pairs
$n(E) = 7$ ← Number of edges

(c) Sum of degrees
$\Sigma d(v) = 2(E)$
$= 2(7)$
$= 14$

Degree of vertex: $d(1) = 2$
$d(2) = 3$
$d(3) = 2$
$d(4) = 3$
$d(5) = 3$
$d(6) = 1$
Sum = 14

The degree of vertex 1 is two, that is the edges which connect vertex 1 to vertex 2 and vertex 1 to vertex 5.

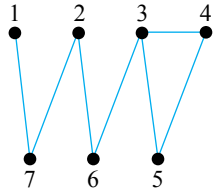
INFO ZONE

The edge for the vertex pair (1, 2) is also the edge for the vertex pair (2, 1).

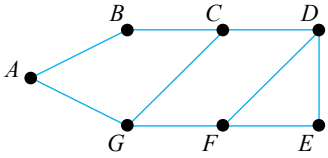
Example 2

State the number of vertices, edges and the sum of degrees for the following simple graphs.

(a)



(b)



Solution:

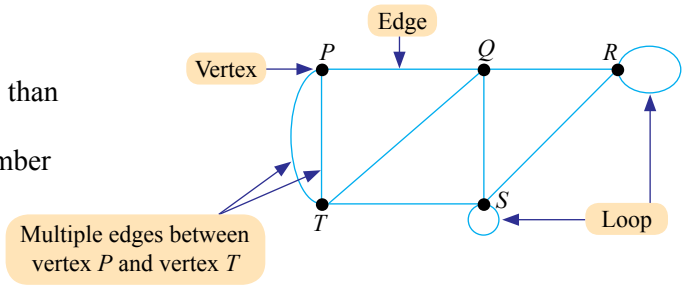
(a) Number of vertices = 7
Number of edges = 7
Sum of degrees = 2 × Number of edges
= 2 × 7
= 14

(b) Number of vertices = 7
Number of edges = 9
Sum of degrees = 2 × Number of edges
= 2 × 9
= 18

What is the meaning of multiple edges and loops of a graph?

Multiple edges

- Involve two vertices.
- The vertices are connected by more than one edge.
- The sum of degrees is twice the number of edges.



Loops

- Involve one vertex.
- The edge is in the form of an arc that starts and ends at the same vertex.
- Each loop adds 2 to the degree.

Let the graph given be denoted by a set of ordered pairs, $G(V, E)$, then

$V = \{A, B, C, D\}$

Multiple edges

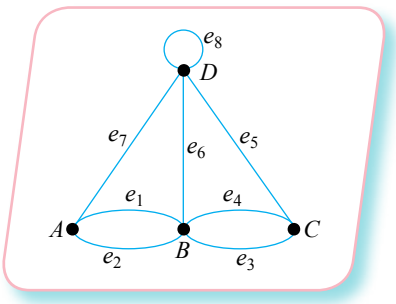
$E = \{(A, B), (A, B), (B, C), (B, C), (C, D), (B, D), (A, D), (D, D)\}$

A loop

$E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}$

Second edge of AB

First edge of AB



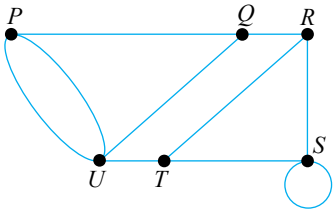
Example 3

The diagram on the right shows a graph with a loop and multiple edges. State

(a) V and $n(V)$

(b) E and $n(E)$

(c) sum of degrees.



Solution:

(a) $V = \{P, Q, R, S, T, U\}$
$n(V) = 6$

(b) $E = \{(P, Q), (P, U), (P, U), (Q, R), (Q, U), (R, S), (R, T), (S, S), (S, T), (T, U)\}$
$n(E) = 10$

MY MEMORY

V = Set of vertices
E = Set of edges

(c) Sum of degrees = 20

Degree of vertex P = 3
Degree of vertex Q = 3
Degree of vertex R = 3
Degree of vertex S = 4
Degree of vertex T = 3
Degree of vertex U = 4

The sum of degrees is 20.

TIPS

The degree of a vertex with a loop in an undirected graph is 2, one in clockwise direction and the other in anticlockwise direction.

Example 4

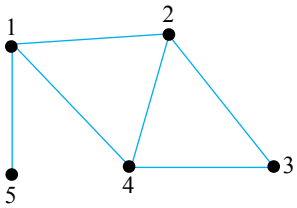
Draw a simple graph based on the given information.

(a) $V = \{1, 2, 3, 4, 5\}$
$E = \{(1, 2), (1, 4), (1, 5), (2, 3), (2, 4), (3, 4)\}$

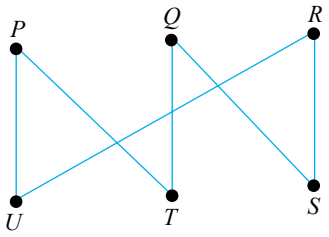
(b) $V = \{P, Q, R, S, T, U\}$
$E = \{(P, U), (P, T), (Q, T), (Q, S), (R, S), (R, U)\}$

Solution:

(a)



(b)



Example 5

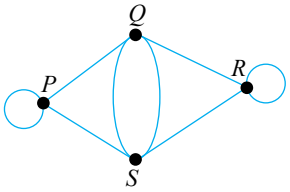
Draw a graph with multiple edges and loops based on the given information.

(a) $V = \{P, Q, R, S\}$
$E = \{(P, P), (P, Q), (P, S), (Q, S), (Q, S), (Q, R), (S, R), (R, R)\}$

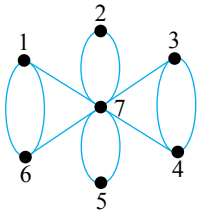
(b) $V = \{1, 2, 3, 4, 5, 6, 7\}$
$E = \{(1, 6), (1, 6), (2, 7), (2, 7), (5, 7), (5, 7), (3, 4), (3, 4), (1, 7), (6, 7), (3, 7), (4, 7)\}$

Solution:

(a)



(b)



TIPS

- The vertex pair for a loop is in the form (a, a).
- The vertex pairs for multiple edges are in the forms (a, b), (a, b).

Example 6

Determine whether a graph with the following degrees of vertices can be drawn.

(a) 3, 2, 2, 1, 3

(b) 2, 1, 1, 3, 3, 2

Solution:

(a) Sum of degrees = 3 + 2 + 2 + 1 + 3
= 11

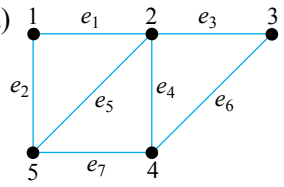
The graph cannot be drawn because the sum of degrees is odd.

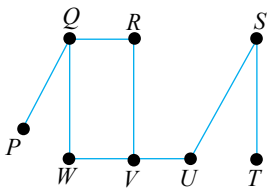
(b) Sum of degrees = 2 + 1 + 1 + 3 + 3 + 2
= 12

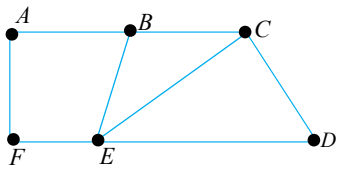
The graph can be drawn because the sum of degrees is even.

Self Practice 5.1a

1. Three simple graphs are given below.

(a) 

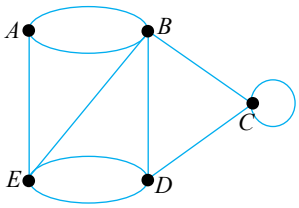
(b) 

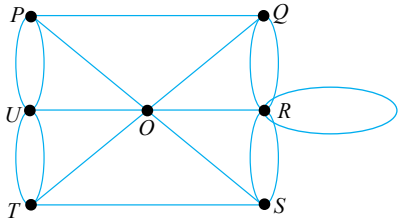
(c) 

For each graph, determine

(i) V and $n(V)$ (ii) E and $n(E)$ (iii) sum of degrees

2. Two graphs with multiple edges and loops are given below.

(a) 

(b) 

For each graph, determine

(i) V and $n(V)$. (ii) E and $n(E)$. (iii) sum of degrees.

3. Draw a simple graph based on the given information.

(a) $V = \{1, 2, 3, 4, 5, 6\}$

$E = \{(1, 6), (2, 6), (3, 6), (3, 4), (3, 5), (4, 5), (5, 6)\}$

(b) $V = \{P, Q, R, S, T, U\}$

$E = \{(P, U), (P, T), (Q, U), (Q, T), (Q, R), (R, S), (R, T), (S, T)\}$

4. Draw a graph with multiple edges and loops based on the given information.

(a) $V = \{P, Q, R, S\}$

$E = \{(P, S), (P, S), (Q, R), (Q, R), (P, P), (S, S), (Q, Q), (R, R), (P, Q), (R, S)\}$

(b) $V = \{1, 2, 3, 4, 5\}$

$E = \{(1, 5), (1, 5), (3, 5), (3, 5), (1, 1), (3, 3), (2, 1), (2, 3), (2, 5), (4, 1), (4, 3), (4, 5)\}$

5. Draw a simple graph with the given degrees of vertices.

(a) 2, 3, 2, 3, 4 (b) 1, 2, 3, 3, 3, 2

6. Draw a graph with loops and multiple edges with the given degrees of vertices.

(a) 3, 3, 4 (b) 2, 2, 4, 6, 6

What is the difference between a directed graph and an undirected graph?

Learning Standard

Compare and contrast
- directed graphs and undirected graphs.
- weighted graphs and unweighted graphs

A **directed graph** is a graph in which a direction is assigned to the edge connecting two vertices. Directed graphs are usually used to represent the flow of a certain process. For example, road maps, airlines networks, electrical circuits, computer networks and organisation charts.

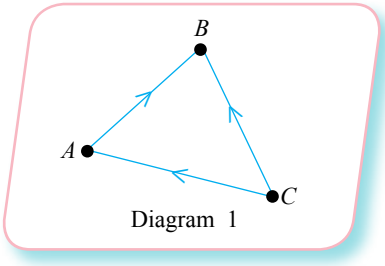


Diagram 1

INTERACTIVE ZONE

The blood circulation system in the human body is also categorised as a directed graph. Why?

Diagram 1 shows a simple directed graph. Based on the directions of the arrows, it can be seen that,

(a) for edge AB, A is the initial vertex and B is the terminal vertex.

(b) for edge CB, C is the initial vertex and B is the terminal vertex.

(c) all vertices are connected in one direction only.

INTERACTIVE ZONE

For electricity networks, power stations, transformer stations, substations and consumers are the vertices, and the cables and wires are the edges. Is an electricity network a directed graph or an undirected graph? Discuss.

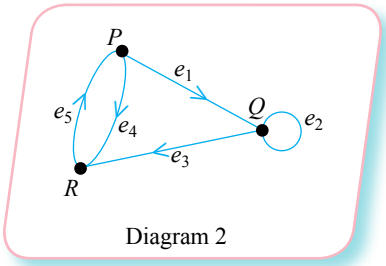


Diagram 2

Diagram 2 shows a directed graph with a loop and multiple edges. Based on the directions of the arrows, it can be seen that

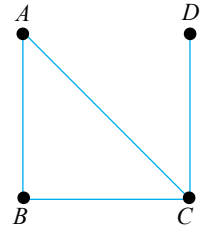
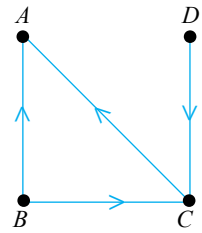
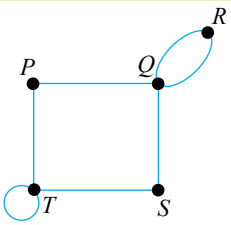
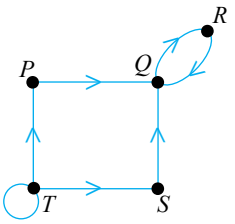
(a) $e_4 = (P, R)$; P is the initial vertex and R is the terminal vertex.

(b) $e_5 = (R, P)$; R is the initial vertex and P is the terminal vertex.

(c) $e_2 = (Q, Q)$; Q is the initial vertex and the terminal vertex because e_2 is a loop.

An **undirected graph** is a simple graph or a graph with loops and multiple edges drawn without any direction being assigned.

Differences between directed graphs and undirected graphs

Type of graph	Graph	Set V and Set E	Degrees
Simple graph	Undirected graph 	$V = \{A, B, C, D\}$ $E = \{(A, B), (A, C), (B, C), (C, D)\}$ The order of the vertices written is not important. Both pairs of vertices, (A, B) and (B, A) represent the edge AB.	$d(A) = 2$, $d(B) = 2$, $d(C) = 3$, $d(D) = 1$ $\Sigma d(V) = 8$
	Directed graph 	$V = \{A, B, C, D\}$ $E = \{(B, A), (C, A), (B, C), (D, C)\}$ The order of the vertices are written according to the direction of the edge. (B, A) and (A, B) represent different edges.	$d_{in}(A) = 2$ and $d_{out}(A) = 0$ Thus, $d(A) = 2 + 0$ $d(A) = 2$ $d_{in}(A)$ means the number of edges 'going into' vertex A. $d_{out}(A)$ means the number of edges 'coming out' from vertex A. $d_{in}(B) = 0$, $d_{out}(B) = 2$ $d_{in}(C) = 2$, $d_{out}(C) = 1$ $d_{in}(D) = 0$, $d_{out}(D) = 1$ $\Sigma d(V) = 8$
Graph with loops and multiple edges	Undirected graph 	$V = \{P, Q, R, S, T\}$ $E = \{(P, T), (P, Q), (Q, R), (Q, R), (Q, S), (S, T), (T, T)\}$	$d(P) = 2$, $d(Q) = 4$ $d(R) = 2$, $d(S) = 2$ $d(T) = 4$ $\Sigma d(V) = 14$
	Directed graph 	$V = \{P, Q, R, S, T\}$ $E = \{(P, Q), (Q, R), (R, Q), (S, Q), (T, S), (T, T), (T, P)\}$	$d_{in}(P) = 1$, $d_{out}(P) = 1$ $d_{in}(Q) = 3$, $d_{out}(Q) = 1$ $d_{in}(R) = 1$, $d_{out}(R) = 1$ $d_{in}(S) = 1$, $d_{out}(S) = 1$ $d_{in}(T) = 1$, $d_{out}(T) = 3$ Loop = 1 Loop = 1 $(T, P) = 1$ $(T, S) = 1$ $\Sigma d(V) = 14$

What are the differences between weighted graphs and unweighted graphs?

	Weighted graph	Unweighted graph
Type of graph	Directed graph and undirected graph	Directed graph and undirected graph
Edge	Associated with a value or a weight	Not associated with a value or a weight
Example	The edge represents: • distance between two cities. • travelling time. • the current in an electrical circuit. • cost.	The edge relates information like: • job hierarchy in an organisation chart. • flow map. • tree map. • bubble map.

Example 7

Draw a directed graph based on the given information.

(a) $V = \{P, Q, R, S, T, U\}$
$E = \{(P, Q), (P, R), (R, Q), (S, R), (S, Q), (S, T)\}$

(b) There is a loop at vertex Q and RS is a multiple edge such that
$d_{in}(P) = 1, d_{out}(P) = 1$
$d_{in}(Q) = 3, d_{out}(Q) = 2$
$d_{in}(R) = 0, d_{out}(R) = 3$
$d_{in}(S) = 3, d_{out}(S) = 1$

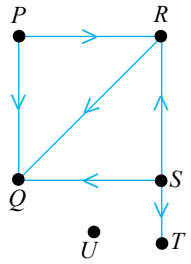
INFO ZONE

For Example 7(a), the vertex U appears in set V but not in set E. This means vertex U is not connected to any other vertices in the graph and it is known as an **isolated vertex**.

Solution:

(a)

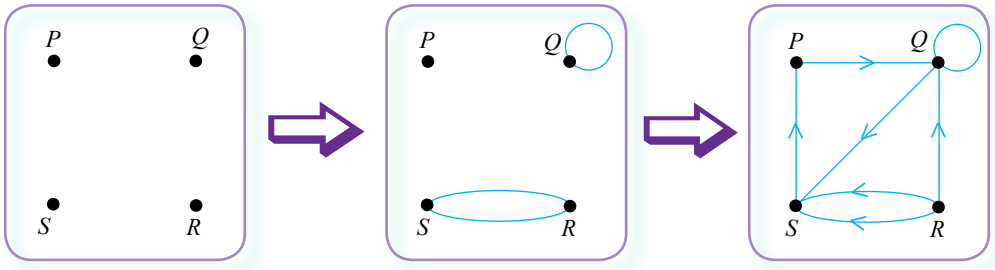
Vertex pair	Direction	
(P, Q)	P to Q	Two edges from vertex P
(P, R)	P to R	
(R, Q)	R to Q	One edge from vertex R
(S, R)	S to R	Three edges from vertex S
(S, Q)	S to Q	
(S, T)	S to T	
		Vertex T only connected to vertex S



(b) Total number of vertices = 4

RS – a multiple edge,
A loop at vertex Q.

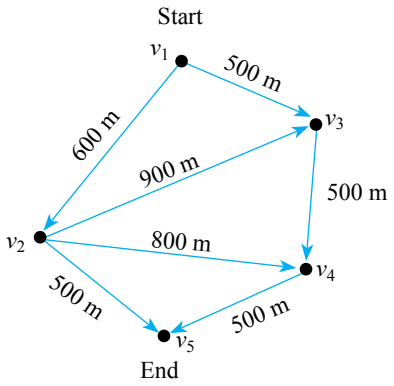
Complete the graph based on the number of edges going into and coming out from each vertex.



Example 8

The diagram on the right shows one-way paths that Izarul can choose for his running practice. Vertex v_1 is the starting position and vertex v_5 is the ending position before he goes home. Determine

(a) the shortest distance from v_1 to v_5.
(b) the longest distance from v_1 to v_5.
(c) the vertices that must be passed through if the distance of the one-way run is between 1.4 km and 2.1 km.



Solution:

(a) Shortest distance
$= v_1 \rightarrow v_2 \rightarrow v_5$
$= (600 + 500)$ m
$= 1\,100$ m
$= 1.1$ km

(b) Longest distance
$= v_1 \rightarrow v_2 \rightarrow v_3 \rightarrow v_4 \rightarrow v_5$
$= (600 + 900 + 500 + 500)$ m
$= 2\,500$ m
$= 2.5$ km

(c) v_1, v_3, v_4, v_5 and
v_1, v_2, v_4, v_5

Self Practice 5.1b

1. State two differences between directed graphs and undirected graphs.

2. What is the meaning of weight in a weighted graph?

3. Draw a directed graph based on the given information.

(a) $V = \{P, Q, R, S, T, U, V\}$
$E = \{(P, Q), (Q, R), (Q, S), (S, P), (S, R), (S, T), (U, T)\}$

(b) (i) There is a loop at vertex Q and a loop at vertex S.
(ii) QS is a multiple edge.
(iii) $d_{in}(P) = 0$, $d_{out}(P) = 2$
$d_{in}(Q) = 4$, $d_{out}(Q) = 2$
$d_{in}(R) = 0$, $d_{out}(R) = 2$
$d_{in}(S) = 4$, $d_{out}(S) = 2$

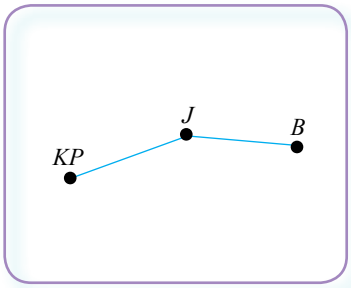
4. (a) Based on the information in Table 1 and Table 2, complete the weighted and undirected graph.

Name of place	Vertex
Kuala Pilah	KP
Bahau	B
Rompin	R
Batu Kikir	BK
Juasseh	J

Table 1

Vertex pair	Weight (km)
(J, B)	11.6
(KP, R)	40
(B, R)	20.7
(B, BK)	11.4
(BK, J)	6.6
(KP, J)	9.3

Table 2



Incomplete graph

(b) Mr Benny and Mr Muruges drive individually from Kuala Pilah to Rompin such that:
(i) Mr Benny uses the shortest route.
(ii) Mr Muruges takes the route which passes through Juasseh and Bahau.
Calculate the difference in distance, in km, for the journeys taken by Mr Benny and Mr Muruges from Kuala Pilah to Rompin.

How do you identify and draw subgraphs and trees?

Learning Standard

Identify and draw subgraphs and trees.

What do you understand about a subgraph?

A **subgraph** is part of a graph or the whole graph redrawn without changing the original positions of the vertices and edges. A graph H is said to be a subgraph of G if,

(a) the vertex set of graph H is a subset of the vertex set of graph G, that is $V(H) \subset V(G)$.
(b) the edge set of graph H is a subset of the edge set of graph G, that is $E(H) \subset E(G)$.
(c) the vertex pairs of the edges of graph H are the same as the edges of graph G.

In short,

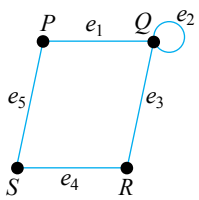
- a vertex in graph G is a subgraph of graph G.
- an edge in graph G along with the vertices it connects is a subgraph of graph G.
- each graph is a subgraph of itself.

INFO ZONE

The symbol $\subseteq$ also stands for subset.

Example 9

Determine whether Diagram 1, Diagram 2, Diagram 3 and Diagram 4 are the subgraphs of graph G.



Graph G

Diagram 1

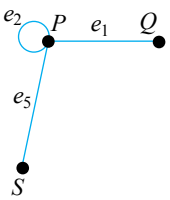


Diagram 2

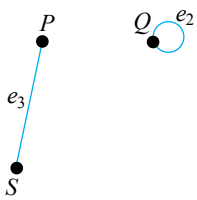


Diagram 3

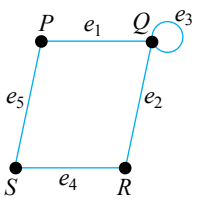


Diagram 4

Solution:

Diagram 1 – Yes, because the vertex pair for edge e_5 is the same.
$\{e_5\} \subset \{e_1, e_2, e_3, e_4, e_5\}$ and $\{P, S\} \subset \{P, Q, R, S\}$

Diagram 2 – No, because the position of loop e_2 is not on vertex Q.

Diagram 3 – No, because the edge connecting vertex P and vertex S is not e_3.

Diagram 4 – No, because the loop and the edge connecting vertex Q and vertex R are wrong.

What do you understand about a tree?

A **tree** of a graph is a subgraph of the graph with the following properties:

(a) A simple graph without loops and multiple edges.
(b) All the vertices are connected and each pair of vertices is connected by only one edge.
(c) Number of edges = Number of vertices – 1
Number of vertices = n
Number of edges = $n - 1$

INFO ZONE

The term tree was introduced by Arthur Cayley, an English mathematician, in the year 1857.

INFO ZONE

A family history chart is an example of a tree.

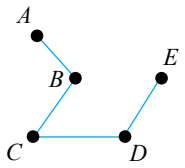


Diagram 1

Diagram 1 is a tree because
- all the vertices are connected.
- every pair of vertices is connected by an edge only.
- there are no loops or multiple edges.
- 5 vertices, 4 edges.

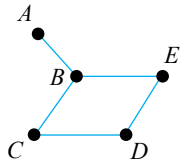
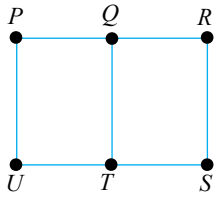


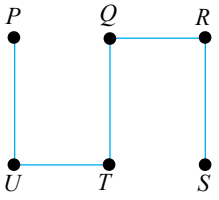
Diagram 2

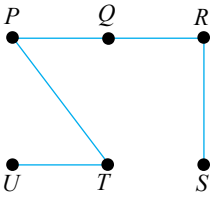
Diagram 2 is not a tree because
- vertex B and vertex E can be connected in two ways.
  (i) $B \rightarrow E$
  (ii) $B \rightarrow C \rightarrow D \rightarrow E$
- 5 vertices, 5 edges.

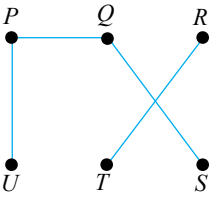
Example 10

Determine whether each of the following graphs is a tree. Justify your answer.

(a) 

(b) 

(c) 

(d) 

Solution:

(a) Not a tree. 6 vertices, 7 edges. Each pair of vertices can be connected in various ways.

(b) A tree. 6 vertices, 5 edges. Each pair of vertices is connected by one edge.

(c) A tree. 6 vertices, 5 edges. Each pair of vertices is connected by one edge.

(d) Not a tree. 6 vertices, 4 edges. Vertex R and vertex T are not connected to the other vertices.

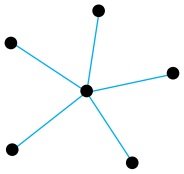
Example 11

Draw a tree for the following information given.

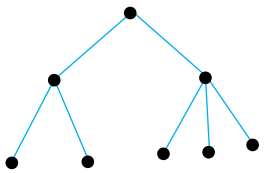
(a) 6 vertices (b) 8 vertices (c) 4 edges (d) 9 edges

Solution:

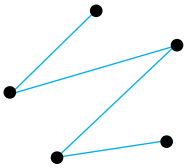
(a) 6 vertices
5 edges



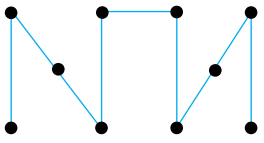
(b) 8 vertices
7 edges



(c) 4 edges
5 vertices



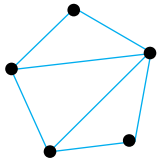
(d) 9 edges
10 vertices



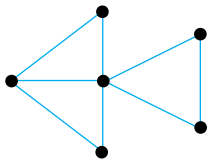
Example 12

Draw two trees based on the graphs given below.

(a)



(b)



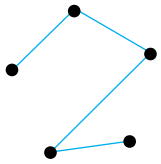
INFO ZONE

Trees are used to determine the shortest path with the condition that the path passes through each vertex once only.

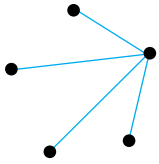
Solution:

(a) 5 vertices
7 edges (exceed by 3)

(i)

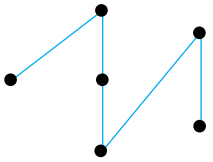


(ii)

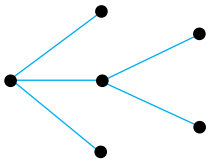


(b) 6 vertices
8 edges (exceed by 3)

(i)

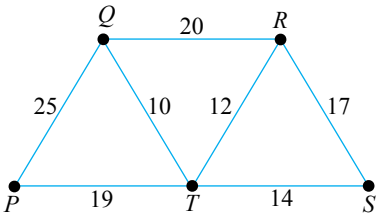


(ii)



Example 13

The following diagram shows an undirected weighted graph. Draw a tree with a minimum total weight.

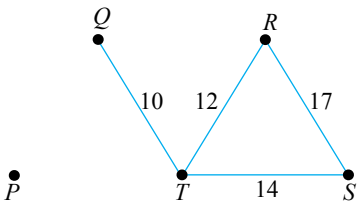


Solution:

Step 1

5 vertices, 7 edges

- 3 edges to be removed.
- Remove edges with the greatest weights (*PQ*, *QR*, *PT*)

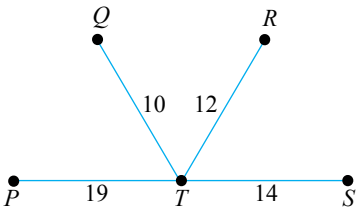


The graph above is not a tree because

- vertex *P* is not connected to the other vertices.
- three edges, *RS*, *ST* and *RT*, connect three vertices only.

Step 2

- Between the weights 19 and 25, keep weight 19 because its weight is smaller.
- Between the weights 12, 14 and 17, remove weight 17.



The graph obtained is a tree.

Minimum total weight of the tree
= 10 + 12 + 14 + 19
= 55

Self Practice 5.1c

1. Determine whether the given diagrams are the subgraphs of graph G.

Graph G	Diagram 1	Diagram 2	Diagram 3	Diagram 4	Diagram 5

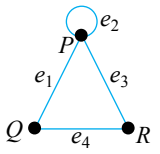
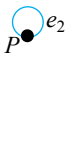
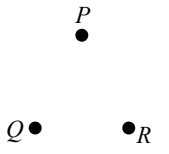
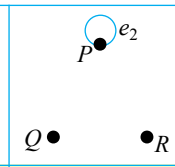
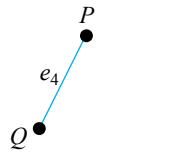
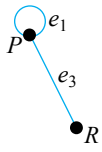
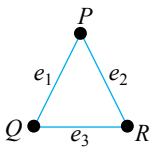
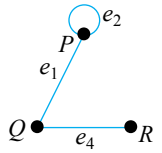
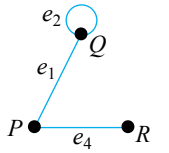
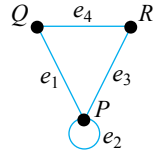
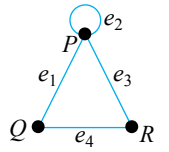
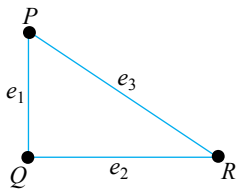
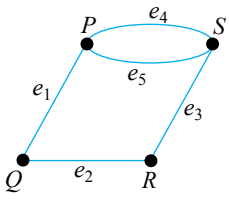
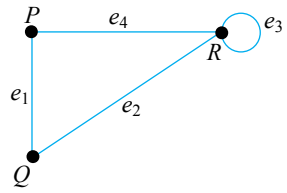

					

Diagram 6	Diagram 7	Diagram 8	Diagram 9	Diagram 10	Diagram 11
					

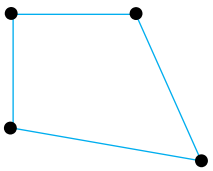
2. Draw five subgraphs for each given graph.

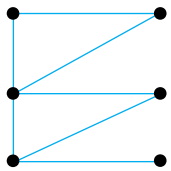
(a) 

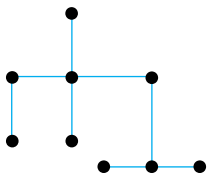
(b) 

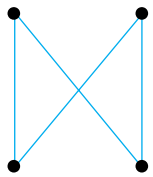
(c) 

3. Identify whether it is a tree or not a tree for the following diagrams.

(a) 

(b) 

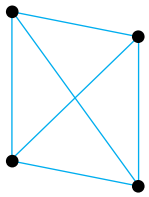
(c) 

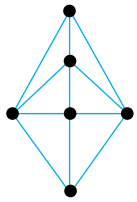
(d) 

4. Draw a tree based on the given information.

(a) 7 vertices (b) 9 vertices (c) 5 edges (d) 9 edges

5. Draw two trees based on the given graphs.

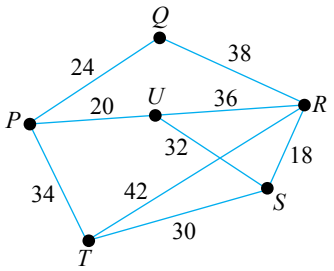
(a) 

(b) 

6. The diagram on the right shows an undirected weighted graph.

(a) Draw a tree with a minimum total weight.

(b) What is the minimum total weight?



How would you represent information in the form of networks?

Learning Standard

Represent information in the form of networks.

A network is a type of unique graph and can be used to represent overlapping and intersecting information. Networks are widely used in almost every area of our daily lives. A network that is drawn and displayed in graphic forms enables the interrelationships between various information or data structures to be understood easily.

Networks can be drawn as:
- directed weighted graph or directed unweighted graph
- undirected weighted graph or undirected unweighted graph

Transportation Networks

Transportation networks can be shown as weighted graphs and unweighted graphs. The weights can represent the distance, travelling time or cost of the journey. The well known navigation system in the weighted transportation networks is the GPS (Global Positioning System).

The diagram on the right shows the train transit network in Kuala Lumpur Sentral (KL Sentral). This undirected graph is an example of transportation networks with the vertices representing names of stations that are connected and the edges representing the types of trains.

Example 14

Mr Voon and his family plan to visit historical places in Melaka. The map shows three alternative routes with the distances and estimated times needed to travel from Tangkak to A Famosa, Melaka.

Assume P is a 46.3-kilometre route, Q is a 50.2-kilometre route and R is a 53.3-kilometre route. In your opinion, why does route P take the longest time compared to the other routes even though route P is the shortest route?

Solution:

Route P takes the longest time because the route passes through crowded town areas and there are more road users compared to the other two routes.

Social Networks

Social networks are becoming more popular among teenagers and adults. Social networks are used in areas like job opportunities, business opportunities, socialising, family relationships, education, social media and connecting with communities around the world.

Even though social networks are main platforms for various activities and are useful, you should be cautious and moderate in using social networks to avoid being distracted and being deceived easily.

Smart Mind

State a social network that you know of.

INTERACTIVE ZONE

Discuss the negative effects of using social networks.

Example 15

The table below shows the data of six pupils and the games that they like. Represent the information in the form of a network.

Name of pupil	Game
Edmund	Badminton, Chess
Azwan	Football, *Sepak takraw*
Rajan	Chess, Football
Aina	Chess, Netball
Maria	Badminton, Netball
Jenny	Netball, Volleyball

Smart Mind

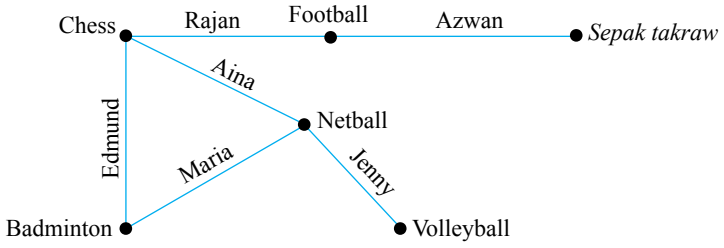
What are the meanings of LTE and 4G which are often used in wireless Internet networks?

INTERACTIVE ZONE

Solving criminal cases is also related to graphs. All evidence obtained are linked to one another. Discuss information that can be represented by the vertices and edges of graph.

Solution:

Let the vertices represent types of games and the edges represent the pupils' names.



TIPS

Choose information that is not repeated as the edges.

INTERACTIVE ZONE

Is the food chain a network? Discuss.

Self Practice 5.1d

Name of place	Vertex
Kuala Sepetang	*KS*
Matang	*M*
Kamunting	*K*
Simpang	*S*
Taiping	*T*
Changkat	*CJ*

Table 1

Vertex pair	Distance (km)
(*KS*, *M*)	8
(*K*, *M*)	8.4
(*K*, *T*)	5.2
(*S*, *T*)	5.9
(*S*, *M*)	3.5
(*S*, *KS*)	11
(*CJ*, *S*)	4.9
(*CJ*, *KS*)	15
(*CJ*, *T*)	10

Table 2

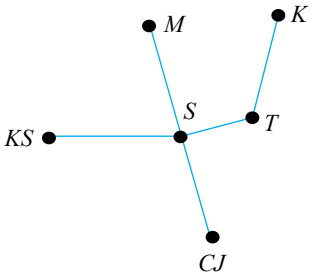


Diagram 1

1. Table 1 and Table 2 show names and distances of six places in Perak. Diagram 1 shows an incomplete undirected graph connecting the six places.

(a) Complete Diagram 1 by drawing an undirected weighted graph.
(b) Draw a tree with a minimum total weight which shows every place being visited once only.
(c) What is the minimum distance of the tree that you have drawn?

2. The table below shows four types of favourite food of several pupils.

Food	Name of pupil
Chicken rice	Mervin, Raj, Helen, Wong, Ain
Nasi lemak	Mervin, Nurul, Atiqah, Ain, Puspa
Fried rice	Helen, Julia, Nurul, Faruk, Puspa
Fried noodles	Faruk, Atiqah, Raj, Wong, Julia

(a) Based on the table, draw a graph with multiple edges.
(b) Between the types of food and the names of pupils, which group will you use to represent the vertices? Justify your answer.
(c) What is the relationship between the sum of degrees of your graph with the total number of food choices?
(d) Between table form and graph form, which form is clearer in showing the relationship between the types of food and the pupils?

3. (a) Draw an organisation chart of your class using your own creativity.
(b) State the type of graph you have used. Is the class organisation chart a network? Justify your answer.

How do you solve problems involving networks?

Learning Standard
Solve problems involving networks.

Example 16

The table below shows the choices of public transportation, estimated travelling time and estimated cost for a journey from Johor Bahru to Kota Bharu.

Type of transportation	Travelling time (24-hour system)	Estimated duration of journey	Estimated expenses
			Price of ticket per person/cost of fuel
Bus	2000 hours	9 hours	RM64 – RM75
Train	(1833 – 1147) hours	17 hours	With bed Child: RM32 – RM38 Adult: RM49 – RM55 Without bed Child: RM26 Adult: RM43
Cab	–	9 hours	RM120 (for one cab)

Based on the table, determine the type of transportation that should be chosen for the situation given below. Justify your answer.

(a) A journey involving an adult without time constraint.

(b) A journey involving an adult with time constraint.

(c) A journey involving two adults and two children.

Solution:

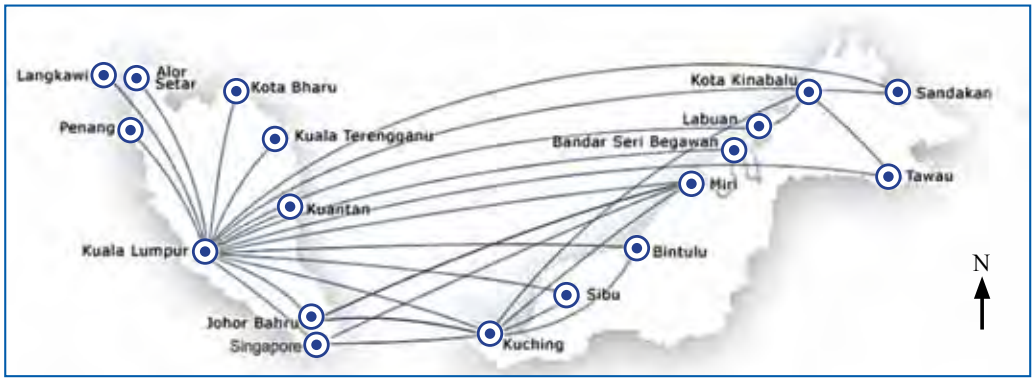
Type of transportation	Price of ticket per person (RM)	
	Minimum	Maximum
Bus	64	75
Train (With bed)	Child = 32 Adult = 49	Child = 38 Adult = 55
Train (Without bed)	Child = 26 Adult = 44	–

(a) Taking a train with bed is the best choice because the difference in price is only RM5 compared to taking a train without bed. The passengers can have a good rest throughout the whole journey. This choice is also cheaper compared to taking a cab.

(b) Taking a bus is the best choice because the duration of journey is shorter than that of a train and it is more economical than taking a cab. For safety purpose, it is not wise for an individual to take a cab for a long journey.

(c) Taking a cab is the most economical choice.

Example 17



The map above shows the domestic flight routes of a private airline in Malaysia.

(a) Mr Joshua works in Kuala Lumpur. He wants to visit his family in Kota Kinabalu. State the best route Mr Joshua can choose.

(b) What are the advantages and disadvantages for the choice of flight made?

Solution:

(a) The direct flight from Kuala Lumpur to Kota Kinabalu.

(b) The direct flight from Kuala Lumpur to Kota Kinabalu saves time and cost. The flight from Kuala Lumpur to Kota Kinabalu with transit takes a longer time and most probably the cost of the journey is also higher.

Self Practice 5.1e

1. Mr Maswi works in Kulai. He plans to visit his family in Miri on a certain weekend. On Friday, Mr Maswi's work ends at 12.30 noon. The table below shows the choices of domestic flight routes of a private airline in Malaysia on Friday and Saturday for that week.

Route	Friday		Saturday	
	Time	Price of ticket	Time	Price of ticket
Johor Bahru - Miri	No flight		(1705 – 1900) hrs	RM259.30
Johor Bahru - Kuching	(1630 – 1755) hrs	RM144.30	(0605 – 0730) hrs	RM174.30
	(1930 – 2055) hrs	RM124.30	(1205 – 1330) hrs	RM154.30
Kuching - Miri	(2010 – 2115) hrs	RM149.00	(0835 – 0940) hrs	RM84.00
	(2155 – 2300) hrs	RM149.00	(1145 – 1250) hrs	RM64.00

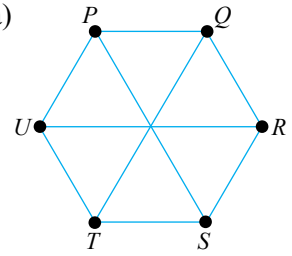
(a) Determine the most economical flight from Johor Bahru to Miri.

(b) Determine the best flight that Mr Maswi can choose if he needs to go back to Peninsular Malaysia on Sunday. Give your reason.

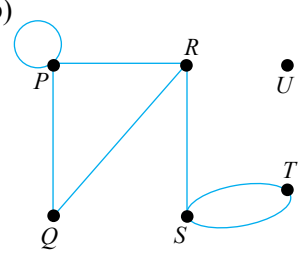
Comprehensive Practice

1. Three graphs are given below.

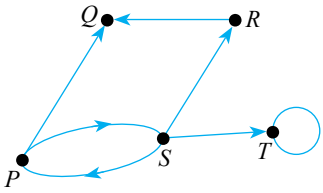
(a)



(b)



(c)



For each graph, determine

(i) set V. (ii) set E. (iii) sum of degrees.

2. Draw an undirected graph based on the given information.

(a) $V = \{P, Q, R, S, T, U\}$

$E = \{(P, Q), (Q, R), (R, S), (S, T)\}$

(b) $V = \{P, Q, R, S, T\}$

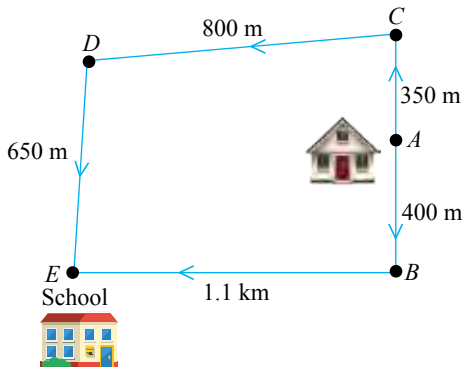
$E = \{(P, Q), (P, Q), (Q, R), (Q, T), (R, S), (S, S), (S, T)\}$

3. (a) Draw an undirected weighted graph to represent the federal roads connecting a few towns as shown in the table. You are advised to use an area map to determine the positions of the towns.

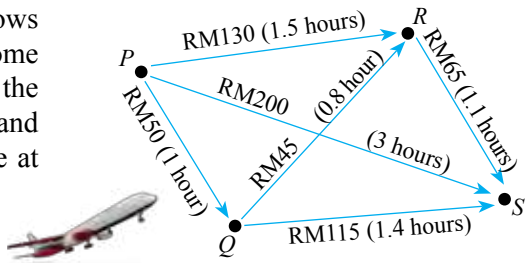
(b) Is the graph that you have drawn a tree? Justify your answer.

Name of town	Distance (km)
Temerloh - Mentakab	9.3
Temerloh - Bandar Bera	30.2
Bandar Bera - Teriang	4.9
Mentakab - Lanchang	21.1
Lanchang - Karak	21.9
Mentakab - Kuala Krau	34.6

4. The directed graph on the right shows the roads connecting Lani's house at A to the school at E. The edge BE is a municipal road and edges AB, AC, CD and DE are roads in the housing estates. Suggest the best route that Lani can choose to cycle to school. Justify your answer.



5. The directed weighted graph on the right shows the prices of tickets and the travel times for some choices of flights of a private airline. Vertex S is the destination of the flight from vertex P. Vertex Q and vertex R are the transit airports. The transit time at each of the airports is 45 minutes.



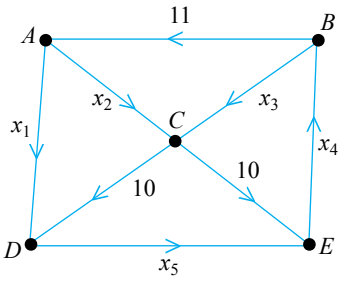
(a) State

(i) the most economical route.

(ii) the route that takes the shortest time.

(b) If you need to go to a destination at optimum cost, state the route that you will choose. Justify your answer.

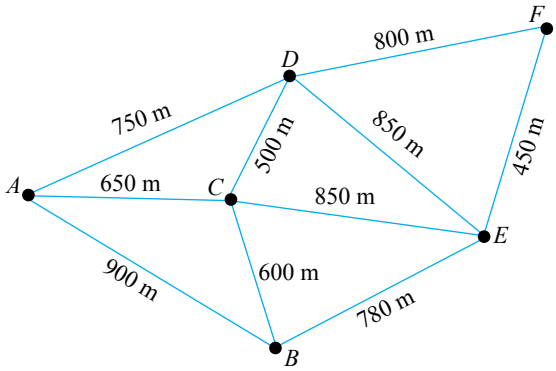
6. Write five linear equations based on the directed graph below. Given $x_5 = 15$, determine the values of x_1, x_2, x_3 and x_4.



TIPS

Use $\Sigma d_{in} = \Sigma d_{out}$ for each vertex.

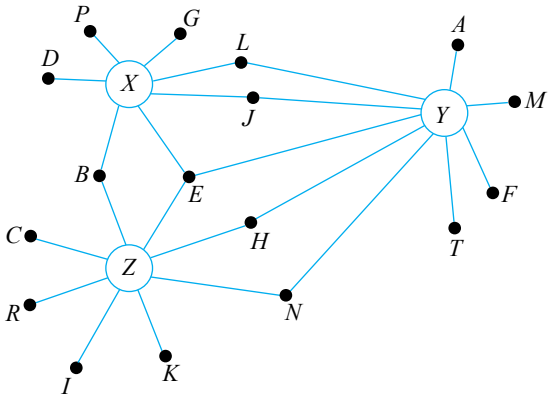
7. The following undirected graph shows six houses in a village. A salesperson needs to visit all the houses starting from house A and finishing at house F.



(a) Draw a directed graph to represent the shortest distance from A to F with the condition that all the paths are taken once only.

(b) Based on your graph, calculate the shortest distance in km.

8. The graph on the right shows the connections between the elements in set X, set Y and set Z, where the universal set, $\xi = X \cup Y \cup Z$.



(a) Represent the graph in a Venn diagram.

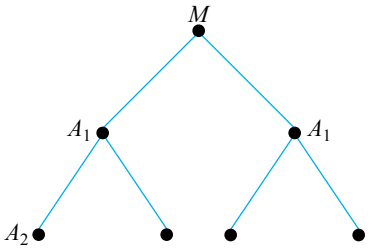
(b) Determine

(i) $(X \cup Y)'$

(ii) $(X \cap Z)' \cap Y'$

(iii) $(Y \cap Z) \cap (X \cap Y)$

9. Mr Ganesan is the manager of an insurance agency. He recruites two active insurance agents to sell the latest insurance scheme valued at RM100 per month. Besides selling insurance policies, each agent needs to recommend at least two new insurance agents. In the incomplete tree on the right, M represents the manager, A_1 represents the first level agents and A_2 represents the second level agents.



(a) If there are 30 agents in January, complete the given tree.

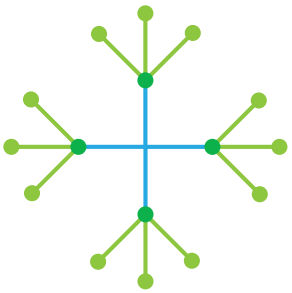
(b) The table on the right shows the percentages of basic commission received by an agent and the manager for an insurance policy sold.

Agent	25%
Manager	2%

(i) Calculate the total basic commission received by Mr Ganesan in January if the minimum number of insurance policies sold by an agent in January is 18.

(ii) What is the minimum number of policies that an agent needs to sell in order to receive a basic commission of at least RM1 000?

10. The diagram on the right shows the growth of a type of cell. It is given that on the first day, there are four cells. On the second day, each cell produces three cells. On the following day, each new cell produces another three new cells. The process of producing new cells repeats at the same rate.



(a) On which day will the total number of cells exceeds 50 for the first time?

(b) Calculate the total number of cells on the fifth day.

(c) Given the life span of a cell is 5 days, calculate the total number of cells on the eighth day.

PROJECT

1. A maze is an example of a network.
Diagram 1 shows an example of a maze whereas Diagram 2 is the corresponding network.

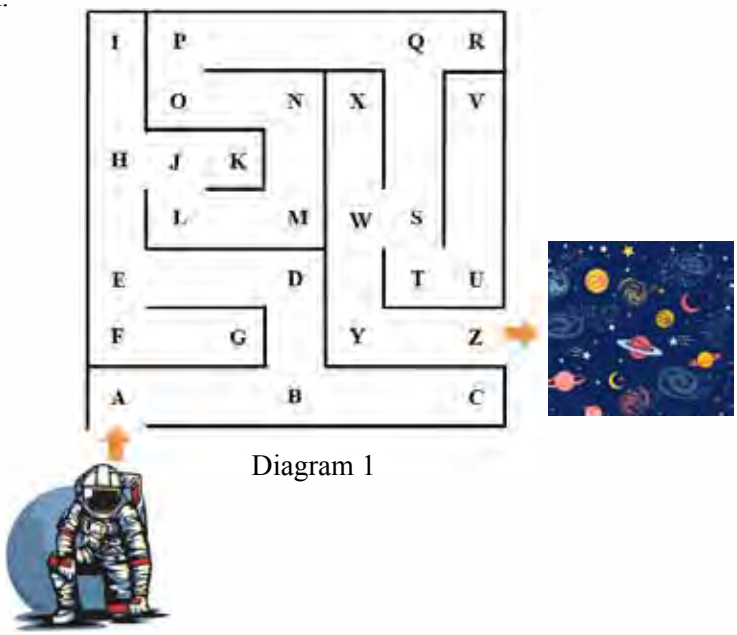


Diagram 1

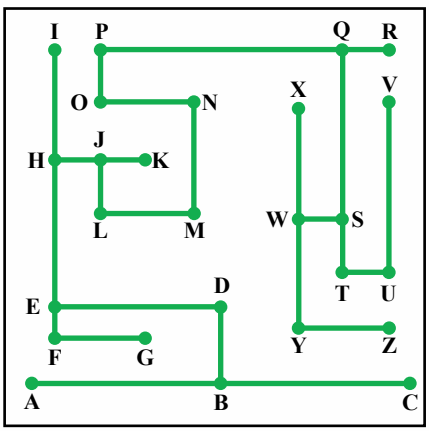


Diagram 2

2. Divide the class into groups.

3. Obtain examples of mazes with different levels of difficulty.

4. Label the vertices with suitable letters as shown in Diagram 1.

5. Draw the corresponding networks.

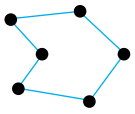
6. Exhibit your project work at the Mathematics Corner.

CONCEPT MAP

Network in Graph Theory

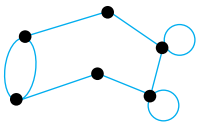
Graph

Simple

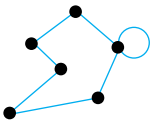


Degree

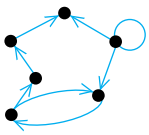
Has loops and multiple edges



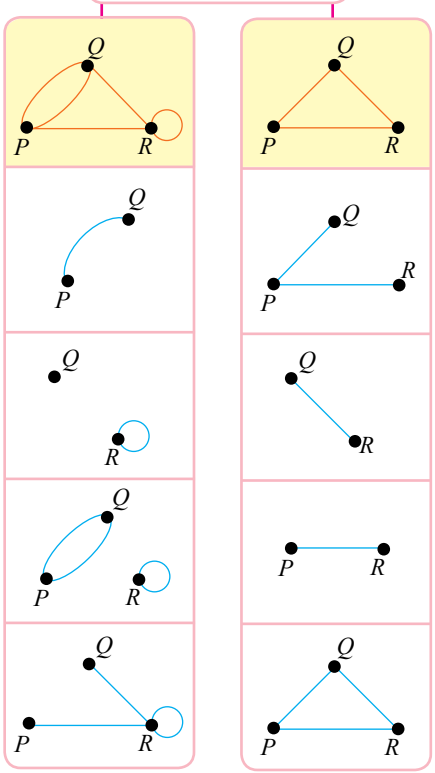
Undirected graph



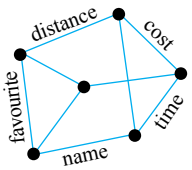
Directed graph



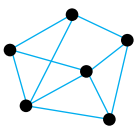
Subgraph



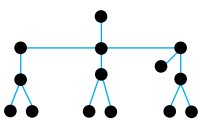
Weighted graph



Unweighted graph



Tree (Simple graph)



Self Reflection

Complete the following statements with the answers provided.

subgraph | weighted graph | degree | simple graph | directed graph | edge | tree | loop

1. An _____ is a line which connects two vertices.
2. The _____ is the number of edges which connects two vertices.
3. A graph without a _____ and without multiple edges is known as a _____.
4. Edges of a _____ are associated with directions.
5. Edges of a _____ are associated with values or information.
6. A _____ is part of a graph or the whole graph redrawn.
7. Every pair of vertices in a _____ is connected by an edge only.

Mathematics Exploration

Gather information and prepare a multimedia presentation on one of the networks given below.

Social

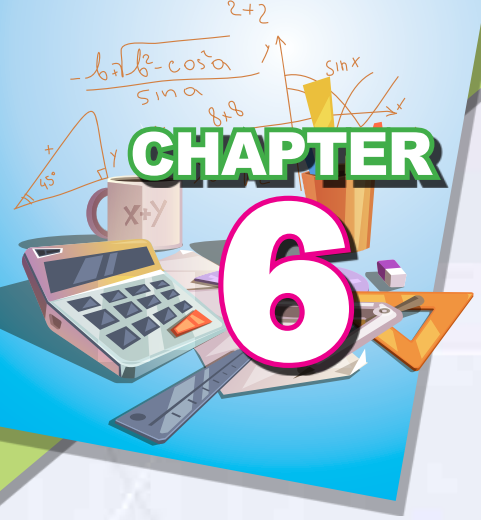
Medicine

Banking

Language

Computer science

Include pictures, video or interesting materials to make your presentation more creative and informative.



Linear Inequalities in Two Variables

You will learn

- Linear Inequalities in Two Variables
- Systems of Linear Inequalities in Two Variables

Business and entrepreneurship are the main pulses of a country's economy to provide career opportunities and to improve standard of living of people. A businessman or an entrepreneur needs to be proficient in communication, leadership, accounting, marketing and most importantly in planning. Careful planning enables a businessman or an entrepreneur to obtain a high return, which is a maximum profit at minimal cost.

Do you know how an entrepreneur plans to make a high profit by minimising expenditures?

Why Study This Chapter?

Knowledge of linear inequalities is fundamental to the field of linear programming. This knowledge is very important in business, corporate world and other areas that require optimum results, which is a maximum profit at minimal cost.

Walking Through Time

Thomas Harriot
(1560 – 1621)

Thomas Harriot was an English astronomer, mathematician and translator. He contributed to the advancement of the scientific field. Thomas Harriot was the first person who introduced the inequality symbols greater than, >, and less than, <.

http://bt.sasbadi.com/m4155

WORD BANK

• linear inequality	• *ketaksamaan linear*
• linear	• *linear*
• variable	• *pemboleh ubah*
• region	• *rantau*
• linear inequality system	• *sistem ketaksamaan linear*

6.1 Linear Inequalities in Two Variables

How do you represent a situation in the form of a linear inequality?

Inequalities are used to describe the relationship between two quantities that are not equal.

Diagram 1

Diagram 1 shows a common warning sign on the road or highway. The warning sign is placed at an entrance of a tunnel. What is the maximum height of a vehicle that can pass through the tunnel?

Diagram 2 shows the maximum mass of a 3-tonne lorry when it is loaded and not loaded. What is the relationship between the mass of an unloaded lorry, the mass of a loaded lorry and the mass of the load?

For the situation in Diagram 1, let the overall height of a vehicle is represented by a variable h, thus $h < 4.75$ m. In this situation, only one quantity is involved, that is the height in metres.

The situation in Diagram 2 involves the mass in kg but in two different conditions. Let,

y = gross vehicle weight (BDM, *berat dengan muatan*)
x = kerb weight (BTM, *berat tanpa muatan*)

Value of load = BDM – BTM
= (7 500 – 3 410) kg
= 4 090 kg

Thus, the situation in Diagram 2 can be stated as

$$y \leqslant x + 4\,090$$

That is, x and y are two variables representing two quantities with the same unit.

Learning Standard

Represent situations in the form of linear inequalities.

MY MEMORY

$>$ greater than
$<$ less than
$\geqslant$ greater than or equal to
$\leqslant$ less than or equal to

Diagram 2

INFO ZONE

A variable is a factor such as an element, a feature or an integer that must be taken into account in a calculation.

MY MEMORY

The highest power of a variable in a linear equation is 1.

MY MEMORY

Linear equation in two variables:
$ax + by = c$

Example 1

Taufik wants to buy some revision books and exercise books at a book exhibition. He finds that the price of a reference book is RM14 and the price of an exercise book is RM9. The maximum amount of money that Taufik can spend is RM100. Represent the above situation in an appropriate form of linear inequality.

Solution:

Let, x = reference book and y = exercise book.

Hence, $14x + 9y \leqslant 100$

The price of a reference book is RM14.

The price of an exercise book is RM9.

The total price of the reference books and the exercise books is less than or equal to RM100.

The linear inequality $14x + 9y \leqslant 100$ can also be written as;

$$14x \leqslant 100 - 9y \text{ or } 9y \leqslant 100 - 14x$$

Example 2

Puan Hidayah wants to buy mathematics posters to hang in the Mathematics Room. The cost of a small poster is RM12.50 and for a large poster is RM18.50. Represent the purchase of both posters in an appropriate form of linear inequality if the allocation from the school is RM150.00.

Solution:

Let,

k = small poster and b = large poster.

Hence, $12.50k + 18.50b \leqslant 150$

$$\frac{25}{2}k + \frac{37}{2}b \leqslant 150$$

Multipy both sides of the inequality by 2 to cancel the fractional denominator:

$$2 \times \frac{25}{2}k + 2 \times \frac{37}{2}b \leqslant 2 \times 150$$

$$25k + 37b \leqslant 300$$

Self Practice 6.1a

1. Represent the given situations in the form of linear inequalities using suitable variables.

(a) Madam Linda Lim wants to prepare fish and prawn dishes in conjunction with the Chinese New Year celebration. The cost of a kilogram of mackerel is RM25 and a kilogram of prawns is RM45. Madam Linda Lim wants to spend RM250 to buy fish and prawns.

(b) Encik Halim wants to rear chickens and ducks on a small scale to generate some extra income for managing his growing family expenses. A chick costs RM2 and a duckling costs RM1.50. Encik Halim has a capital of RM500 to purchase the chicks and ducklings.

(c) Madam Letchumy wants to contribute RM50 for two types of *kuih* in conjunction with Canteen Day at her son's school. The cost of a curry puff is 30 sen and a *kuih kasturi* is 40 sen.

(d) Puan Yati sells *nasi lemak*. The selling prices of a packet of *nasi lemak* with egg is RM1.50 and a packet of *nasi lemak* with chicken is RM3.50. The total daily sales of *nasi lemak* should exceed RM120 in order to make a minimum profit.

How do you verify the conjecture about points in the region that satisfy a linear inequality?

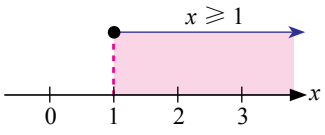


Diagram 1

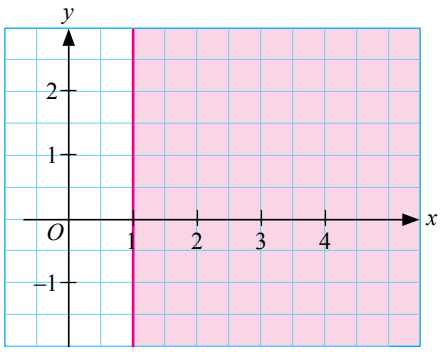


Diagram 2

Learning Standard

Make and verify the conjecture about the points in the region and the solution of certain linear inequalities.

In Form 1, you learned how to represent an inequality in one variable using a number line as in Diagram 1. Do you know that an inequality can also be represented on a Cartesian plane by shading the region that satisfies the inequality as in Diagram 2? All the x-coordinates in the shaded region satisfy the inequality $x \geqslant 1$.

TIPS

Inequality sign	Type of line
$>$, $<$	Dashed line
$\geqslant$, $\leqslant$	Solid line

Diagram 3 shows the types of regions generated on a Cartesian plane when a straight line is drawn.

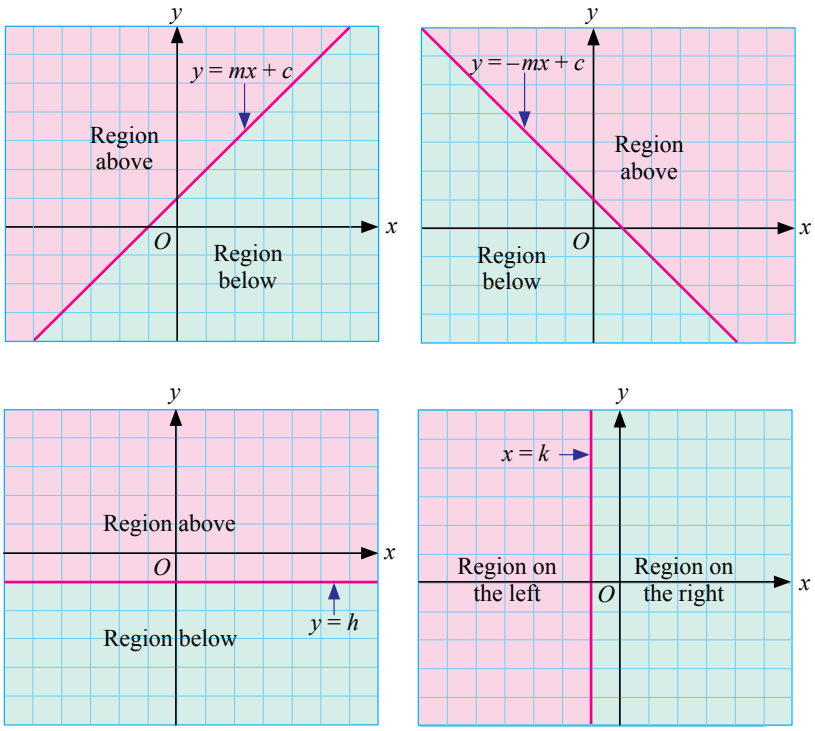


Diagram 3

MY MEMORY

General form of a straight line:
$y = mx + c$
m = gradient
c = y-intercept

Smart Mind

The gradient of the straight line $y = h$ is zero. Why?

INTERACTIVE ZONE

Why is the gradient of the straight line $x = k$ undefined? Discuss.

Mind Stimulation 1

Aim: To verify the conjecture about the points in the region that satisfy a linear inequality.

Materials: Activity sheets, graph papers

Scan the QR Code to carry out this activity.
http://bt.sasbadi.com/m4159

Steps:

1. Divide the class into groups.
2. Each group is given a piece of graph paper and an activity sheet.
3. Pupils are asked to draw a straight line that represents the given linear equation for $-5 \leqslant x \leqslant 5$ on the graph paper and to plot the points obtained in the table on the activity sheet. (Example 1)

Point	y-coordinate	Value of $x + 2$	Position of point (From graph)			Point that satisfies		
			On the straight line	Region above	Region below	$y = x + 2$	$y > x + 2$	$y < x + 2$
(–5, 4)	4	$-5 + 2 = -3$		✓			$(4 > -3)$	
(1, 3)	3	$1 + 2 = 3$	✓			$(3 = 3)$		
(0, –2)	–2	$0 + 2 = 2$			✓			$(-2 < 2)$
(4, 7)								
(–3, 0)								
(3, 5)								

Example 1

4. Three Stray, One Stay activity can be carried out so that all pupils have the opportunity to explain the results.

Discussion:
What is the relationship between the position of a point on the straight line, in the region above or in the region below with the given linear equation or linear inequality?

From the activity in Mind Stimulation 1, it is found that:

- All the points that lie on the straight line satisfy the equation $y = mx + c$.
- All the points that lie in the region above satisfy the inequality $y > mx + c$.
- All the points that lie in the region below satisfy the inequality $y < mx + c$.

Example 3

Draw the straight line $y = -2x + 6$ for $-1 \leq x \leq 5$. Plot the points (1, −2), (4, −2), (0, 1), (1, 4), (4, 3) and (2, 6). Determine whether the points plotted satisfy $y = -2x + 6$, $y > -2x + 6$ or $y < -2x + 6$.

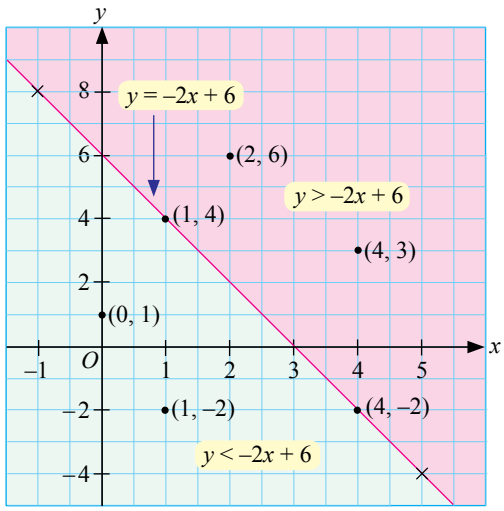
Solution:

$y = -2x + 6$

x	-1	5
y	8	-4

When $x = -1$,
$y = -2(-1) + 6$
$y = 8$

When $x = 5$,
$y = -2(5) + 6$
$y = -4$



- Points (1, 4) and (4, −2) satisfy the equation $y = -2x + 6$.
- Points (2, 6) and (4, 3) satisfy the inequality $y > -2x + 6$.
- Points (0, 1) and (1, −2) satisfy the inequality $y < -2x + 6$.

Example 4

Given the linear equation $y = -3x + 6$, without drawing the graph of the straight line, determine whether the given points satisfy $y = -3x + 6$, $y > -3x + 6$ or $y < -3x + 6$.

(a) (2, 5) (b) (1, 2) (c) (−1, 9) (d) (0, 8)

Solution:

(a) (2, 5)

y	$-3x + 6$
5	$-3(2) + 6$ $= 0$
$5 > 0$	
thus, point (2, 5) satisfies $y > -3x + 6$	

(b) (1, 2)

y	$-3x + 6$
2	$-3(1) + 6$ $= 3$
$2 < 3$	
thus, point (1, 2) satisfies $y < -3x + 6$	

(c) (−1, 9)

y	$-3x + 6$
9	$-3(-1) + 6$ $= 9$
$9 = 9$	
thus, point (−1, 9) satisfies $y = -3x + 6$	

(d) (0, 8)

y	$-3x + 6$
8	$-3(0) + 6$ $= 6$
$8 > 6$	
thus, point (0, 8) satisfies $y > -3x + 6$	

Self Practice 6.1b

1. Draw the straight line $y = \frac{2}{3}x - 2$ for $0 \leqslant x \leqslant 3$. Plot the points (3, 1), (2, –2), (1.5, –1), (3, –2) and (1, –1). Determine whether the points plotted satisfy $y = \frac{2}{3}x - 2$, $y > \frac{2}{3}x - 2$ or $y < \frac{2}{3}x - 2$.

2. Draw the straight line $y = -\frac{1}{2}x + 2$ for $-4 \leqslant x \leqslant 6$. Plot the points (–3, 5), (–3, 1), (1, –2), (2, 1) and (4, 5). Determine whether the points plotted satisfy $y = -\frac{1}{2}x + 2$, $y > -\frac{1}{2}x + 2$ or $y < -\frac{1}{2}x + 2$.

3. Given the linear equation $y = 4x - 5$, without drawing the graph of the straight line, determine whether the given points satisfy $y = 4x - 5$, $y > 4x - 5$ or $y < 4x - 5$.

(a) (2, 4) (b) (3, 7) (c) (0, –6) (d) (–2, 0) (e) (4, 5)

4. Given the linear equation $y = -3x + 4$, without drawing the graph of the straight line, determine whether the given points satisfy $y = -3x + 4$, $y > -3x + 4$ or $y < -3x + 4$.

(a) (–2, 3) (b) (1, 1) (c) (–1, 8) (d) (0, 1) (e) (–0.5, 7)

How do you determine and shade the region that satisfies a linear inequality?

Learning Standard

Determine and shade the region that satisfies a linear inequality.

You have learned that if a straight line representing a linear equation $y = mx + c$ is drawn on a Cartesian plane, all the points on the Cartesian plane can be categorised into three groups, which are;

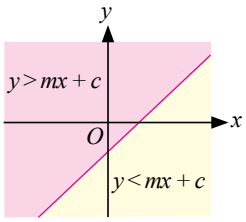


Diagram 1

- Points that lie on the straight line satisfy the equation $y = mx + c$.
- Points located in the region above the straight line satisfy the inequality $y > mx + c$.
- Points located in the region below the straight line satisfy the inequality $y < mx + c$.

For the straight lines $y = h$ and $x = k$ that are drawn on a Cartesian plane where h and k are constants, all the points on the Cartesian plane can also be categorised as follows:

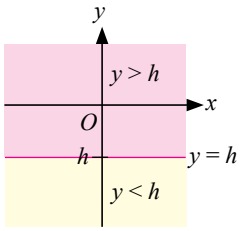
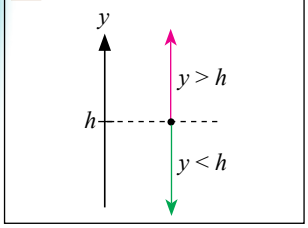


Diagram 2

- Points that lie on the straight line satisfy the equation $y = h$.
- Points located in the region above the straight line satisfy the inequality $y > h$.
- Points located in the region below the straight line satisfy the inequality $y < h$.

MY MEMORY



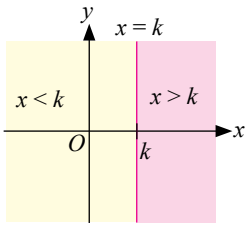
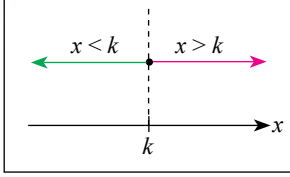


Diagram 3

- Points that lie on the straight line satisfy the equation $x = k$.
- Points located in the right region of the straight line satisfy the inequality $x > k$.
- Points located in the left region of the straight line satisfy the inequality $x < k$.

MY MEMORY



What is the relationship between a point on a Cartesian plane with the inequality $y > mx + c$, $y < mx + c$, $y \geqslant mx + c$ or $y \leqslant mx + c$?

$>$, $<$

- Points that lie in the region above or below a straight line $y = mx + c$.
- The straight line is drawn using a dashed line.

$\geqslant$, $\leqslant$

- Points that lie on the straight line $y = mx + c$ including the region above or below.
- The straight line is drawn using a solid line.

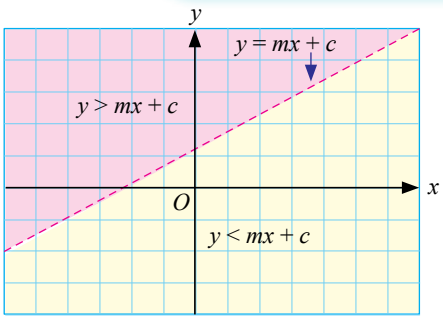


Diagram 4

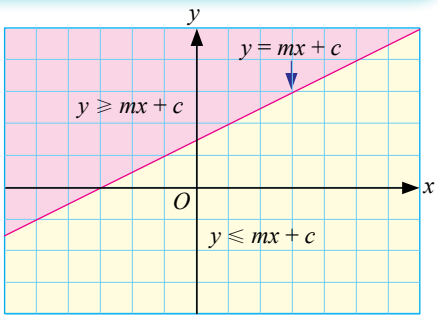


Diagram 5

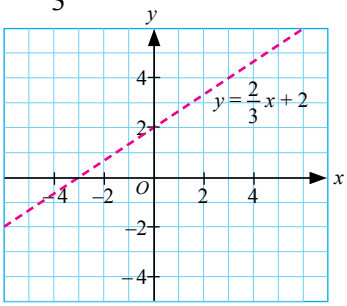
The regions do not include points that lie on the straight line $y = mx + c$.

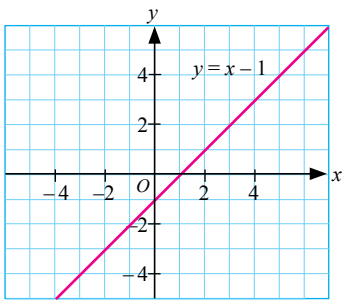
The regions include points that lie on the straight line $y = mx + c$.

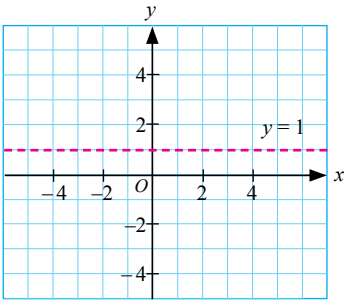
Example 5

Shade the region that represents each of the following inequalities.

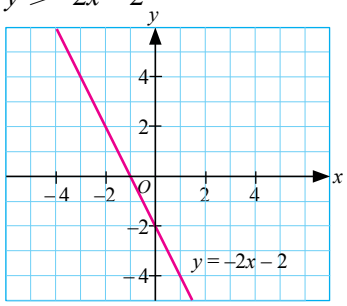
(a) $y > \frac{2}{3}x + 2$ (b) $y \leqslant x - 1$ (c) $y < 1$



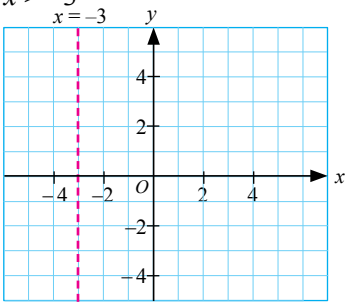




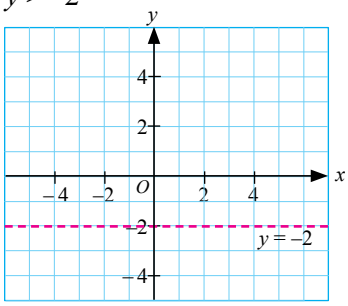
(d) $y \geq -2x - 2$



(e) $x > -3$

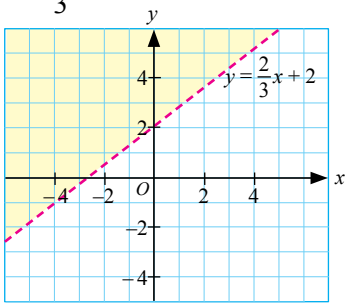


(f) $y > -2$

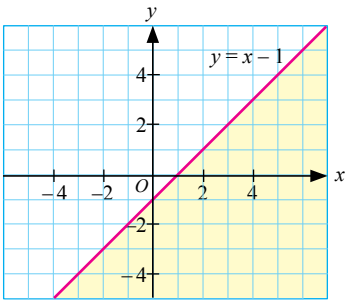


Solution:

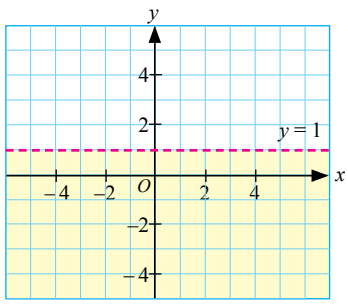
(a) $y > \frac{2}{3}x + 2$



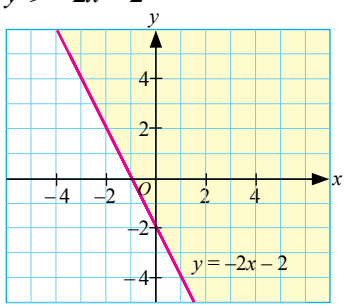
(b) $y \leq x - 1$



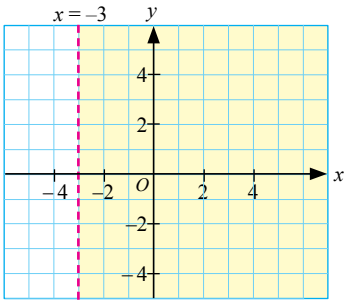
(c) $y < 1$



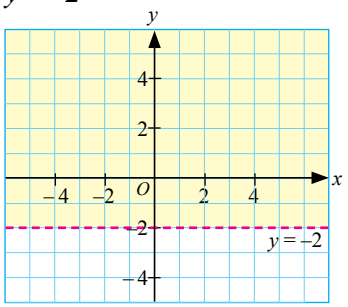
(d) $y \geq -2x - 2$



(e) $x > -3$



(f) $y > -2$



Example 6

Draw the graphs and shade the region that represents each of the following inequalities.

(a) $y \leq 2x + 3$ (b) $y > x + 5$ (c) $y \leq 2x$

(d) $x - y > 4$ (e) $y \geq 0$ (f) $x < 4$

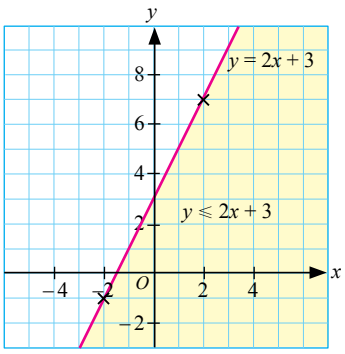
TIPS

Convert the given linear inequality to the form of a linear equation to draw the straight line.

Solution:

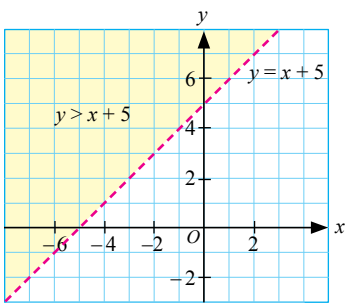
(a) $y = 2x + 3$

x	−2	2
y	−1	7



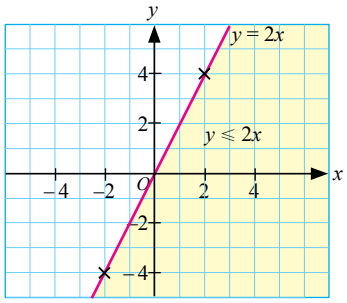
(b) $y = x + 5$

x	0	−5
y	5	0



(c) $y = 2x$

x	-2	2
y	-4	4



(d) $x - y = 4$

$x - y > 4$

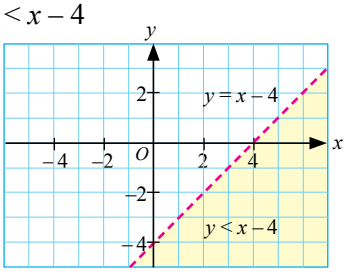
$(\times -1)$: $-x + y < -4$

$y < x - 4$

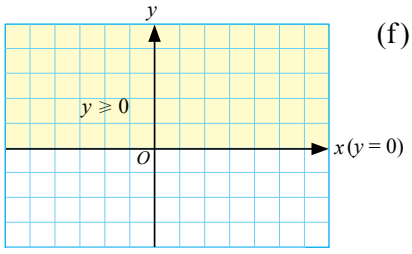
$y = x - 4$

x	0	4
y	-4	0

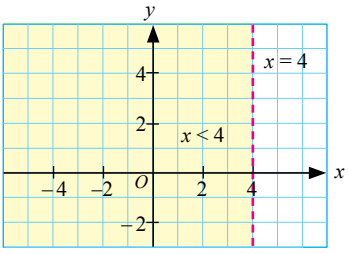
Change the coefficient of y to a positive value so that it is easier to choose the region.



(e) $y = 0$ (x-axis)



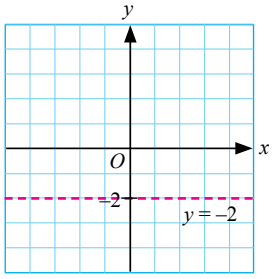
(f) $x = 4$



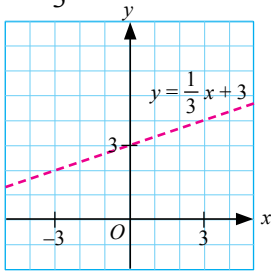
Self Practice 6.1c

1. Shade the region that represents each of the following inequalities.

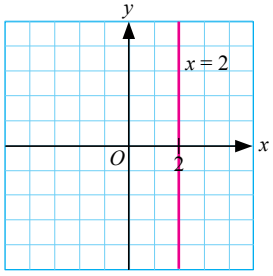
(a) $y < -2$



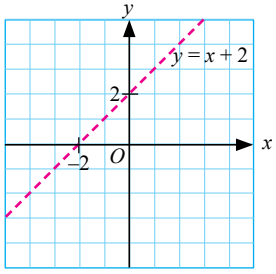
(b) $y < \frac{1}{3}x + 3$



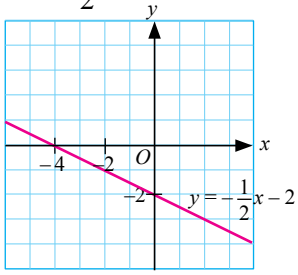
(c) $x \leqslant 2$



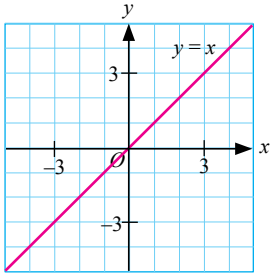
(d) $y > x + 2$



(e) $y \geqslant -\frac{1}{2}x - 2$



(f) $y \geqslant x$



2. Draw the graphs and shade the region that represents each of the following inequalities.

(a) $x \leqslant 0$ (b) $y > \frac{1}{2}x$ (c) $x + y \geqslant -3$ (d) $2y < x + 4$

(e) $y \leqslant -x + 2$ (f) $2y + x \geqslant 2$ (g) $x - y \geqslant 2$ (h) $x - y < -3$

6.2 Systems of Linear Inequalities in Two Variables

What is the meaning of system of linear inequalities?

Learning Standard

Represent situations in the form of system of linear inequalities.

A travel agency needs to transport 150 tourists and luggage with a total mass of 2 000 kg. The agency provides buses and vans to transport the tourists. A bus can carry 32 passengers and luggage with 350 kg and a van can carry eight passengers and luggage with 100 kg. The maximum number of buses provided is four. What is the maximum number of buses and vans required at a minimal cost?

The above problem can be solved by constructing several related linear inequalities and determining the region that satisfies all the linear inequalities. A combination of two or more linear inequalities is known as a **system of linear inequalities**.

How do you determine the appropriate inequality for a certain situation?

Example of situation	Linear inequality
y is greater than x	$y > x$
y is less than x	$y < x$
y is not less than x	$y \geqslant x$
y is not more than x	$y \leqslant x$
y is at least k times x	$y \geqslant kx$
y is at most k times x	$y \leqslant kx$
Maximum of y is k	$y \leqslant k$
Minimum of y is k	$y \geqslant k$
Sum of x and y is greater than k	$x + y > k$
Difference between y and x is less than k	$y - x < k$
y is more than x by at least k	$y - x \geqslant k$

Example 7

SMK Seri Permata is hosting a Leadership Camp during the mid-year holidays. y Form 4 pupils and x Form 5 pupils participate in the camp. The pupils are selected according to the following conditions:

(a) The total number of participants for the camp is at most 40.

(b) The number of Form 5 pupils is not less than the number of Form 4 pupils.

(c) At least 15 Form 4 pupils participate in the camp.

Write all the linear inequalities that satisfy the above conditions.

Indicator

$x \geqslant 0$ and $y \geqslant 0$ because x and y represent quantities.

Solution:

(a) $x + y \leqslant 40$ (b) $x \geqslant y$ (c) $y \geqslant 15$

Example 8

A computer shop sells refill ink of brands X and Y for printers. The seller needs to place an order of x number of refill ink of brand X and y number of refill ink of brand Y which cost RM14 and RM18 respectively from a supplier. The conditions for the order are as follows:

(a) The total number of ink ordered does not exceed 25 units.

(b) The number of ink of brand Y is at least twice that of brand X.

(c) The number of ink of brand Y is not more than 8 units.

(d) The total purchase does not exceed RM400.

Write all the linear inequalities that satisfy the above conditions.

Solution:

(a) $x + y \leqslant 25$ (b) $y \geqslant 2x$ (c) $y \leqslant 8$ (d) $14x + 18y \leqslant 400$

Self Practice 6.2a

1. Mr Wong bought two types of shirts from a supplier to sell in his shop. Each shirt of brand X costs RM8 and each shirt of brand Y costs RM12. The conditions for the purchase of the shirts are as follows:

(a) The total number of shirts bought is not more than 50.
(b) The number of shirts of brand X is at least twice the number of shirts of brand Y.
(c) The total purchase does not exceed RM850.

Write all the linear inequalities that satisfy the above conditions.

2. A factory produces two types of sports shoes. The sports shoes of type X are for children and type Y are for adults. The conditions for the production of both types of sports shoes in a week are as follows:

(a) The maximum production of shoes is 500 pairs.
(b) The number of sports shoes for children is at most three times the number of sports shoes for adults.
(c) The minimum production of sports shoes for adults is 200 pairs.

Write all the linear inequalities that satisfy the above conditions.

3. Encik Musa has a piece of land for growing vegetables. He wants to grow two types of chilli plants, green chilli and *cili padi*. The conditions for growing the two types of chilli plants are as follows:

(a) The total number of chilli plants that can be grown is at most 250.

(b) The number of green chilli plants is at least three times the number of *cili padi* plants.

(c) The minimum number of green chilli plants is 100.

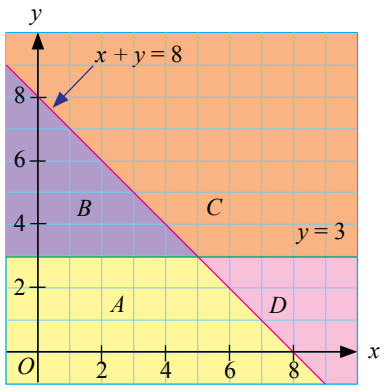
Write all the linear inequalities that satisfy the above conditions.

How do you determine the conjecture about the points in the region that satisfy a system of linear inequalities?

Learning Standard

Make and verify the conjecture about the points in the region and solution of linear inequality system.

The points that satisfy all the linear inequalities in a system of linear inequalities can be determined by identifying the appropriate region.



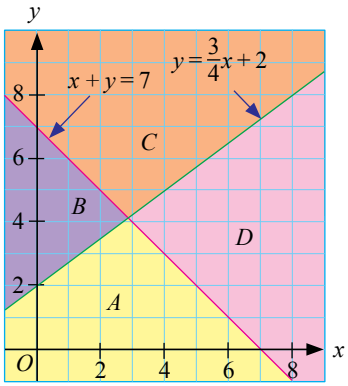
The above diagram shows the regions in a system of linear inequalities. Only one region from regions A, B, C and D satisfies both the linear inequalities $x + y \leq 8$ and $y \geq 3$. You can determine the region by substituting at least one point in the region into the system of linear inequalities.

Region	Point	Inequality $x + y \leq 8$	True/False	Inequality $y \geq 3$	True/False
A	(2, 1)	$2 + 1 \leq 8$	True	$1 \geq 3$	False
	(4, 2)	$4 + 2 \leq 8$	True	$2 \geq 3$	False
B	(2, 5)	$2 + 5 \leq 8$	True	$5 \geq 3$	True
	(3, 4)	$3 + 4 \leq 8$	True	$4 \geq 3$	True
C	(2, 7)	$2 + 7 \leq 8$	False	$7 \geq 3$	True
	(7, 4)	$7 + 4 \leq 8$	False	$4 \geq 3$	True
D	(7, 2)	$7 + 2 \leq 8$	False	$2 \geq 3$	False
	(8, 1)	$8 + 1 \leq 8$	False	$1 \geq 3$	False

From the table, it is found that only points from region B satisfy both the inequalities that are tested. Thus, region B satisfies the inequalities $x + y \leq 8$ and $y \geq 3$.

Example 9

Based on the diagram, determine the region that satisfies the inequalities $y \leq \frac{3}{4}x + 2$ and $x + y \geq 7$.



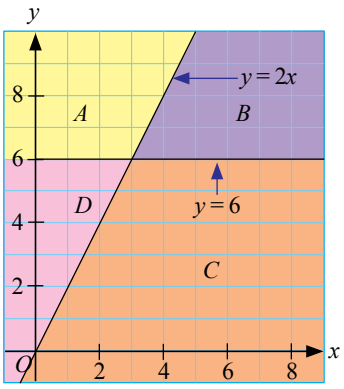
Solution:

Region	Point	$y \leq \frac{3}{4}x + 2$	True/ False	$x + y \geq 7$	True/ False
A	(4, 2)	$2 \leq \frac{3}{4}(4) + 2$	True	$4 + 2 \geq 7$	False
B	(1, 5)	$5 \leq \frac{3}{4}(1) + 2$	False	$1 + 5 \geq 7$	False
C	(4, 7)	$7 \leq \frac{3}{4}(4) + 2$	False	$4 + 7 \geq 7$	True
D	(8, 6)	$6 \leq \frac{3}{4}(8) + 2$	True	$8 + 6 \geq 7$	True

Region D satisfies both the inequalities $y \leq \frac{3}{4}x + 2$ and $x + y \geq 7$.

Self Practice 6.2b

1. Based on the diagram, determine the region that satisfies each of the following systems of linear inequalities.



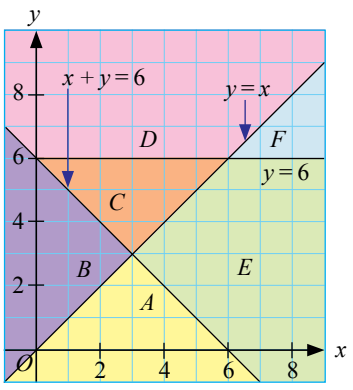
(a) $y \geq 2x$ and $y \leq 6$.

(b) $y \geq 2x$ and $y \geq 6$.

(c) $y \leq 2x$ and $y \leq 6$.

(d) $y \leq 2x$ and $y \geq 6$.

2. Based on the diagram, determine the region that satisfies each of the following systems of linear inequalities.



(a) $y \leq 6, y \leq x$ and $x + y \geq 6$.

(b) $y \leq 6, y \geq x$ and $x + y \geq 6$.

(c) $y \leq 6, y \leq x$ and $x + y \leq 6$.

(d) $y \geq 6, y \geq x$ and $x + y \geq 6$.

How do you determine and shade the region that satisfies a system of linear inequalities?

Learning Standard

Determine and shade the region that satisfies a linear inequality system.

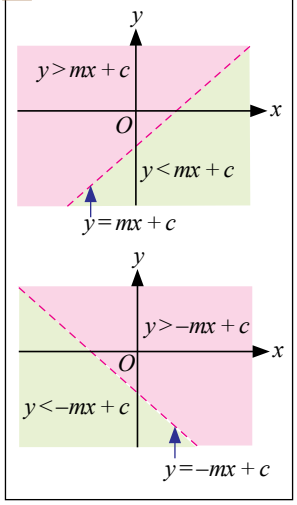
The region that satisfies a system of linear inequalities can be determined by the following steps:

(a) Mark the region involved for each linear inequality with different and easily spotted markings.

(b) Identify the **common region** for all the markings involved.

(c) Shade the common region completely. Make sure that the shading is not outside the common region.

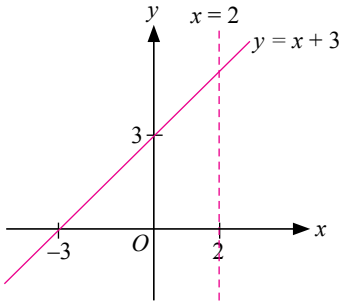
MY MEMORY



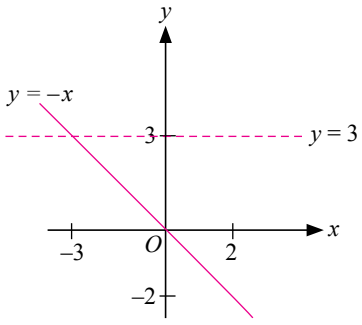
Example 10

Shade the region that satisfies each of the following systems of linear inequalities.

(a) $y \leqslant x + 3$ and $x < 2$



(b) $y \geqslant -x$, $y < 3$ and $y \geqslant 0$

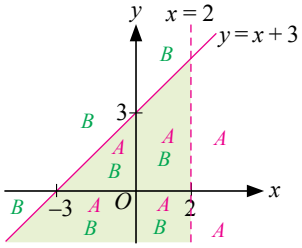


Solution:

(a) (i) Mark the region that satisfies $y \leqslant x + 3$ with the letter A.

(ii) Mark the region that satisfies $x < 2$ with the letter B.

(iii) Shade the common region marked by both the letters A and B.

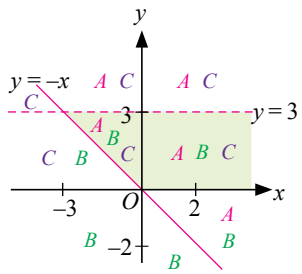


(b) (i) A represents the region $y \geqslant -x$.

B represents the region $y < 3$.

C represents the region $y \geqslant 0$.

$y \geqslant 0$ is the region above straight line $y = 0$, that is the x-axis.



(ii) Shade the common region marked with the three letters A, B and C.

Example 11

Draw and shade the region that satisfies the system of linear inequalities $2y \geqslant x$, $x + y < 4$ and $x \geqslant 0$.

Solution:

(a) Convert the linear inequalities to linear equations and draw the straight lines that represent the equations.

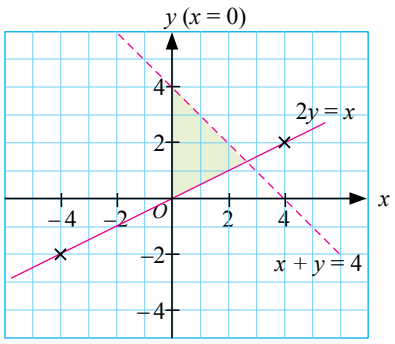
$2y = x$

x	-4	4
y	-2	2

$x + y = 4$

x	0	4
y	4	0

$x = 0$ (y-axis)



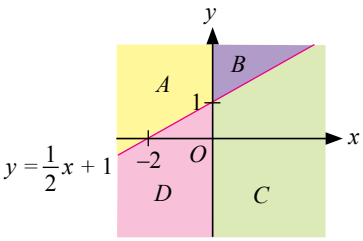
(b) Draw the straight lines of the equations involved.

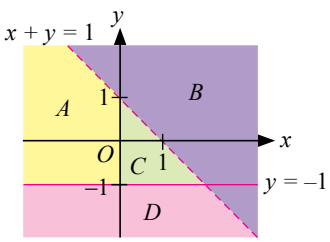
(c) Identify the common region and then shade the region.

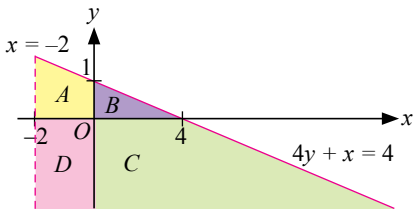
Self Practice 6.2c

1. Identify the region that satisfies each of the following systems of linear inequalities.

(a) $y \leqslant \frac{1}{2}x + 1$ and $x \geqslant 0$ (b) $x + y < 1$, $y \geqslant -1$ and $x \geqslant 0$ (c) $4y + x \leqslant 4$, $x > -2$, $x \leqslant 0$ and $y \geqslant 0$

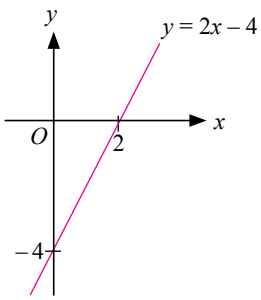


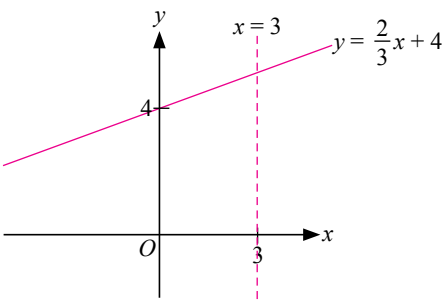




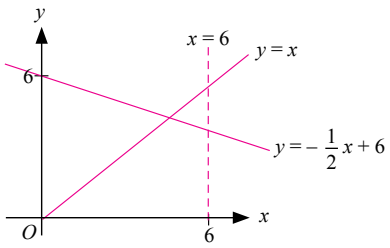
2. Shade the region that satisfies each of the following systems of linear inequalities.

(a) $y \geqslant 2x - 4$, $x \geqslant 0$ and $y \leqslant 0$ (b) $y \leqslant \frac{2}{3}x + 4$, $x < 3$, $x \geqslant 0$ and $y \geqslant 0$

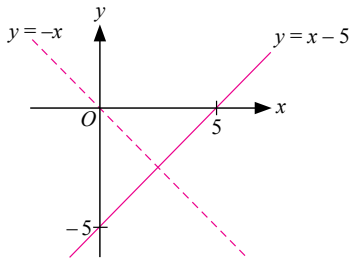




(c) $y \leqslant -\frac{1}{2}x + 6$, $y \leqslant x$, $x < 6$ and $y \geqslant 0$



(d) $y < -x$, $y \leqslant x - 5$ and $y > -5$



3. Sketch and shade the region that satisfies each of the following systems of linear inequalities.

(a) $y < -2x + 6$, $x \geqslant 0$ and $y \geqslant 0$

(b) $y \geqslant -\frac{1}{2}x + 2$, $y \leqslant x + 2$ and $x < 4$

(c) $y \leqslant -x + 8$, $y \geqslant -2x + 8$ and $x < 4$

(d) $y - x \leqslant 6$, $y \geqslant x$, $y \leqslant -x + 6$ and $y \geqslant 0$

How do you solve problems involving system of linear inequalities in two variables?

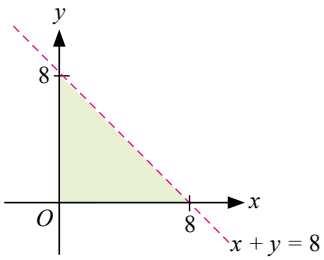
Learning Standard

Solve problems involving systems of linear inequalities in two variables.

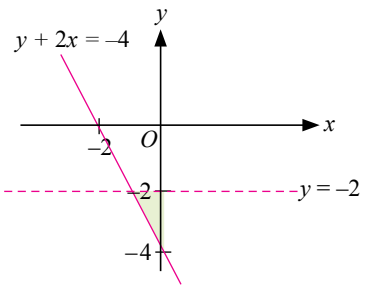
Example 12

State three linear inequalities that define the shaded region in each of the following diagrams.

(a)



(b)



TIPS

Linear equations for
- x-axis ⇨ $y = 0$
- y-axis ⇨ $x = 0$

Solution:

(a) The three straight lines involved are $x + y = 8$, the x-axis and the y-axis.

(i) The shaded region is below the straight line $x + y = 8$ and is drawn with a dashed line, thus $x + y < 8$.

(ii) The shaded region is above the x-axis, thus $y \geqslant 0$.

(iii) The shaded region is to the right of the y-axis, thus $x \geqslant 0$.

The three linear inequalities that satisfy the shaded region are $x + y < 8$, $y \geqslant 0$ and $x \geqslant 0$.

(b) The three straight lines involved are $y + 2x = -4$, $y = -2$ and the y-axis.

(i) The shaded region is above the straight line $y + 2x = -4$ and is drawn with a solid line, thus $y + 2x \geqslant -4$.

(ii) The shaded region is below the straight line $y = -2$ and is drawn with a dashed line, thus $y < -2$.

(iii) The shaded region is to the left of the y-axis, thus $x \leqslant 0$.

The three linear inequalities that satisfy the shaded region are $y + 2x \geqslant -4$, $y < -2$ and $x \leqslant 0$.

Example 13

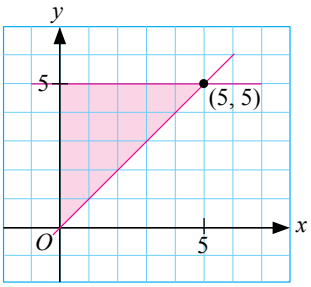
Write three linear inequalities that satisfy the shaded region in each diagram below.

MY MEMORY

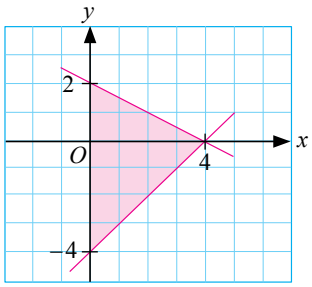
Gradient

$m = \frac{y_2 - y_1}{x_2 - x_1}$

(a)

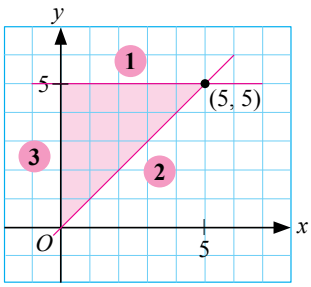


(b)



Solution:

(a)



(i) Equation 1

The straight line is parallel to the x-axis.

Thus, $y = 5$

(ii) Equation 2

Gradient, $m = \frac{5-0}{5-0} = 1$

y-intercept, $c = 0$

Thus, $y = x$

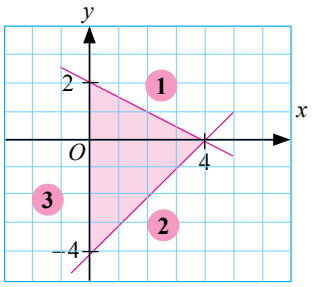
(iii) Equation 3

The straight line is the y-axis.

Thus, $x = 0$

The three linear inequalities that satisfy the shaded region are $y \leqslant 5$, $y \geqslant x$ and $x \geqslant 0$.

(b)



(i) Equation 1

Gradient, $m = \frac{2-0}{0-4} = -\frac{1}{2}$

y-intercept, $c = 2$

Thus, $y = -\frac{1}{2}x + 2$

(ii) Equation 2

Gradient, $m = \frac{-4-0}{0-4} = 1$

y-intercept, $c = -4$

Thus, $y = x - 4$

(iii) Equation 3

The straight line is the y-axis.

Thus, $x = 0$

The three linear inequalities that satisfy the shaded region are $y \leqslant -\frac{1}{2}x + 2$, $y \geqslant x - 4$ and $x \geqslant 0$.

Example 14

Madam Carol needs to select at most 20 pupils for a choir competition. The number of girls is at least twice the number of boys.

(a) Write two linear inequalities other than $x \geqslant 0$ and $y \geqslant 0$, which represent the conditions of the selection of the competitors.

(b) Draw and shade the region that satisfies the above system of linear inequalities.

(c) From the graph,

(i) determine the minimum and maximum number of girls when the number of boys is five.

(ii) determine whether the conditions of the selection are adhered to if eight boys want to participate in the choir competition. Justify your answer.

Solution:

(a) Let x = number of girls and y = number of boys

$x + y \leqslant 20$

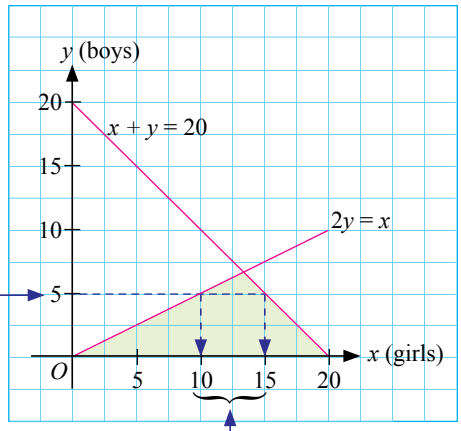
$x \geqslant 2y$ or $2y \leqslant x$ ← Make y the subject of the formula so that the region can be marked correctly.

(b) $x + y = 20$

x	0	20
y	20	0

$2y = x$

x	0	20
y	0	10



(c) (i) $y = 5$ boys
$x = 10$ girls (minimum)
$x = 15$ girls (maximum)

(ii) No, because the value of $y = 8$ is outside the shaded region.

Indicator

The graph of an inequality is drawn in the first quadrant only because the situation involves the variables x and y which represent quantities.

INFO ZONE

A system of linear inequalities is the basis used in the field of business to obtain the maximum profit using the minimum cost that involves limited resources such as manpower, raw materials and finance.

Smart Mind

Construct appropriate linear inequalities for the following conditions.

(a) y is at most three times x.

(b) The sum of x and y is less than 100.

(c) x exceeds y by at least 20.

(d) The difference between y and x is less than 50.

Example 15

The maximum number of passengers on a train in a theme park is 30. As a security measure, the required number of adults is always greater than or equal to the number of children.

(a) Write two linear inequalities, other than $x \geq 0$ and $y \geq 0$, which represent the given situation.

(b) Draw and shade the region that satisfies the above system of linear inequalities.

(c) From the graph, determine the maximum number of children that is allowed to board the train.

(d) If there are 18 children, can all the children board the train at the same time? Justify your answer.

Solution:

Understanding the problem

(a) Determine two linear inequalities according to the conditions in the situation.

(b) Draw and shade the region for the system of linear inequalities.

(c) Determine the related value from the graph.

Planning a strategy

(a) Let x = number of children and y = number of adults

(b) A graph of linear inequalities is drawn in the first quadrant.

(c) The value is determined from the common region.

Implementing the strategy

(a) $x + y \leq 30$

$y \geq x$

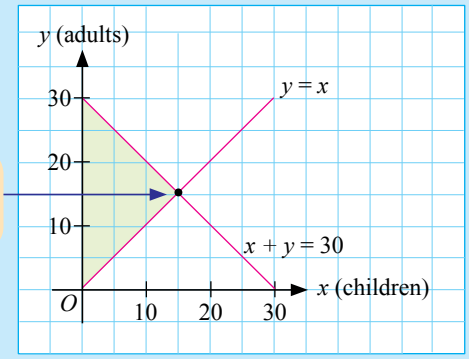
(b) $x + y = 30$

x	0	30
y	30	0

$y = x$

x	0	30
y	0	30

Maximum number of children



(c) The maximum number of children is 15.

(d) No, because the maximum number of children is only 15.

or

No, because the value of $x = 18$ is outside the shaded region.

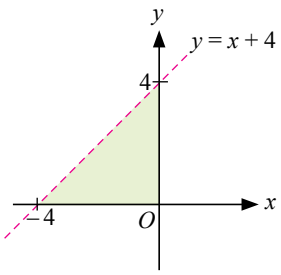
Conclusion

All the points in the common region satisfy the system of linear inequalities constructed based on the given conditions in the situation.

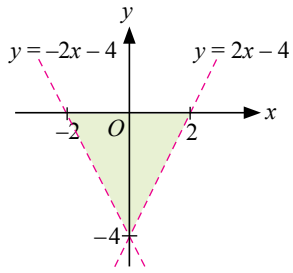
Self Practice 6.2d

1. State three linear inequalities that define the shaded region in each of the following diagrams.

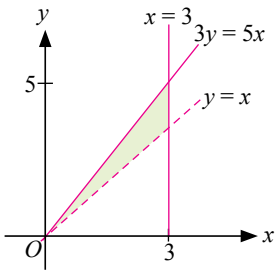
(a)



(b)

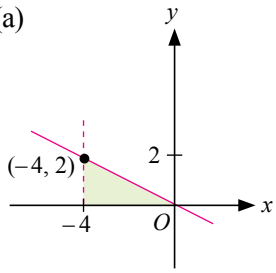


(c)

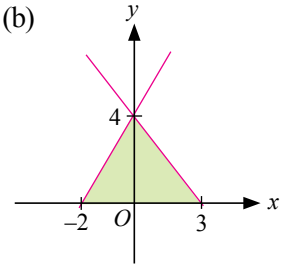


2. Write the inequalities that satisfy the shaded region in each of the following diagrams.

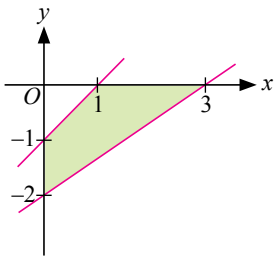
(a)



(b)



(c)



3. Mr Timothy wants to buy x doughnuts and y curry puffs to be donated to the school in conjunction with Canteen Day. The total number of both types of pastries is at most 150 and the number of doughnuts is at least twice the number of curry puffs.

(a) Write two linear inequalities, other than $x \geqslant 0$ and $y \geqslant 0$, which represent the conditions of the purchase of the pastries by Mr Timothy.

(b) Draw and shade the region that satisfies the above system of linear inequalities.

(c) From the graph, determine

(i) the maximum number of curry puffs purchased.

(ii) the minimum and maximum number of doughnuts that Mr Timothy can buy if he buys 25 curry puffs.

4. Mrs Kiran Kaur needs to buy curtain fabrics for her new house. She buys x metres of floral fabrics and y metres of abstract fabrics. The total length of both types of fabrics is not exceeding 120 metres. The length of the abstract fabrics is at least one third of the length of the floral fabrics.

(a) Write two linear inequalities, other than $x \geqslant 0$ and $y \geqslant 0$, which represent the conditions of the purchase of curtain fabrics by Mrs Kiran Kaur.

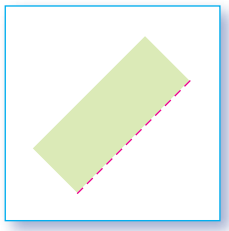
(b) Draw and shade the region that satisfies the above system of linear inequalities.

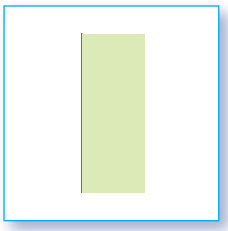
(c) From the graph, determine the maximum length, in metres, of the floral fabrics purchased.

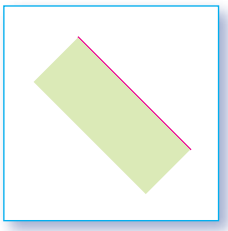
(d) Mrs Kiran Kaur buys 60 metres of abstract fabrics and 80 metres of floral fabrics. Does the above purchase satisfy the system of linear inequalities that you have constructed? Justify your answer.

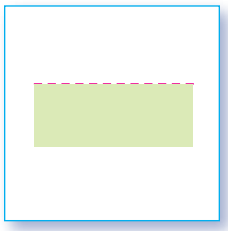
Comprehensive Practice

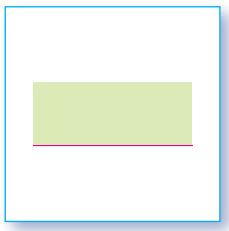
1. Write the linear inequalities that match the given sketch of the regions.

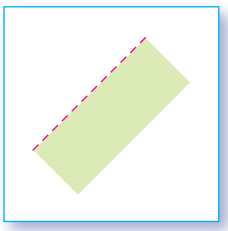
(a) 

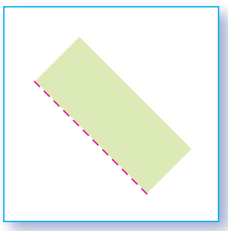
(b) 

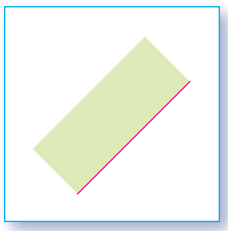
(c) 

(d) 

(e) 

(f) 

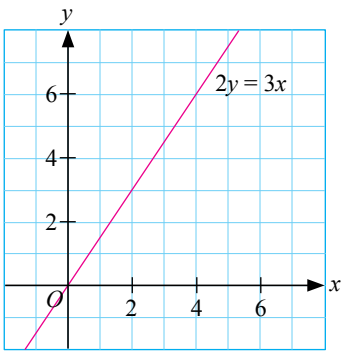
(g) 

(h) 

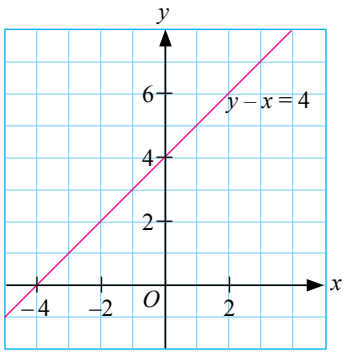
$y > -x - 3$	$y < 4$	$y \leqslant 4 - x$	$x \geqslant 0$	$y \geqslant 0$
$\frac{1}{2}y - x \geqslant 4$	$y < 2x - 5$	$3y + x > 4$	$x \leqslant 2 - y$	$y < -1$
$2y > x + 5$	$-y > 8 - 2x$	$y + x \leqslant 2$	$2y < x$	$2y \geqslant x$
$y \leqslant -\frac{1}{2}x$	$x \geqslant -5$	$-y \leqslant 4 - x$	$y \geqslant 10$	$y - x > 8$

2. Shade the common region for each of the following systems of linear inequalities.

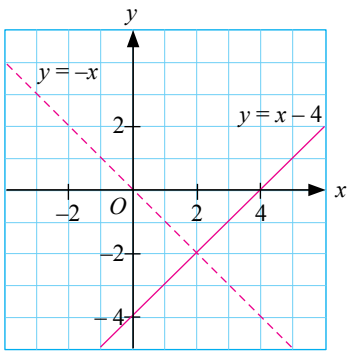
(a) $2y \leqslant 3x$, $x < 4$ and $y \geqslant 0$



(b) $y - x \geqslant 4$, $y < 4$ and $x > -4$

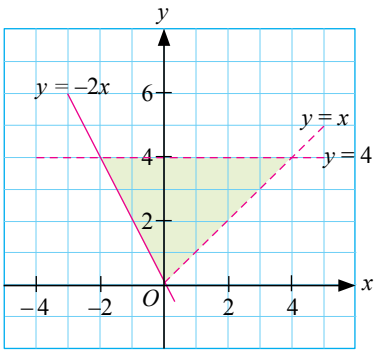


(c) $y > -x$, $y \geqslant x - 4$, $y \leqslant 2$ and $y \geqslant 0$

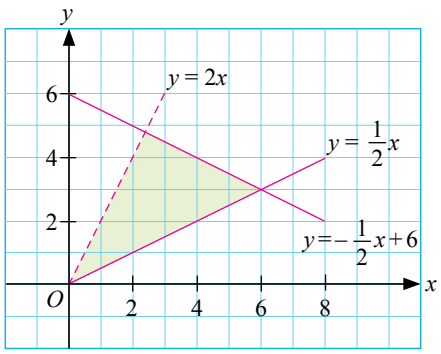


3. Write three linear inequalities that satisfy the shaded region in each of the following diagrams.

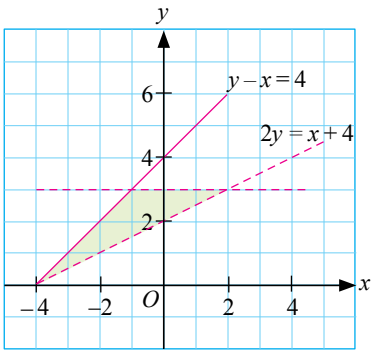
(a)



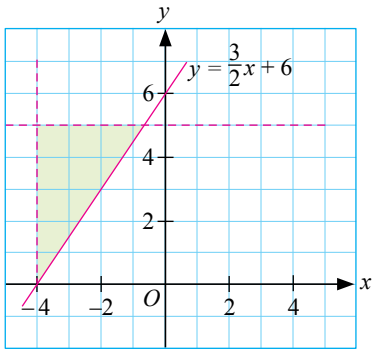
(b)



(c)



(d)



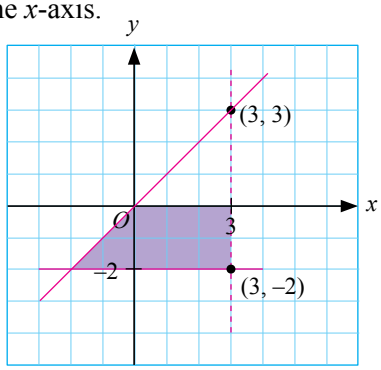
4. Draw and shade the common region for each of the following systems of linear inequalities.

(a) $y + x \geqslant 10$, $y \geqslant x$ and $y < 10$

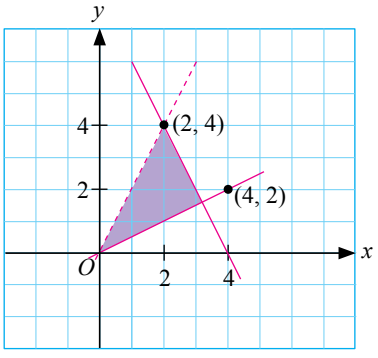
(b) $y \leqslant x + 6$, $y \geqslant \frac{2}{3}x + 4$, $y > -x$ and $x \leqslant 0$

5. Write the linear inequalities that satisfy the image of the shaded region under a reflection in the x-axis.

(a)

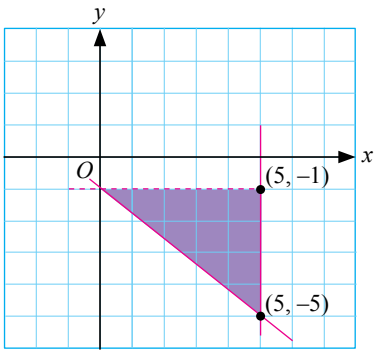


(b)

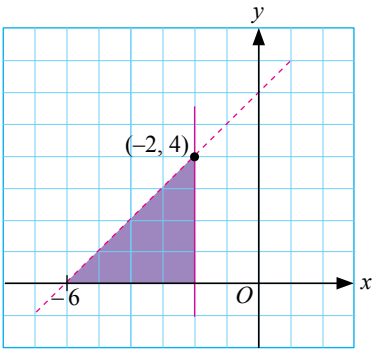


6. Write the linear inequalities that satisfy the image of the shaded region under a reflection in the y-axis.

(a)



(b)



7. Puan Jasmin is a seamstress. She sews two types of *baju kurung*; *baju kurung pesak* and *baju kurung moden*. Assuming Puan Jasmin sews x pairs of *baju kurung pesak* and y pairs of *baju kurung moden* in a certain month. The information below is related to both types of *baju kurung* sewn by Puan Jasmin.

- The total number of *baju kurung* sewn is at most 40 pairs.
- The maximum number of *baju kurung pesak* is 25 pairs.
- The minimum number of *baju kurung moden* is 10 pairs.

(a) Based on the above information, write three linear inequalities, other than $x \geq 0$ and $y \geq 0$, which represent the above situation.

(b) Draw and shade the common region that satisfies the linear inequalities constructed.

(c) From the graph, determine the minimum and maximum number of *baju kurung moden* that may be sewn if the number of *baju kurung pesak* is 10 pairs.

(d) The cost of sewing a pair of *baju kurung pesak* is RM50 and a pair of *baju kurung moden* is RM75. Based on the common region, calculate the maximum income that can be earned by Puan Jasmin if she successfully sewed 15 pairs of *baju kurung pesak*.

8. Encik Aiman sells x metres of batik cloth that he purchased from supplier X and y metres of batik cloth from supplier Y. The total length of the batik cloth purchased is at most 1 000 metres. The batik cloth supplied by supplier Y is at least half of that supplied by supplier X.

(a) Based on the above purchase information, write two related linear inequalities other than $x \geq 0$ and $y \geq 0$.

(b) Draw and shade the common region that satisfies the linear inequalities constructed.

(c) From the graph, determine the minimum and maximum lengths in metres of batik cloth that is supplied by supplier Y if Encik Aiman purchased 500 metres of batik cloth from supplier X.

(d) Supplier X faces a shortage of stock. Encik Aiman is forced to purchase at least $\frac{3}{4}$ of the batik cloth from supplier Y.

(i) Write a linear inequality representing the above situation.

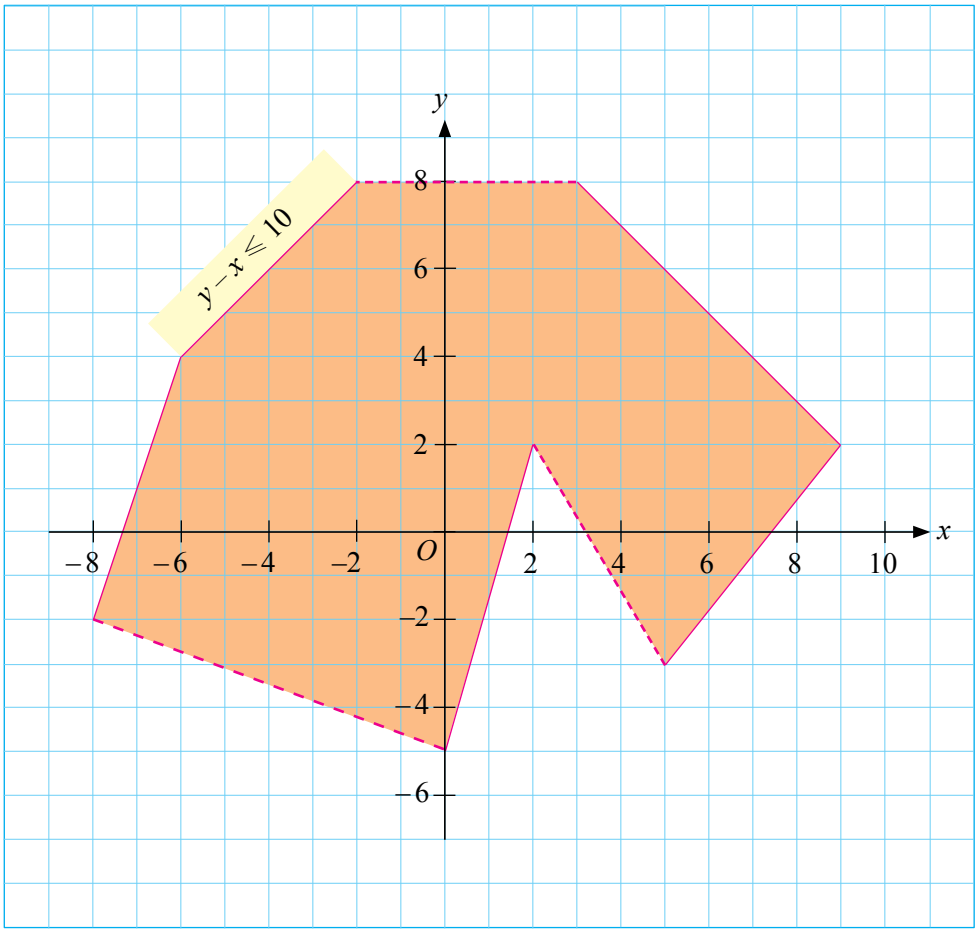
(ii) Draw a straight line representing the inequality in (d)(i).

PROJECT

1. Divide the class into groups.
2. Each group is required to draw an octagon on a Cartesian plane using a grid paper (Example 1) and write the linear inequalities that represent the shaded region on a separate sheet of paper.
3. All the materials supplied are put into envelopes and distributed to the other groups.
4. Each group is required to match the correct linear inequalities by the sides of the polygon (Example 1) in a certain period of time.
5. The leader or a representative from the original group (provider) is required to check the matching of the linear inequalities and give the score.

TIPS

Write the linear inequalities in the general form.

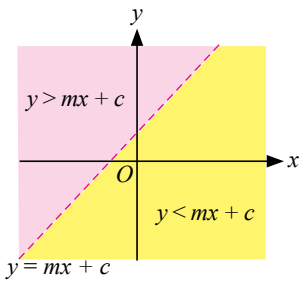


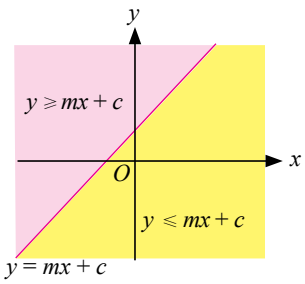
Example 1

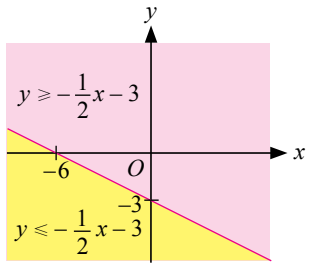
CONCEPT MAP

Linear Inequalities in Two Variables

Dashed line ⇨ points on the straight line $y = mx + c$ are not included in the region.
Solid line ⇨ points on the straight line $y = mx + c$ are included in the region.

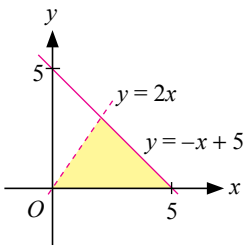




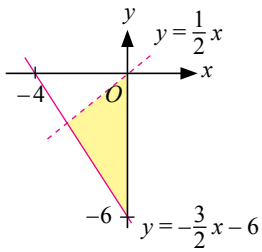


Systems of Linear Inequalities in Two Variables

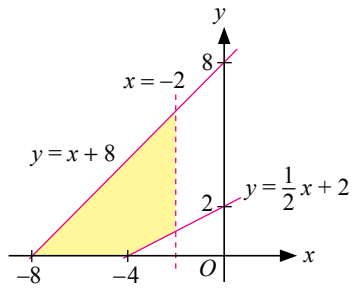
Common region ⇨ region that satisfies all the linear inequalities involved in a system of linear inequalities.



1. $y \geqslant 0$
2. $y \leqslant -x + 5$
3. $y < 2x$



1. $x \leqslant 0$
2. $y < \frac{1}{2}x$
3. $y \geqslant -\frac{3}{2}x - 6$



1. $y \geqslant 0$
2. $y \geqslant \frac{1}{2}x + 2$
3. $y \leqslant x + 8$
4. $x < -2$

Self Reflection

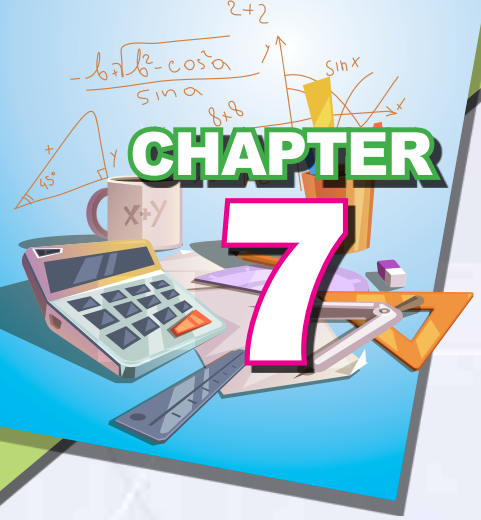
1. The _____ lines are used to draw the linear inequalities in the form $y > mx + c$ and $y < mx + c$.

2. The _____ lines are used to draw the _____ inequalities in the form $y \geqslant mx + c$ and $y \leqslant mx + c$.

3. A region that satisfies a _____ of linear inequalities is known as the _____ region.

Mathematics Exploration

1. Download a dynamic geometry software.
2. Type a linear inequality. You can click on the keyboard at the bottom of the display.
3. Press Enter each time you want to add another linear inequality to form a common region.
4. You can also see the common region for a combination of linear inequalities and non-linear inequalities.



CHAPTER

7 Graphs of Motion

You will learn

- Distance-Time Graphs
- Speed-Time Graphs

Mohamad Ridzuan Puzi, our national Paralympic athlete, created history by setting a world record in the 100 m T36 men's event and the Asian Games record in the long jump event by winning three overall gold medals. The best record in the blue ribbon event, that is 11.87 s, is a new world record. Ridzuan also broke the long jump Asian Games record with his best personal record of 5.59 m and won a gold medal in the 400-metre event. Mohamad Ridzuan Puzi became the first national athlete to be crowned the 2018 Best Asian Para Athlete (Male) in a special ceremony held in the United Arab Emirates (UAE) on 5 February 2019.

What is the technique used by runners to win a certain race?

Why Study This Chapter?

Knowledge about motion is important in the automotive field, sports science, physics, engineering and astronomy.

WORD BANK

• distance-time graph	• *graf jarak-masa*
• speed-time graph	• *graf laju-masa*
• distance	• *jarak*
• speed	• *laju*
• uniform speed	• *laju seragam*
• deceleration	• *nyahpecutan*
• acceleration	• *pecutan*

Walking Through Time

Nicholas Oresme
(1323 – 1382)

Nicholas Oresme was an important mathematician and scientist from France in the 14th century. He used the rectangular coordinate system and is said to be the first person who produced speed-time graphs.

http://bt.sasbadi.com/m4183

7.1 Distance-Time Graphs

What do you understand about distance-time graphs?

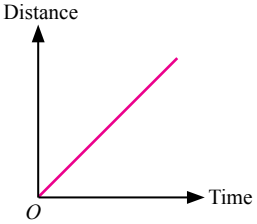
Learning Standard

Draw distance-time graphs.

Have you ever used public transport to go to a certain destination? The ticket for the journey, especially flight ticket, has a display of departure time and the estimated time of arrival at your destination. For example, the estimated duration of a domestic flight from Kuala Lumpur International Airport 2 (KLIA2) to Miri is 2 hours 20 minutes. Do you know how the duration is estimated?

Do you still remember the relationship between speed, distance and time that you have studied in Form 2?

Speed is a rate which involves distance and time. The relationship between speed and time can be represented by a distance-time graph. A distance-time graph enables the motion of an object to be represented in the graphic form which is easy to be understood.



In a distance-time graph:

- the vertical axis represents distance
- the horizontal axis represents time
- the gradient of the graph represents the **rate of change in distance with respect to time**, that is **speed**.

How do you draw a distance-time graph?

A distance-time graph can be drawn if the following information regarding the motion is obtained.

(a) Distance-time table

(b) Equation that represents the relationship between distance and time

Draw a distance-time graph based on a distance-time table

Example 1

Azree wants to be a track cyclist and hopes to make Malaysia well-known in the world like Azizulhasni Awang, our national track cycling champion. The table below shows the distance and the time taken by Azreen during the training.

Time (minutes)	0	30	60	90	120
Distance (km)	0	10	20	30	40

Draw a distance-time graph based on the above table.

Malaysiaku

Azizulhasni Awang is also nicknamed *The Pocket Rocketman*. Why?

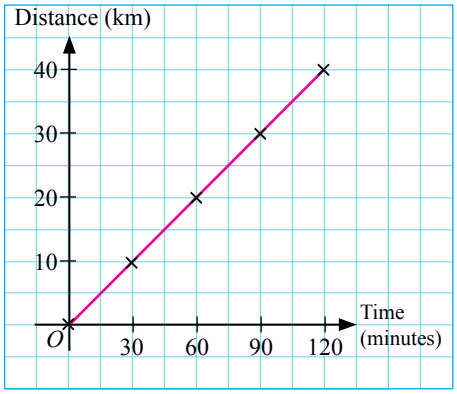
Solution:

Steps

(a) Choose suitable scales to represent the given distance and time.

(b) Plot a point that represents each pair of values of distance and time on a grid paper or graph paper.

(c) Join the points plotted using a ruler to obtain the distance-time graph as shown.



Draw distance-time graphs based on equations that represent the relationship between distance and time

Example 2

Mr Selva drove his car for a distance of 240 km from Kuala Lumpur to Kuantan in 3 hours to visit his mother. The distance, s km, that Mr Selva travelled in t hours is given by the equation $s = 80t$. Draw a distance-time graph to represent Mr Selva's journey from Kuala Lumpur to Kuantan.

Solution:

Steps

(a) Construct a distance-time table as shown below using the equation $s = 80t$.

Time, t (hours)	0	1	2	3
Distance, s (km)	0	80	160	240

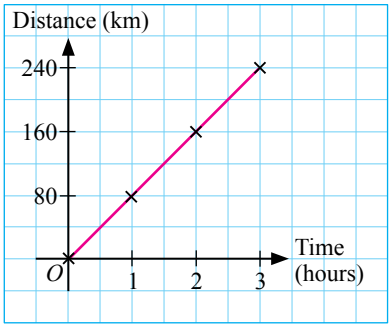
$s = 80t$ when $t = 0$, $s = 80(0)$, $s = 0$

$s = 80t$ when $t = 3$, $s = 80(3)$, $s = 240$

TIPS

A straight line can be drawn by joining at least two of the plotted points.

(b) Draw the distance-time graph as shown by joining the plotted points.



Example 3

Jaswinder Singh takes part in an 8-kilometre cross country event organised by the school during National Sports Day. The relationship between distance from the finishing line and the time of the run is $s = 8 - \frac{1}{5}t$, where s is the distance in km and t is the time in minutes. Draw a distance-time graph to represent Jaswinder Singh's run for the duration $0 \leqslant t \leqslant 40$.

Solution:

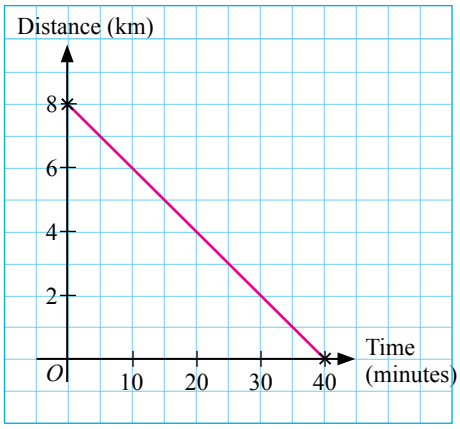
Given $s = 8 - \frac{1}{5}t$

Time, t (minutes)	0	40
Distance, s (km)	8	0

2 points that satisfy the equation are enough to draw the straight line.

$s = 8 - \frac{1}{5}t$
when $t = 0$,
$s = 8 - \frac{1}{5}(0)$
$s = 8$

$s = 8 - \frac{1}{5}t$
when $t = 40$,
$s = 8 - \frac{1}{5}(40)$
$s = 0$



Self Practice 7.1a

1. The table below shows the time taken by Haji Ali to walk from his house to the mosque for prayers. He needs 20 minutes to walk to the mosque which is located 300 m from his house. Draw a distance-time graph based on the given table.

Time (minutes)	0	5	10	15	20
Distance (metres)	0	75	150	225	300

2. Encik Nyambek drives his car to his office which is located 45 km from his house in Bekenu. The table below shows the time taken by Encik Nyambek to reach his office in Miri from Bekenu. Draw a distance-time graph based on the given table.

Time (hours)	0	0.2	0.4	0.6	0.8	1.0
Distance (km)	0	9	18	27	36	45

3. The motion of a particle for a certain period is represented by $s = 8t + 5$ where s is the distance in cm and t is the time in seconds. Draw a distance-time graph to represent the motion of the particle for a period of 5 seconds.

4. Leong cycles to Zainal's house which is located 1.6 km from his house. Leong's journey from Zainal's house is given by the equation $s = 1.6 - 0.2t$ where s is the distance in km and t is the time in minutes. Draw a distance-time graph to represent Leong's journey for the period $0 \leqslant t \leqslant 8$.

How do you interpret distance-time graphs?

Study the two distance-time graphs below.

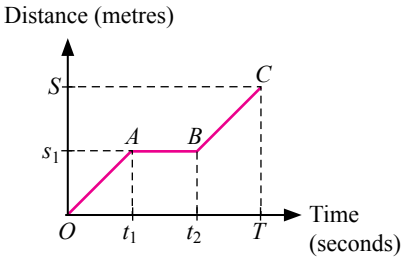


Diagram 1

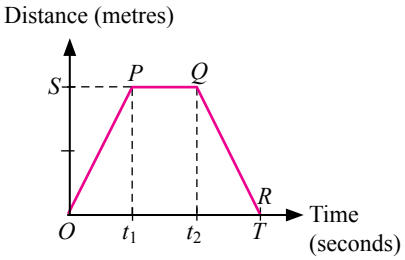


Diagram 2

Diagram 1 shows a motion from O to C for a distance of S metres in a period of T seconds.	Diagram 2 shows a motion from O to R passing through P and Q.
OA ⇨ **positive gradient** of graph ⇨ **gradient** of OA represents **speed** of motion ⇨ motion at **uniform speed** ⇨ motion for a distance of s_1 metres in a period of t_1 seconds	OP ⇨ **positive gradient** of graph ⇨ motion for a distance of S metres in a period of t_1 seconds
AB ⇨ **zero gradient** ⇨ no change in distance means motion stops (**stationary**) ⇨ stationary for the period $(t_2 - t_1)$ seconds	PQ ⇨ **zero gradient** ⇨ no change in distance (**stationary**) ⇨ stationary for the period $(t_2 - t_1)$ seconds
BC ⇨ positive gradient ⇨ motion continues to C	QR ⇨ **negative gradient** ⇨ negative speed shows object moves back to original place or moves in the opposite direction
OC ⇨ motion of S metres in a period of T seconds	OR ⇨ motion of $2S$ metres (to and fro) in a period of T seconds

Learning Standard

Interpret distance-time graphs and describe the motion based on the graphs.

MY MEMORY

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

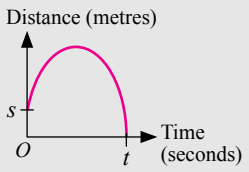
$$\text{Gradient} = \frac{\text{Vertical distance}}{\text{Horizontal distance}}$$

TIPS

Positive gradient and negative gradient of a distance-time graph indicate directions of motion.

Smart Mind

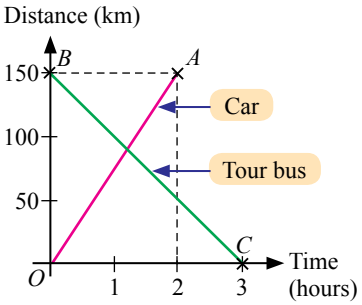
The distance-time graph below shows the motion of an object.



What type of equation will give us the distance-time graph as shown above? Discuss and state one example of the motion.

Example 4

The distance-time graph shows the motion of a car and a tour bus. Graph OA represents the motion of the car from Puchong to Melaka. Graph BC represents the motion of the tour bus from Melaka to Puchong. Determine the difference in speed, in km h^{-1}, of the two vehicles.



Solution:

Speed of car

$$= \frac{\text{difference in distance travelled}}{\text{corresponding difference in time}}$$

$$= \frac{(150 - 0)\text{ km}}{(2 - 0)\text{ hours}}$$

$$= 75 \text{ km per hour}$$

$$= 75 \text{ km h}^{-1}$$

Speed of tour bus $= \dfrac{\text{difference in distance travelled}}{\text{corresponding difference in time}}$

$$= \frac{(0 - 150)\text{ km}}{(3 - 0)\text{ hours}}$$

$$= -50 \text{ km per hour}$$

$$= 50 \text{ km h}^{-1}$$

The negative sign means the tour bus and the car move in the opposite direction.

Hence, difference in speed $= (75 - 50)$ km h^{-1}

$= 25$ km h^{-1}

TIPS

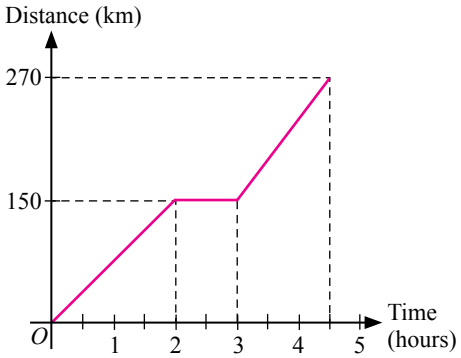
km h^{-1} can also be written as km/h.

Example 5

The distance-time graph shows the motion of a car for a period of 4.5 hours.

(a) Determine
  (i) the duration when the car is stationary.
  (ii) the speed, in km h^{-1}, of the car in the first hour.

(b) Describe the motion of the car for the last 90 minutes.



Solution:

(a) (i) Stationary period = period the car stops

$= (3 - 2)$ hour

$= 1$ hour

(ii) Speed of car in the first hour = speed of car in the first 2 hours

$$= \frac{(150 - 0)\text{ km}}{(2 - 0)\text{ hours}}$$

$$= 75 \text{ km h}^{-1}$$

From the graph, the distance travelled in the first hour is not known. Therefore, the speed is determined by the distance travelled in the first 2 hours. (The gradients on a straight line are the same).

(b) Speed of car $= \dfrac{(270 - 150)\text{ km}}{(4.5 - 3)\text{ h}}$

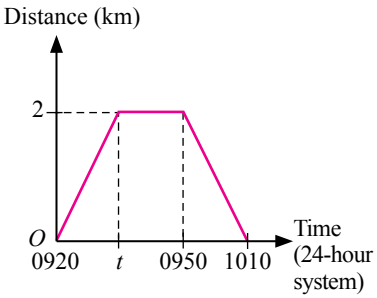
last 90 minutes

$= 80$ km h^{-1}

The car travels for 120 km with a speed of 80 km h^{-1} in the last 90 minutes.

Example 6

Sahana cycles to the post office to send Hari Raya Aidilfitri greeting cards to her close friends. The distance-time graph shows Sahana's journey from her house to the post office and back.



(a) Determine

(i) the total distance, in km, for Sahana's whole journey.

(ii) the value of t, if Sahana cycles with a speed of 8 km h^{-1} to the post office.

(b) Describe Sahana's journey from the post office back to her house.

Solution:

(a) (i) Total distance travelled = 2 km + 2 km
= 4 km

(ii) $\text{Time} = \dfrac{\text{Distance}}{\text{Speed}}$

$= \dfrac{2 \text{ km}}{8 \text{ km h}^{-1}}$

= 0.25 hour

= 15 minutes

Thus, t = 0920 + 0015
= 0935

(b) Rate of change in distance $= \dfrac{(0 - 2) \text{ km}}{\left(\dfrac{1010 - 0950}{60}\right) \text{hour}}$

1 minute $= \frac{1}{60}$ hour

$= -6 \text{ km h}^{-1}$

$= 6 \text{ km h}^{-1}$

Sahana cycles for a distance of 2 km in 20 minutes with a speed of 6 km h^{-1}.

TIPS

$\text{Speed} = \dfrac{\text{Distance}}{\text{Time}}$

$\text{Time} = \dfrac{\text{Distance}}{\text{Speed}}$

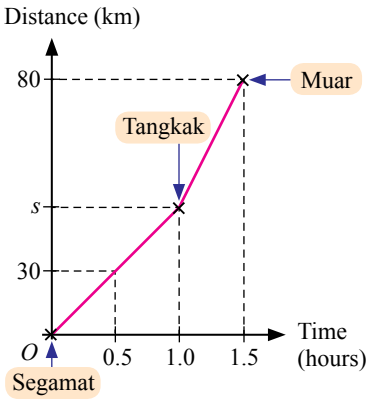
Motion with different speeds

The speed of a motion usually changes in a journey. In such situation, **average speed** is used.

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}}$$

Example 7

Zabedah wants to visit her friend in Muar. The distance-time graph shows her journey by car from Segamat to Muar passing through Tangkak.



(a) Calculate the average speed, in km h^{-1}, of Zabedah's journey from Segamat to Muar.

(b) If the rate of change in distance of the car from Segamat to Tangkak is 50 km h^{-1}, calculate the distance, in km, between Tangkak and Muar.

(c) Describe the motion of the car from Segamat to Muar.

Solution:

(a) Average speed $= \dfrac{\text{Total distance}}{\text{Total time}}$

$= \dfrac{80 \text{ km}}{1.5 \text{ h}}$

$= 53\dfrac{1}{3} \text{ km h}^{-1}$

TIPS

$\text{Speed} = \dfrac{\text{Distance}}{\text{Time}}$

Distance = Speed × Time

(b) Total distance = Average speed × Total time

$= 50 \text{ km h}^{-1} \times 1 \text{ h}$

$= 50 \text{ km}$ ← Distance between Segamat and Tangkak

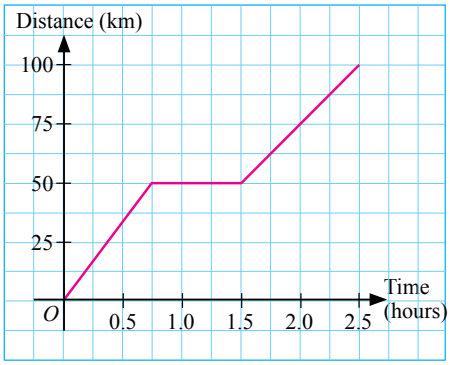
(c) The car moves for a distance of 80 km in 1.5 hours with an average speed of $53\dfrac{1}{3} \text{ km h}^{-1}$.

Distance between Tangkak and Muar = (80 – 50) km

= 30 km

Self Practice 7.1b

1. The distance-time graph shows the journey of Encik Rejab and his family by car from Kota Kinabalu to Keningau to celebrate *Pesta Kaamatan*.



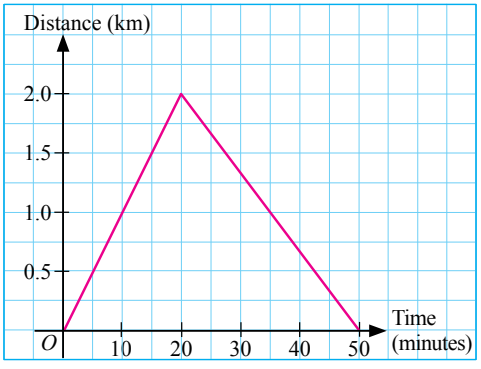
(a) Calculate the speed, in km h^{-1}, of the car for the last 1 hour.

(b) Describe the motion of Encik Rejab's car for the period of 45 minutes after travelling the first 50 km of the journey.

(c) (i) Calculate the average speed, in km h^{-1}, for the journey from Kota Kinabalu to Keningau.

(ii) Hence, describe the motion of the car for the whole journey.

2. Encik Rashid exercises every day to stay healthy. The distance-time graph shows the distance and time of Encik Rashid's run from his house to the playground and then back to his house.

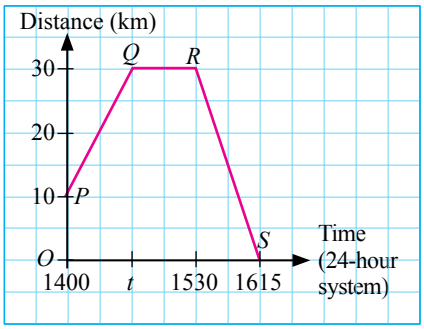


(a) Calculate the difference between the speed of Encik Rashid's run from his house to the playground and the speed of his run from the playground back to his house in km h^{-1}.

(b) Calculate the average speed, in km h^{-1}, for Encik Rashid's whole run.

(c) Describe Encik Rashid's motion for the period of 50 minutes.

3. The distance-time graph shows Puan Rozita's journey by car for a period of $2\frac{1}{4}$ hours. PQ represents Puan Rozita's journey from her workplace to a market and RS represents the return journey to her house.



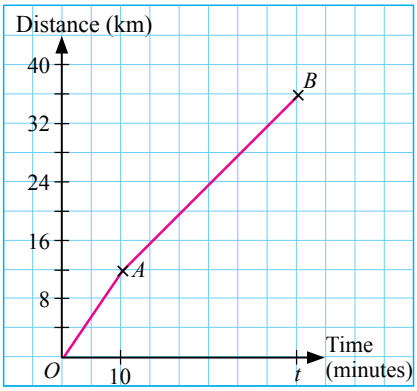
(a) Calculate the value of t, if the speed of the car for Puan Rozita's journey from her workplace to the market is 50 km h^{-1}.

(b) Describe the motion of the car represented by

(i) the straight line QR.

(ii) the straight line RS.

4. Encik Yusri works in a law firm. Every day Encik Yusri sends his child to school on his way to workplace by car. OA represents the journey from his house to school and AB represents the journey from the school to his workplace.



(a) Calculate the value of t, if the rate of change in distance of the car from the school to his workplace is 48 km h^{-1}.

(b) Describe the motion of the car for the whole journey from his house to his workplace.

How do you solve problems involving distance-time graphs?

Example 8

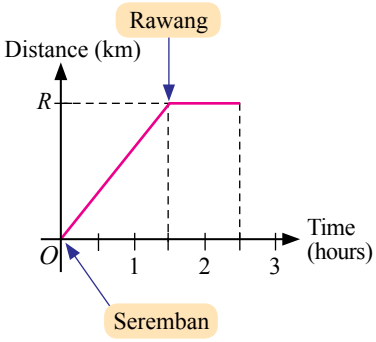
Learning Standard

Solve problems involving distance-time graphs.

The incomplete distance-time graph shows Mr Tan's journey from Seremban to Lumut. Mr Tan stops at Rawang for lunch and a short break before he continues his journey to Lumut.

(a) If the average speed of Mr Tan's car from Seremban to Rawang is $66\frac{2}{3}$ km h^{-1}, calculate the distance, in km, between Seremban and Rawang.

(b) The distance between Seremban and Lumut is 300 km and Mr Tan drives at an average speed of 80 km h^{-1} to reach Lumut from Rawang. Complete the given distance-time graph to represent Mr Tan's whole journey.



Solution:

Understanding problem

(a) Calculate the distance between Seremban and Rawang in km.

(b) Complete the distance-time graph from Rawang to Lumut.

Planning strategy

(a) $\text{Speed} = \dfrac{\text{Distance}}{\text{Time}}$

$\text{Distance} = \text{Speed} \times \text{Time}$

(b) • Determine the distance between Rawang and Lumut.

• $\text{Time} = \dfrac{\text{Distance}}{\text{Time}}$

• Complete the distance-time graph.

Carrying out strategy

(a) $\text{Distance} = \text{Speed} \times \text{Time}$

$= 66\frac{2}{3}\ \text{km h}^{-1} \times 1.5\ \text{hours}$

$= 100\ \text{km}$

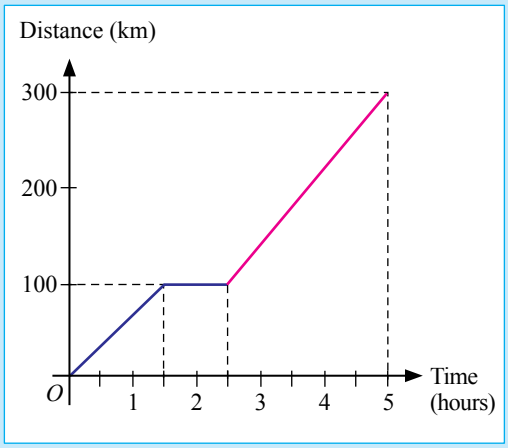
(b) Distance between Rawang and Lumut

$= 300\ \text{km} - 100\ \text{km} = 200\ \text{km}$

$\text{Time} = \dfrac{\text{Distance}}{\text{Speed}}$

$= \dfrac{200\ \text{km}}{80\ \text{km h}^{-1}}$

$= 2.5\ \text{hours}$



Conclusion

(a) The distance between Seremban and Rawang is 100 km.

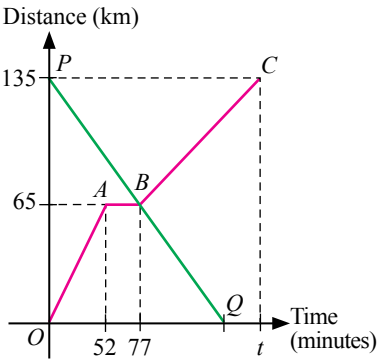
(b) The distance between Rawang and Lumut is 200 km and the time taken is 2.5 hours.

Example 9

The distance-time graph shows the journey of two cars between Kuala Lipis and Cameron Highlands. Graph *PBQ* represents Encik Manaf's journey together with his family from Cameron Highlands to Kuala Lipis to attend his cousin's wedding. Graph *OABC* represents the journey of Encik Raven's family from Kuala Lipis to Cameron Highlands for a holiday.

(a) The rates of change in distance for *OA* and *BC* are the same. Calculate the value of *t*.

(b) The average speed of Encik Manaf's journey is 72 km h^{-1}. Calculate the difference in time, in minutes, for the two journeys to reach their respective destinations.



Solution:

Understanding problem

(a) Calculate t, that is time in minutes.

(b) Difference in time of the two cars' journey to reach their respective destinations.

Planning strategy

(a) Gradient of OA = Gradient of BC

(b) • Time taken by Encik Raven = t.
• Determine the time taken by Encik Manaf.
• Time = $\frac{\text{Distance}}{\text{Speed}}$

Carrying out strategy

(a) $\frac{(65-0)\text{ km}}{(52-0)\text{ min}} = \frac{(135-65)\text{ km}}{(t-77)\text{ min}}$

$$\frac{65}{52} = \frac{70}{t-77}$$

$$t - 77 = \frac{70(52)}{65}$$

$$t = 56 + 77$$

$$t = 133$$

(b) • Total time for Encik Raven's journey, t = 133 minutes.
• Total time for Encik Manaf's journey in minutes.

$$\text{Time} = \frac{135\text{ km}}{72\text{ km h}^{-1}}$$

$$= 1.875\text{ hours} \times 60$$
$$= 112.5\text{ minutes}$$

• Difference in time = 133 – 112.5
= 20.5 minutes

Conclusion

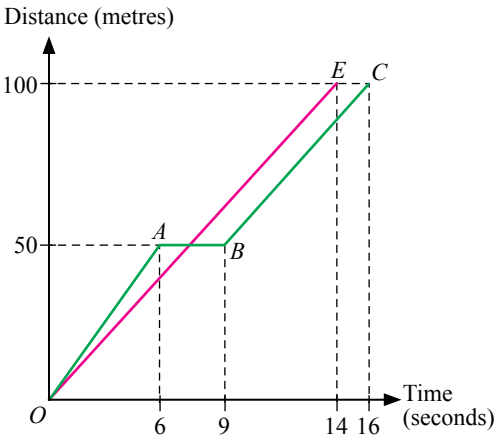
(a) t = 133
(b) The difference in time for the two journeys to reach their respective destinations is 20.5 minutes.

Self Practice 7.1c

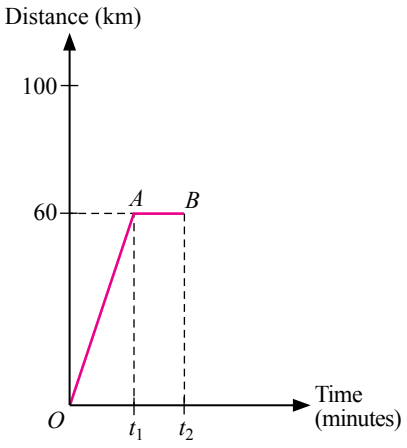
1. The distance-time graph shows the time taken by the two best participants in the 100-metre event during the Sports Championship of SMK Sinar Harapan. Graph OE represents Rizal's run and graph $OABC$ represents Jeffery's run. AB represents the time elapsed before Jeffery continued his run after a fall.

(a) Calculate the time lost, in seconds, by Jeffery in the competition.

(b) Did Jeffry have the chance to become the champion in the 100-metre event if he did not fall down and manage to maintain his speed for the whole run? Justify your answer.



2. The incomplete distance-time graph shows Encik Jumali's journey for a distance of 100 km.



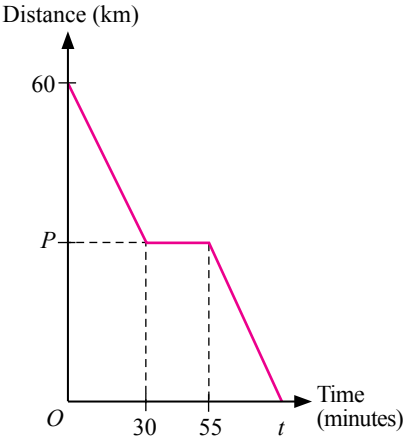
(a) It is given that the rate of change in distance for the first 60 km is 72 km h^{-1}. Calculate the value of t_1.

(b) If Encik Jumali's car is stationary for 20 minutes, calculate the value of t_2.

(c) His journey continues from B to its destination with an average speed of 75 km h^{-1}. Complete the distance-time graph for the whole journey of Encik Jumali.

(d) If the journey from O starts at 9:30 in the morning, calculate the time Encik Jumali arrives at his destination.

3. Encik Jamal goes to Padang Besar with his family. During the return journey to Jitra, they stop at Bukit Kayu Hitam for a tea break. The distance-time graph shows the return journey from Padang Besar to Jitra.



(a) Calculate the duration in which Encik Jamal's car is stationary.

(b) It is given that the average speed for the journey from Padang Besar to Bukit Kayu Hitam is 66 km h^{-1}.

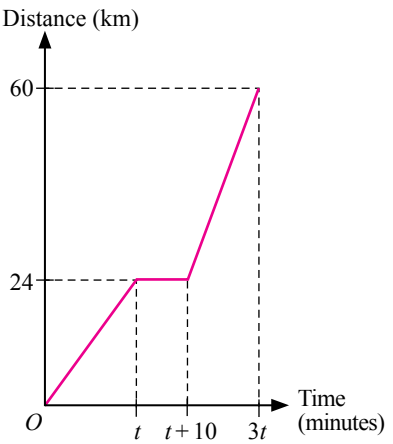
(i) Determine the value of P.

(ii) Calculate the distance between Padang Besar and Bukit Kayu Hitam.

(c) If Encik Jamal drives at an average speed of 64.8 km h^{-1} for the return journey from Bukit Kayu Hitam to his house in Jitra, calculate the value of t.

(d) Calculate the average speed, in km h^{-1}, for the whole journey.

4. The distance-time graph shows Mr Moorthy's journey by car for a distance of 60 km in $3t$ minutes. It is given that the rate of change in distance before and after the stationary period are the same.



(a) Calculate the value of t.

(b) Calculate the average speed, in km h^{-1}, for the whole journey of Mr Moorthy.

(c) Describe the motion of the car after the stationary period.

7.2 Speed-Time Graphs

What do you understand about speed-time graphs?

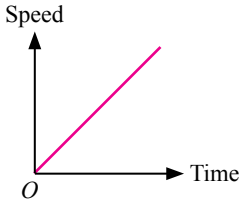
Learning Standard

Draw speed-time graphs.

Have you observed the motion of the needle in a speedometer of a car when your parents drive the car? The needle shows that the value of the speed changes when the oil pedal or brake pedal is pressed. Study the diagrams of the speedometer below.

Diagram 1 Diagram 2 Diagram 3 Diagram 4

The needle in Diagram 1 shows a value of 0, which means the vehicle is stationary. The increase in value as shown in Diagram 2 and Diagram 3 means the speed of the vehicle is increasing. The speed in Diagram 4 is decreasing compared to the speed shown in Diagram 3 for a certain period. The rate of change of speed of a motion can be shown by drawing a speed-time graph.



In a speed-time graph:
- the vertical axis represents the speed of a motion.
- the horizontal axis represents the time taken.
- the gradient of graph represents the **rate of change of speed with respect to time**, that is **acceleration**.

How do you draw a speed-time graph?

A speed-time graph can be drawn if the following information about the motion is obtained.

(a) Speed-time table
(b) Equation that represents the relationship between speed and time

How do you draw a speed-time graph based on a speed-time table?

Example 10

The table below shows the change of speed of Encik Azizul's car for a period of 5 seconds.

Time (seconds)	0	1	2	3	4	5
Speed ($m\ s^{-1}$)	0	10	20	30	40	50

Draw a speed-time graph based on the given table.

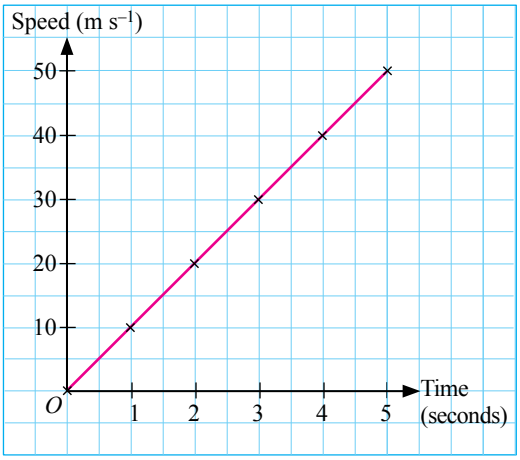
Solution:

Steps

(a) Choose appropriate scales to represent the given speed and time.

(b) Plot a point to represent each pair of values of speed and time on a grid paper or graph paper.

(c) Join the plotted points using a ruler to obtain the speed-time graph as shown.



Example 11

The rate of change of speed of an aeroplane that is landing is given by the equation $v = 800 - 1\,600t$ where v is the speed in km h^{-1} and t is time in hours. Draw a speed-time graph to represent the landing of the aeroplane for the period $0 \leqslant t \leqslant 0.5$.

Solution:

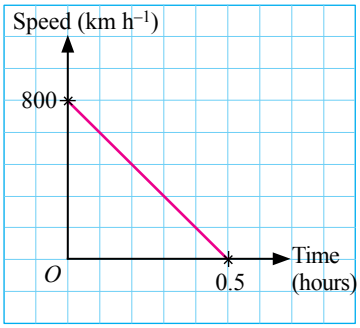
Steps

(a) Construct a speed-time table as shown below using the equation $v = 800 - 1\,600t$.

Time, t (hours)	0	0.5
Speed, v (km h^{-1})	800	0

$v = 800 - 1\,600t$ when $t = 0$, $v = 800 - 1600(0)$, $v = 800$

$v = 800 - 1\,600t$ when $t = 0.5$, $v = 800 - 1\,600(0.5)$, $v = 0$



(b) Draw the speed-time graph by plotting the points based on the table constructed.

Self Practice 7.2a

1. Draw a speed-time graph based on the given table.

(a)

Time (seconds)	0	1	2	3	4	5
Speed (m s^{-1})	3	4	5	6	7	8

(b)

Time (minutes)	0	1	2	3	4
Speed (km min^{-1})	30	25	20	15	10

2. Draw a speed-time graph by constructing a speed-time table for each of the following equations. It is given that v is the speed in m s^{-1} and t is the time in seconds.

(a) $v = 60 - 2t$; $0 \leqslant t \leqslant 30$.

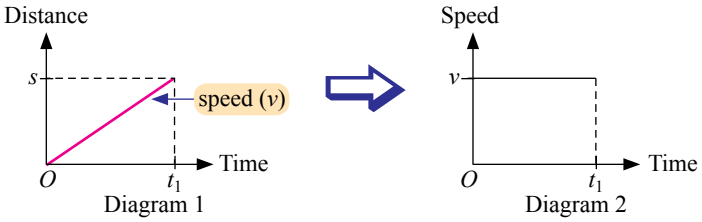
(b) $v = 3t$; $0 \leqslant t \leqslant 5$.

What is the relationship between the area under a speed-time graph and the distance travelled?

Learning Standard

Make a relationship between the area under speed-time graph and the distance travelled, and hence determine the distance.

The diagrams below show two graphs:



From the gradient of the distance-time graph in Diagram 1, we can determine the speed of the motion. This information from the distance-time graph can be used to draw the speed-time graph as shown in Diagram 2. Do you know that the distance, s, travelled by a motion can be determined from the speed-time graph?

Mind Stimulation 1

Aim: To determine the relationship between the area under speed-time graph and the distance travelled.

Steps:

1. Divide the class into groups.
2. Read and understand each of the given statements. Calculate the average speed in km h^{-1}.
3. Sketch the speed-time graphs based on the given statements.
4. Calculate the area under the speed-time graph and the distance travelled like example (a).

Statement	Speed-time graph	Area under the graph	Distance travelled
(a) Encik Faizal drives a distance of 200 km in 4 hours. Speed = $\frac{200\text{ km}}{4\text{ hours}}$ = 50 km h^{-1}	Speed (km h^{-1}) 50 O 4 Time (hours)	Area = 4 h × 50 km h^{-1} = 200 km	Distance travelled = speed × time = 50 km h^{-1} × 4 h = 200 km
(b) A tour bus moves 150 km in 2 hours.			
(c) Mrs Malini cycles 8 km in 40 minutes.			
(d) Mr Gomez runs 4 km in 30 minutes.			

Discussion:

1. What is the relationship between the area under speed-time graph and the distance travelled?
2. Present your group's findings through Gallery Walk activity.
3. Are your group's findings the same as the other groups' findings?

From the activity in Mind Stimulation 1, it is found that:

> The area under a speed-time graph is the same as the distance travelled for the same time interval.

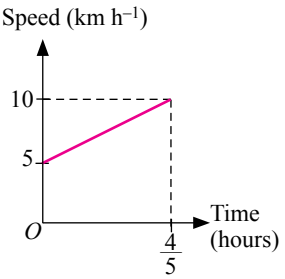
In general,

> For a speed-time graph:
> Area under the graph = Distance travelled

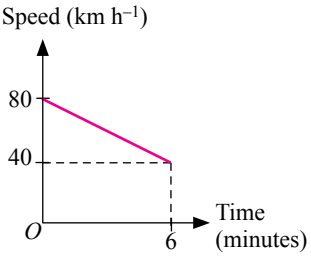
Example 12

Calculate the distance travelled in each motion based on the following speed-time graphs.

(a)



(b)

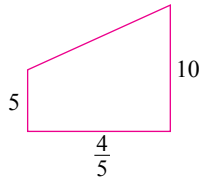


Indicator

Make sure the unit for time used in the speed is the same as the unit for time.

Solution:

(a)

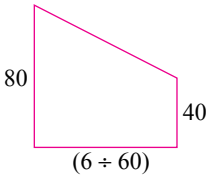


Distance = area of trapezium

$= \frac{1}{2} \times \left(\frac{4}{5}\right)$ h $\times (5 + 10)$ km h^{-1}

$= 6$ km

(b)



Distance = area of trapezium

$= \frac{1}{2} \times \left(\frac{6}{60}\right)$ h $\times (40 + 80)$ km h^{-1}

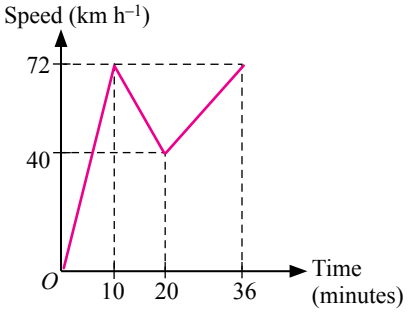
$= 6$ km

Example 13

The speed-time graph shows the speed of Mrs Liew's car for a period of 36 minutes. Calculate

(a) the total distance, in km, travelled by Mrs Liew's car for the period of 36 minutes.

(b) the average speed, in km h^{-1}, of Mrs Liew's car for the period of 36 minutes.



Solution:

(a) Total distance
= area under the graph
$= \left[\frac{1}{2} \times \left(\frac{10}{60}\right) \text{ h} \times 72 \text{ km h}^{-1}\right] + \left[\frac{1}{2} \times \left(\frac{10}{60}\right) \text{ h} \times (40 + 72) \text{ km h}^{-1}\right] + \left[\frac{1}{2} \times \left(\frac{16}{60}\right) \text{ h} \times (40 + 72) \text{ km h}^{-1}\right]$
$= \left(6 + \frac{28}{3} + \frac{224}{15}\right) \text{ km}$
= 30.27 km

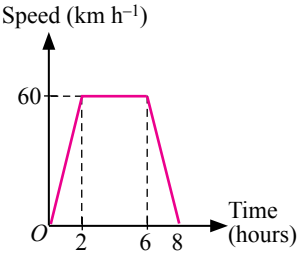
(b) Average speed $= \frac{\text{Total distance}}{\text{Total time}}$
$= \frac{30.27 \text{ km}}{(36 \div 60) \text{ h}}$
$= 50.45 \text{ km h}^{-1}$

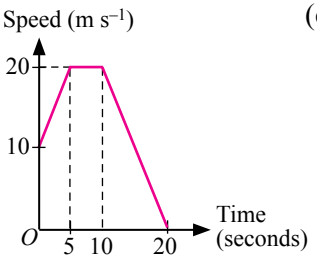
MY MEMORY

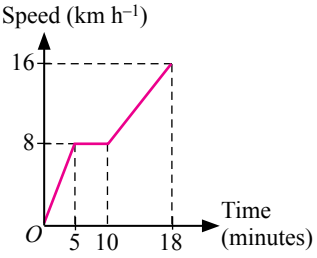
60 minutes = 1 hour
1 minute = $\frac{1}{60}$ hour

Self Practice 7.2b

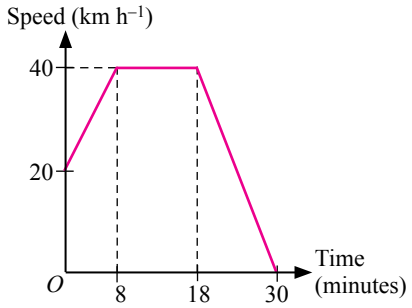
1. Calculate the distance travelled, in km, in each motion based on the given speed-time graph.

(a)


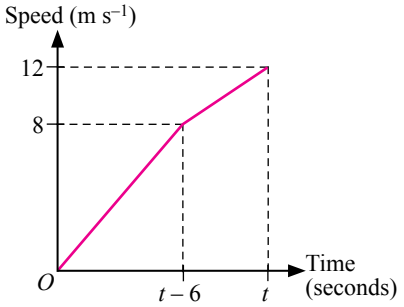
(b)


(c)


2. The speed-time graph shows the speed of Encik Mustaffa's motorcycle for a period of 30 minutes when he fetches his child after extra class. Calculate
(a) the total distance, in km, for a period of 30 minutes.
(b) the average speed, in km h^{-1}, for a period of 30 minutes.

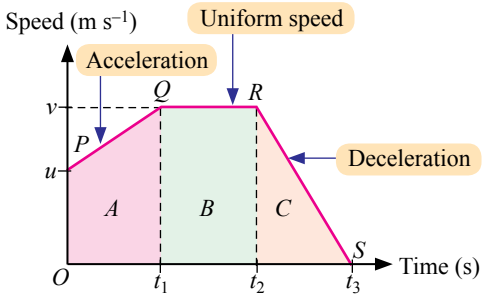


3. Sarves competes in the 100-metre event during the sports event in his school. The speed-time graph shows the speed of Sarves up to the finishing line. Calculate
(a) the value of t.
(b) the average speed of Sarves in km h^{-1}.



How do you interpret speed-time graphs?

Study the speed-time graph below.



The interpretation of the speed-time graph:

PQ ⇨ The **speed of object increases** from u m s^{-1} to v m s^{-1}.

⇨ The gradient of graph is positive, hence the rate of change of speed is positive.

⇨ **Acceleration** = $\dfrac{\text{Change of speed}}{\text{Change in time}}$

⇨ The **area** of trapezium A, that is the area under graph PQ represents the **distance** travelled in the period of t_1 seconds.

QR ⇨ There is **no change of speed** (zero gradient).

⇨ The object moves at a **uniform speed**.

⇨ The area of rectangle B, that is the area under graph QR represents the distance travelled in the period of $(t_2 - t_1)$ seconds.

RS ⇨ **Speed of the object decreases**.

⇨ The gradient of graph is negative, hence the rate of change of speed is negative.

⇨ **Deceleration** = $\dfrac{\text{Change of speed}}{\text{Change in time}}$

⇨ There is no change in the direction, that is the motion of the object remains in the same direction.

⇨ The area of triangle C, that is the area under graph RS represents the distance travelled in the period of $(t_3 - t_2)$ seconds.

Learning Standard

Interpret speed-time graphs and describe the movement based on the graphs.

INFO ZONE

Distance
- The length of the space between two points

Displacement
- **Vector** distance from a fixed point measured in **a certain direction**

INFO ZONE

Speed
- Rate of change in distance with respect to time.
- Speed = $\dfrac{\text{Distance}}{\text{Time}}$

Velocity
- Rate of change in displacement with respect to time.
- Velocity = $\dfrac{\text{Displacement}}{\text{Time}}$

MY MEMORY

Speed = $\dfrac{\text{Distance}}{\text{Time}}$

Acceleration = $\dfrac{\text{Speed}}{\text{Time}}$

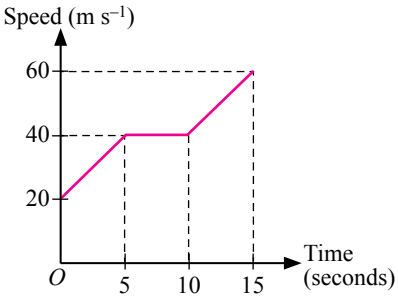
Indicator

The direction of motion of an object remains the same during acceleration or deceleration.

CHAPTER 7

Example 14

The speed-time graph shows the motion of Puan Salina's car for a period of 15 seconds.



(a) Calculate the rate of change of speed, in m s^{-2}, for the first 5 seconds.

(b) Describe the motion of the car for the second 5 seconds.

(c) Calculate the total distance, in metres, travelled in the period of 15 seconds.

Solution:

(a) Rate of change of speed $= \dfrac{\text{Change of speed}}{\text{Change in time}}$

$= \dfrac{(40 - 20)\text{ m s}^{-1}}{(5 - 0)\text{ s}}$

$= 4\text{ m s}^{-2}$

(b) The car moves at a uniform speed of 40 m s^{-1} for the period of 5 seconds.

(c) Total distance travelled = area under the graph

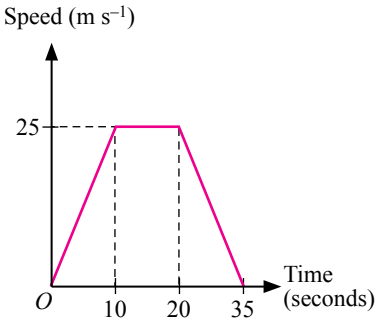
$= \left[\frac{1}{2} \times 5 \times (20 + 40)\right] + [(10 - 5) \times 40] + \left[\frac{1}{2} \times (15 - 10) \times (40 + 60)\right]$

$= (150 + 200 + 250)\text{ m}$

$= 600\text{ m}$

Example 15

Mr Daniel Wong drives his car to a convenience store to buy newspaper. The speed-time graph shows the motion of the car from his house to the road junction before reaching his destination.



(a) Describe the motion of Mr Daniel Wong's car for the first 10 seconds.

(b) What happens to the motion of Mr Daniel Wong's car from the 10th second till the 20th second?

(c) Calculate the rate of change of speed, in m s^{-2}, for the last 5 seconds.

(d) Calculate the distance, in metres, travelled during deceleration and describe the motion of the car for the period.

Solution:

(a) Rate of change of speed for the first 10 seconds

$= \dfrac{\text{Change of speed}}{\text{Change in time}}$

$= \dfrac{(25 - 0)\text{ m s}^{-1}}{10 - 0\text{ s}}$

$= 2.5\text{ m s}^{-2}$

The car accelerates at a rate of 2.5 m s^{-2} for the first 10 seconds.

(b) Mr Daniel Wong's car moves at a uniform speed of 25 m s^{-1} from the 10th second till the 20th second.

(c) Rate of change of speed $= \dfrac{(0 - 25)\text{ m s}^{-1}}{(35 - 20)\text{ s}}$

$= -\dfrac{5}{3}\text{ m s}^{-2}$

The rates of change in speed for the last 5 seconds and for the last 15 seconds are the same.

(d) Distance travelled during deceleration = distance travelled in the last 15 seconds

$= \dfrac{1}{2} \times (35 - 20) \times 25$

$= \left[\dfrac{1}{2} \times 15 \times 25\right]\text{ m}$

$= 187.5\text{ m}$

The answer can be written as
- acceleration $= -\dfrac{5}{3}\text{ m s}^{-2}$ or
- deceleration $= \dfrac{5}{3}\text{ m s}^{-2}$

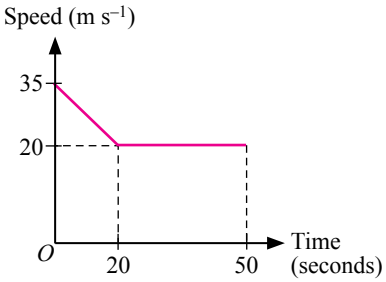
The car travels for 187.5 m in 15 seconds with a deceleration of $\dfrac{5}{3}\text{ m s}^{-2}$.

Self Practice 7.2c

1. The speed-time graph shows the motion of a motorcycle for a period of 50 seconds. Describe the motion of the motorcycle

(a) for the first 20 seconds.

(b) when it moves at a uniform speed.

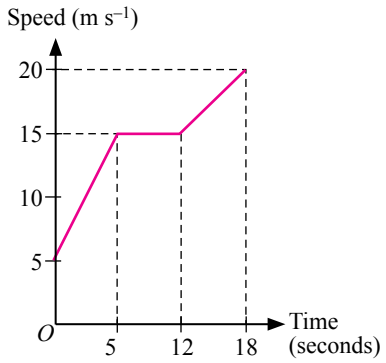


2. The speed-time graph shows the motion of a particle for a period of 18 seconds.

(a) Calculate the acceleration, in m s^{-2}, of the particle for the last 6 seconds.

(b) Calculate the total distance, in metres, travelled by the particle in the period of 18 seconds.

(c) Describe the motion of the particle when it moves at a uniform speed.

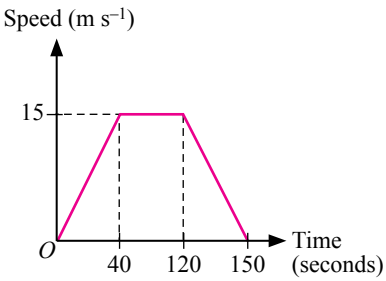


3. Encik Merisat visits his friend who lives in the same housing estate by car. The speed-time graph shows the journey of Encik Merisat to his friend's house.

(a) Calculate the rate of change of speed , in m s^{-2}, of his car for the first 20 seconds.

(b) Calculate the distance, in metres, travelled at a uniform speed.

(c) Describe Encik Merisat's journey for the period of 2.5 minutes.



How do you solve problems involving speed-time graphs?

Example 16

Learning Standard

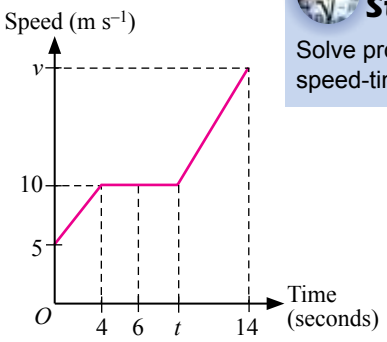
Solve problems involving speed-time graphs.

The speed-time graph shows the motion of a car for a period of 14 seconds. Calculate

(a) the average speed, in m s^{-1}, for the first 6 seconds.

(b) the value of t, if the distance travelled by the car for the first 4 seconds is half the distance travelled at a uniform speed.

(c) the value of v, if the acceleration for the last 2 seconds is 3.5 m s^{-2}.



Solution:

Understanding the problem

(a) Average speed for the first 6 seconds.

(b) Value of t, that is the time when moving at a uniform speed.

(c) Value of v, that is the final speed when the acceleration is 3.5 m s^{-2}.

Planning a strategy

(a) Average speed $= \dfrac{\text{Total distance}}{\text{Total time}}$

(b) Distance for the first 4 seconds
$= \frac{1}{2}$ (distance travelled at a uniform speed)

(c) Acceleration $= \dfrac{\text{Change of speed}}{\text{Change in time}}$

Carrying out strategy

(a) Total distance travelled for the first 6 seconds

$$= \left[\frac{1}{2} \times 4 \times (5 + 10)\right] + [(6 - 4) \times 10]$$
$$= (30 + 20) \text{ m}$$
$$= 50 \text{ m}$$

$$\text{Average speed} = \frac{50 \text{ m}}{6 \text{ s}}$$
$$= \frac{25}{3} \text{ m s}^{-1}$$

(b) Area of trapezium $= \frac{1}{2}$ (area of rectangle)

$$\frac{1}{2} \times 4 \times (5 + 10) = \frac{1}{2} \times (t - 4) \times 10$$
$$30 = 5t - 20$$
$$50 = 5t$$
$$t = 10$$

(c) Acceleration $= 3.5 \text{ m s}^{-2}$

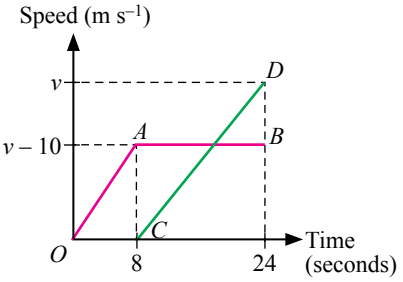
$$\frac{v - 10}{14 - 10} = 3.5$$
$$\frac{v - 10}{4} = 3.5$$
$$v - 10 = 14$$
$$v = 24$$

Conclusion

(a) The average speed for the first 6 seconds is $\frac{25}{3}$ m s^{-1}.
(b) $t = 10$
(c) $v = 24$

Example 17

The speed-time graph shows the motion of two vehicles. Graph OAB represents the motion of Encik Zabadi's car and graph CD represents the motion of a taxi driven by Mr Low. The difference between the distance travelled by the car and the taxi in the period of 24 seconds is 160 m. Calculate the value of v.



Solution:

Understanding the problem

- Value of v, that is the final speed of the taxi in the period of 16 seconds.

Planning a strategy

- Distance travelled by the car – distance travelled by the taxi = 160 m

Carrying out strategy

Distance travelled by the car (OAB) – distance travelled by the taxi (CD) = 160

$$\left[\frac{1}{2}\times(v-10)\times(24+16)\right]-\left[\frac{1}{2}\times(24-8)\times(v)\right]=160$$

$$\left[\frac{1}{2}\times(v-10)\times 40\right]-\left[\frac{1}{2}\times 16\times v\right]=160$$

$$20v-200-8v=160$$

$$12v=360$$

$$v=30$$

Checking Answer

- Distance travelled by the car $=\frac{1}{2}\times(30-10)\times(24+16)=400$ m
- Distance travelled by the taxi $=\frac{1}{2}\times 16\times 30=240$ m
- Difference in distance = 400 – 240 = 160 m

Conclusion

Value of $v = 30$

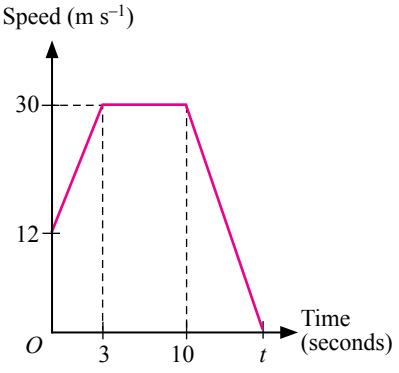
Example 18

The speed-time graph shows the motion of a van for a period of t seconds. Calculate

(a) the rate of change of speed, in m s^{-2}, for the first 3 seconds.

(b) the distance, in metres, travelled for the first 10 seconds.

(c) the value of t, if the magnitude of the rate of change of speed after 10 seconds is the same as the magnitude of the rate of change of speed for the first 3 seconds.

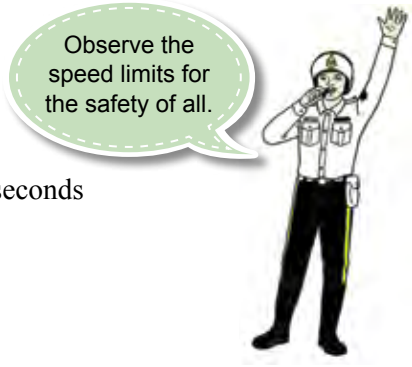


Solution:

(a) Rate of change of speed $=\dfrac{(30-12)\text{ m s}^{-1}}{(3-0)\text{ s}}$

$= 6\text{ m s}^{-2}$

(b) Distance travelled $= \left[\frac{1}{2} \times 3 \times (12 + 30)\right] + [(10 - 3) \times 30]$
$= (63 + 210)$ m
$= 273$ m



(c) Acceleration after 10 seconds = acceleration for the first 3 seconds

$$-\left[\frac{(0 - 30)\text{ m s}^{-1}}{(t - 10)\text{ s}}\right] = \frac{(30 - 12)\text{ m s}^{-1}}{(3 - 0)\text{ s}}$$

$$-\left(\frac{-30}{t - 10}\right) = \frac{18}{3}$$

$$\frac{30}{t - 10} = 6$$

$$\frac{30}{6} = t - 10$$

$$t = 15$$

The rate of change of speed is in the same direction (magnitude).

INFO ZONE

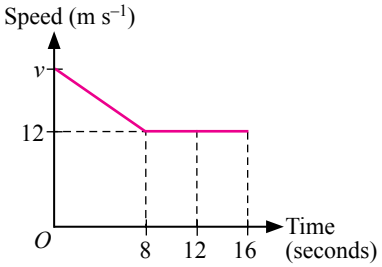
Magnitude
- distance travelled in a certain direction.

Self Practice 7.2d

1. The speed-time graph shows the motion of Dion Johan's car for a period of 16 seconds. Calculate

(a) the distance travelled, in metres, at a uniform speed.

(b) the value of v, if the average speed of the car for the first 12 seconds is 14 m s^{-1}.

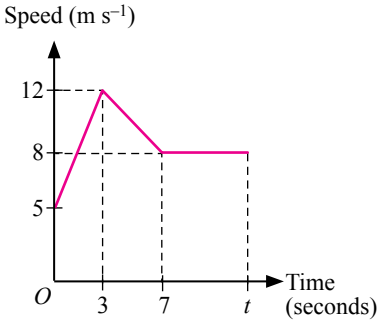


2. The speed-time graph shows the motion of the motorcycle ridden by Abit Lusang for a period of t seconds. Calculate

(a) the deceleration of the motion in m s^{-2}.

(b) the distance, in metres, when the rate of change of speed is positive.

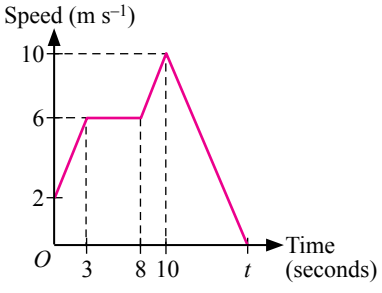
(c) the value of t, if the total distance in t seconds is 121.5 m.



3. The speed-time graph shows the motion of a car for a period of t seconds. Calculate

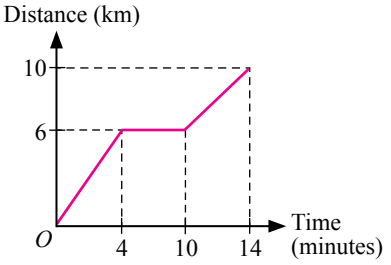
(a) the total distance, in metres, when the rate of change of speed is positive.

(b) the value of t, if the magnitude of the rate of change of speed from the 8th second till the 10th second is the same as the magnitude of the rate of change of speed after 10 seconds.

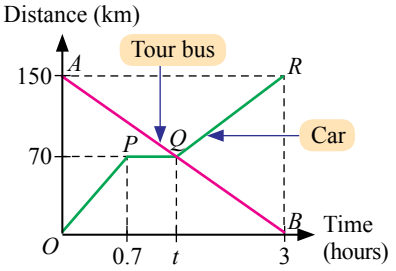


Comprehensive Practice

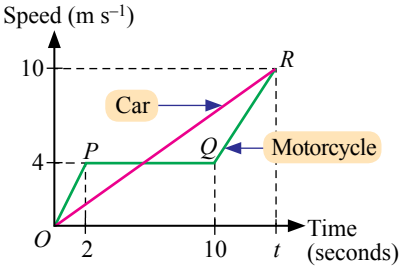
1. The distance-time graph shows the motion of an express bus for a period of 14 minutes. Calculate
(a) the duration when the bus is stationary.
(b) the rate of change in distance, in km h^{-1}, of the bus for the last 4 minutes.
(c) the average speed, in km h^{-1}, of the bus for the period of 14 minutes.



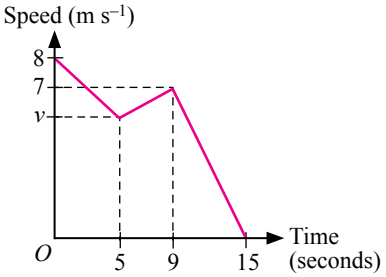
2. A car and a tour bus travel for 150 km in 3 hours. The distance-time graph shows the motion of the car and the tour bus. Calculate
(a) the rate of change in distance, in km h^{-1}, of the car for the first 24 minutes.
(b) the value of t.
(c) the rate of change in distance, in km h^{-1}, of the car for the last 80 km.



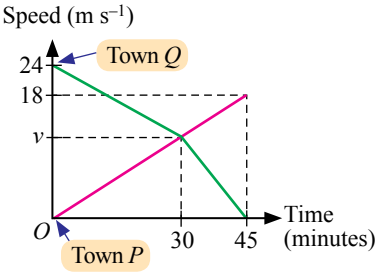
3. The speed-time graph shows the motion of a car and a motorcycle. Calculate
(a) the duration when the motorcycle travels at a uniform speed.
(b) the value of t, if the distances travelled by the car and the motorcycle are the same for the period of t seconds.



4. The speed-time graph shows the motion of a particle for a period of 15 seconds. Calculate
(a) the rate of change of speed, in m s^{-2}, of a particle for the last 6 seconds.
(b) the value of v, if the ratio of the distance travelled in the first 5 seconds to the last 6 seconds is 5 : 3.
(c) the average speed, in km h^{-1}, of the particle for the period of 15 seconds.



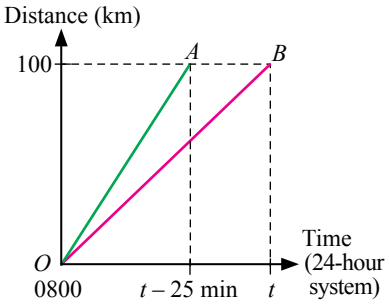
5. The speed-time graph shows the motion of two cars for a period of 45 minutes. Puan Nisha drives from Town P to Town Q and Puan Farah drives in the opposite direction. Calculate
(a) the value of v, if the rate of change of speed of Puan Farah's car for the first 30 minutes is the same as the acceleration of Puan Nisha's car for the period of 45 minutes.



(b) the distance from Town Q, in km, when the two vehicles meet.
(c) the time, in minutes, taken by Puan Nisha to reach Town Q if the acceleration of her car does not change.

6. The distance-time graph shows the motion of two cars for a distance of 100 km. Graph OA represents the motion of the car driven by Mr Lee at an average speed of v km h^{-1} and graph OB represents the motion of the car driven by Encik Dollah at an average speed of $(v - 20)$ km h^{-1}. Calculate

(a) the value of v, if the difference between the time taken by Mr Lee and Encik Dollah to reach the destination is 25 minutes.
(b) the time, in the 24-hour system, Mr Lee reaches his destination.



7. (a) (i) Diagram 1 shows the distance-time graph of car A for a period of t seconds. It is given that the average speed of car A is 25 m s^{-1}. Calculate the value of t.

(ii) Describe the motion of car A for the period of t seconds.

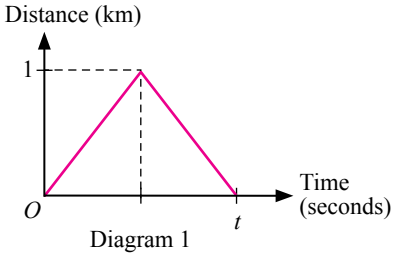


Diagram 1

(b) Diagram 2 shows the speed-time graph of car B. It is given that the uniform speed, v, of car B is the same as the average speed of car A and both vehicles travel the same distance. If the values of t, in seconds, for both graphs are the same, calculate the duration, in minutes, during which car B moves at a uniform speed.

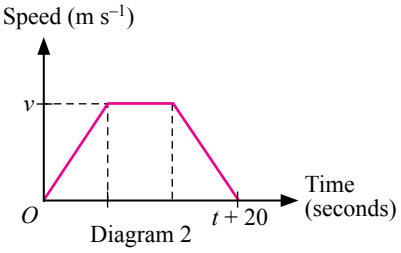


Diagram 2

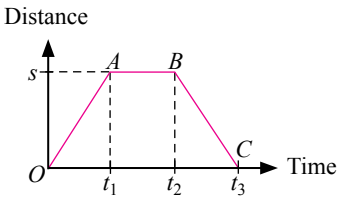
PROJECT

1. Collect information about the various speed limits in the area where you stay.
2. Record the speed limits according to specific areas. For example, 30 km h^{-1} at the surrounding areas of a school.
3. What is the implication if a driver does not follow the speed limit?
4. Prepare a report with photos and present your findings using multimedia.

CONCEPT MAP

Graphs of Motion

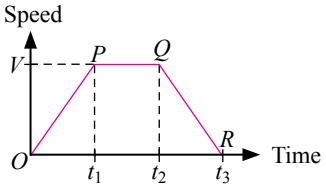
Distance-Time Graphs



- Gradient = $\frac{\text{Change in distance}}{\text{Change in time}}$

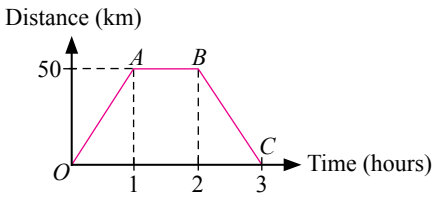
  Rate of change in distance with respect to time = speed
- *OA* ⇨ positive gradient
  (motion towards destination)
- *AB* ⇨ zero gradient
  (stationary)
- *BC* ⇨ negative gradient
  (motion towards origin)
- Positive and negative speeds indicate opposite direction of motion.

Speed-Time Graphs



- Distance travelled = Area under graph
- Gradient = $\frac{\text{Change of speed}}{\text{Change in time}}$

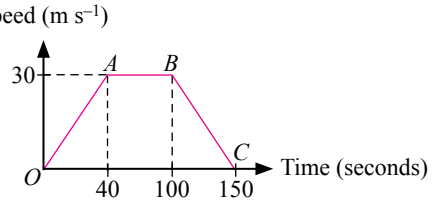
  Rate of change of speed with respect to time = acceleration
- *OP* ⇨ positive gradient
  ⇨ speed increases
  ⇨ acceleration
- *PQ* ⇨ zero gradient
  ⇨ no change of speed
  ⇨ uniform speed
- *QR* ⇨ negative gradient
  ⇨ speed decreases
  ⇨ deceleration



(a) Speed of the object in the first hour or in the last hour

$= \frac{50 - 0}{1 - 0} = 50 \text{ km h}^{-1}$

(b) The object is stationary for 1 hour (*AB*)



(a) Acceleration in the first 40 seconds

$= \frac{30 - 0}{40 - 0} = 0.75 \text{ m s}^{-2}$

(b) Acceleration in the last 50 seconds

$= \frac{0 - 30}{150 - 100} = -0.6 \text{ m s}^{-2}$

(c) Uniform speed for 1 minute (*AB*)

Average speed = $\frac{\text{Total distance}}{\text{Total time}}$

Self Reflection

Instructions

1. Fill in the boxes with the correct answers using capital letters.
2. Fill in the mystery word puzzle according to the numbered boxes.
3. How do you feel with the completed mystery word puzzle?

(a) $\dfrac{\text{Change in distance}}{\text{Change in time}} =$ ☐☐☐☐☐

(b) For a distance-time graph, zero gradient means the state of ☐☐☐☐[4]☐☐☐☐[1]

(c) $\dfrac{\text{Change of speed}}{\text{Change in } \square\square[6]\square\square} = \text{Acceleration}$

(d) For a speed-time graph, negative gradient means ☐☐☐☐[3]☐☐☐☐☐[5]

(e) A uniform speed of a motion occurs when the value of the ☐☐[2]☐☐☐☐☐ is zero in a speed-time graph.

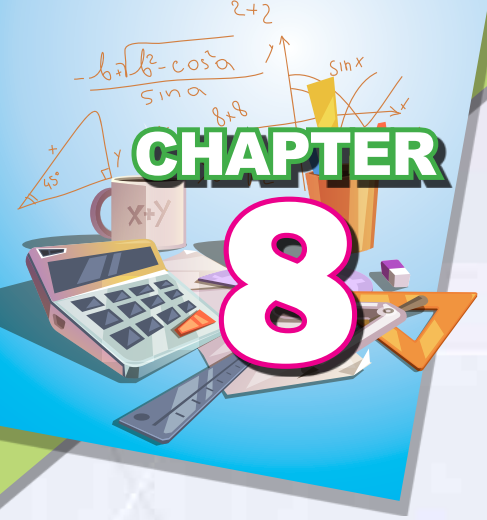
(f) Average [7]☐☐☐☐☐ $= \dfrac{\text{Total distance}}{\text{Total time}}$

Mystery Word Puzzle

4		2	6		6	2	3	2	1	7	4	2	5

Mathematics Exploration

1. Divide the class into groups.
2. Each group is required to collect information about high-speed vehicles that travel on the surface of the earth, in the air and outer space from various sources.
3. The information obtained should include the history of the invention, applications as well as side effects.
4. Exhibit your group's findings at the Mathematics Corner for the benefits of others.



CHAPTER 8

Measures of Dispersion for Ungrouped Data

You will learn

- Dispersion
- Measures of Dispersion

Kuala Lumpur's 29th SEA games were officially held from 19 to 30 August 2017. There were at least 4 646 athletes participated in 404 events. Malaysia became the winner with 145 gold medals.

Do you know how many medals Malaysian athletes had gained according to the events being contested?

Why Study This Chapter?

Statistics help us conduct research effectively, improve on critical and analytical thinking skills, as well as enable us to make the right decisions based on data. Therefore, mastering statistical knowledge enhances our skills in making improvements and data-driven predictions. These skills are extremely important in our career and daily life.

ES	GOLD	SILVER	BRONZE	TOTAL
AYSIA	145	92	86	323
LAND	72	86	88	246
NAM	58	50	60	168
APORE	57	58	73	188
ONESIA	38			
IPPINES	24			
NMAR	7			
BODIA	3			
S				
NEI				

WORD BANK

• ungrouped data	• *data tak terkumpul*
• interquartile range	• *julat antara kuartil*
• range	• *julat*
• outlier	• *pencilan*
• stem-and-leaf plot	• *plot batang-dan-daun*
• box plot	• *plot kotak*
• dot plot	• *plot titik*
• standard deviation	• *sisihan piawai*
• measure of dispersion	• *sukatan serakan*
• variance	• *varians*

Walking Through Time

Sir Ronald Fisher Aylmer
(1890 – 1962)

Sir Ronald Fisher Aylmer had made important contributions to the field of statistics, including pioneering the development of analysis of variance. He is also known as "a genius who created the foundation for modern statistical science".

http://bt.sasbadi.com/m4211

8.1 Dispersion

What is the meaning of dispersion?

Learning Standard
Explain the meaning of dispersion.

Measures of dispersion are important measurements in statistics. The measures of dispersion give us an idea of how the values of a set of data are scattered.

Dispersion is small if the data set has a small range and vice versa.

Measures of dispersion of a set of data are quantitative measures such as range, interquartile range, variance and standard deviation.

Mind Stimulation 1

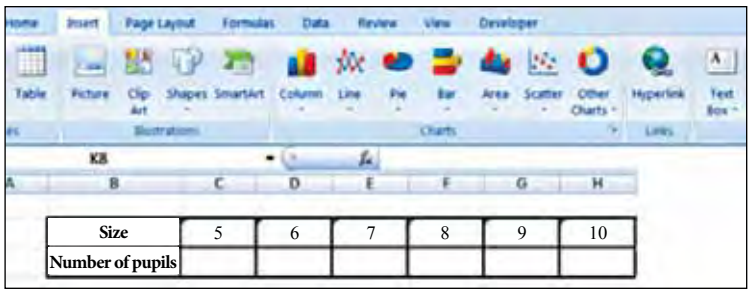
Aim: To explain the meaning of dispersion.

Steps:

1. Divide the class into groups.
2. Using the interview method, collect information about the shoe sizes of your friends in the school. Collect at least 20 data.
3. Using an electronic spreadsheet, construct a table for the information obtained.

Size of shoes	5	6	7	8	9	10
Number of pupils						

4. Complete the following frequency table in the electronic spreadsheet.



MY MEMORY

To represent data ethically and avoid confusion
- The scale used in the representations must be uniformed and must start from 0.
- The data displayed must be accurate.

5. Using the same electronic spreadsheet or manual, construct the following charts.
   (a) dot plot (scatter plot) (edit your displayed results if necessary)
   (b) stem-and-leaf plot
6. Print and display your group's results at the Mathematics Corner.

7. During Gallery Walk activity, obtain information from other groups and complete the table below.

Group	1	2	3	4	5	6	7
Largest shoe size							
Smallest shoe size							
Difference in shoe size							

Discussion:

Based on your group's frequency table in Step 7,

(a) state whether differences in shoe size for each group are the same.

(b) discuss the factors that cause these differences in shoe size.

From the activity in Mind Stimulation 1, it is found that:

The distributions of the data are different. To understand the dispersion of data, the difference between the largest value and the smallest value is taken into consideration. If the difference between the values is large, it indicates that the data is widely dispersed and vice versa.

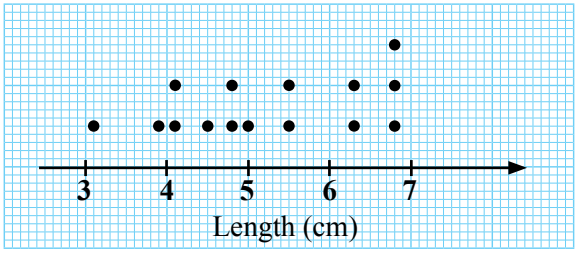
Example 1

(a) The table below shows the masses, in kg, of 20 pupils.

52	60	62	47	55	48	70	67	71	66
48	50	64	51	66	79	62	65	78	72

State the difference in mass, in kg, of the pupils.

(b) The diagram below shows a dot plot of the length, in cm, of a sample of several types of insects.



INFO ZONE

A value is a data or results obtained from an observation.

State the difference in size, in cm, of the longest insect and the shortest insect.

Solution:

(a) Largest mass = 79 kg
Smallest mass = 47 kg
Difference in mass = 79 kg – 47 kg
= 32 kg

(b) Length of the longest insect = 6.8 cm
Length of the shortest insect = 3.1 cm
Difference in length = 6.8 cm – 3.1 cm
= 3.7 cm

Self Practice 8.1a

1. A study about the boiling points, in °C, of 10 types of chemicals is conducted. The results of the study are recorded as below.

112	60	75	81	150	92	108	90	45	132

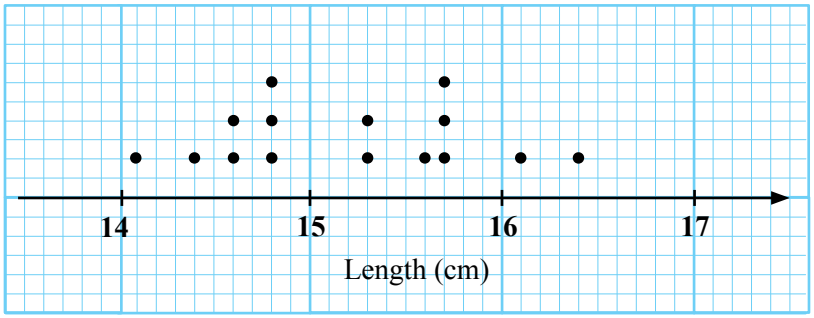
(a) Determine the lowest and the highest temperatures.
(b) Determine the difference between the highest and the lowest temperatures.

2. The durations spent on social media, in minutes, of 12 teenagers on a particular day are recorded as follows.

100	120	80	60	90	30	40	100	60	90	120	60

It is found that the duration spent on social media can be written in the form of $p \leqslant \text{time} \leqslant q$. State the values of p and q.

3. The diagram shows a dot plot of the lengths, in cm, of samples of several leaves.



What is the difference in length between the longest and the shortest leaf?

How do you compare and interpret dispersion of two or more sets of data based on stem-and-leaf plots and dot plots?

Learning Standard

Compare and interpret dispersion of two or more sets of data based on the stem-and-leaf plots and dot plots, and hence make conclusion.

A stem-and-leaf plot is a way to show the distributions of a set of data. Through the stem-and-leaf plot, we can see whether the data is more likely to appear or least likely to appear.

What are the steps to plot a stem-and-leaf plot?

The diagram below shows the marks obtained by a group of 36 pupils in an Accounting test.

27	34	37	39	42	43	46	48	52
29	35	37	40	42	44	47	49	52
31	35	38	40	42	44	47	49	53
32	36	38	41	42	45	47	52	54

If this data is not organised in the table, then we cannot see the dispersion immediately. We set the tens digit as the stem and the units digit as the leaf to plot the stem-and-leaf plot for the given data.

Stem	Leaf
2	7 9
3	1 2 4 5 5 6 7 7 8 8 9
4	0 0 1 2 2 2 2 3 4 4 5 6 7 7 7 8 9 9
5	2 2 2 3 4

Key: 2 | 7 means 27 marks

INFO ZONE

Leaf digits are arranged from the smallest value to the largest value.

From the stem-and-leaf plot, we find that the marks which appear the most are in the range of 40 to 49.

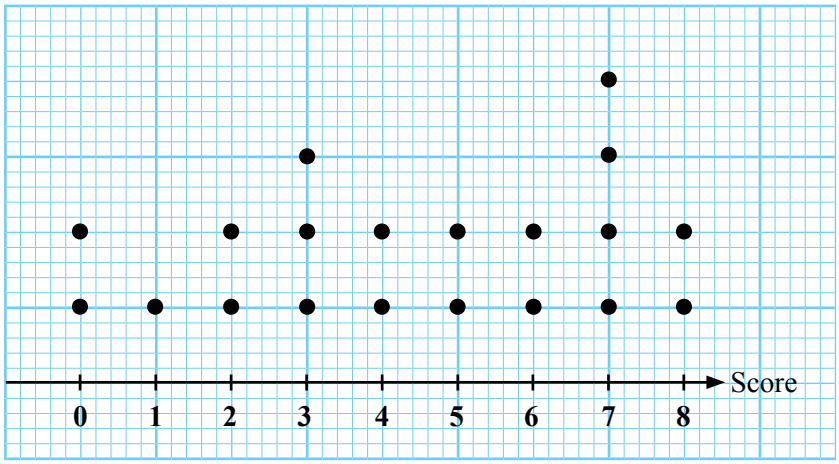
What do you understand about a dot plot?

A dot plot is a statistical chart that contains points plotted using a uniform scale. Each point represents a value.

The following data shows the scores obtained by a group of 20 pupils in a Biology quiz.

0	6	4	7	6
2	0	1	7	7
4	5	2	3	8
7	8	3	5	3

The diagram below shows a dot plot for the above data. Each point represents a value.



INFO ZONE

The dot plot is appropriate for small data set.

In the above dot plot, the highest value is 8, the lowest value is 0 and most pupils get a score of 7.

How do you compare and interpret the dispersion of two sets of data based on a stem-and-leaf plot?

Mind Stimulation 2

Aim: To compare and interpret dispersion of two sets of data based on a stem-and-leaf plot.

Steps:

1. Divide the class into groups.
2. The following data shows the marks scored by Class 4 Budi pupils in April and May monthly History tests.

Marks of April monthly test

32	41	44	51	58
35	42	46	53	58
35	43	48	54	58
39	43	48	54	60
41	44	49	56	61

Marks of May monthly test

34	46	55	63	69
38	46	55	65	71
40	49	55	66	73
40	52	59	68	75
43	53	59	68	77

3. Complete the following stem-and-leaf plot.

Marks of April monthly test		Marks of May monthly test
9 5 5 2	3	4 8
	4	
	5	
	6	
	7	

Discussion:

Based on the above stem-and-leaf plot, which test shows better pupils' achievement? Justify your answer.

From the activity in Mind Stimulation 2, it is found that:

The pupils' achievement in May monthly test is better because there are more pupils scoring higher marks in the test compared to April monthly test.

In general,

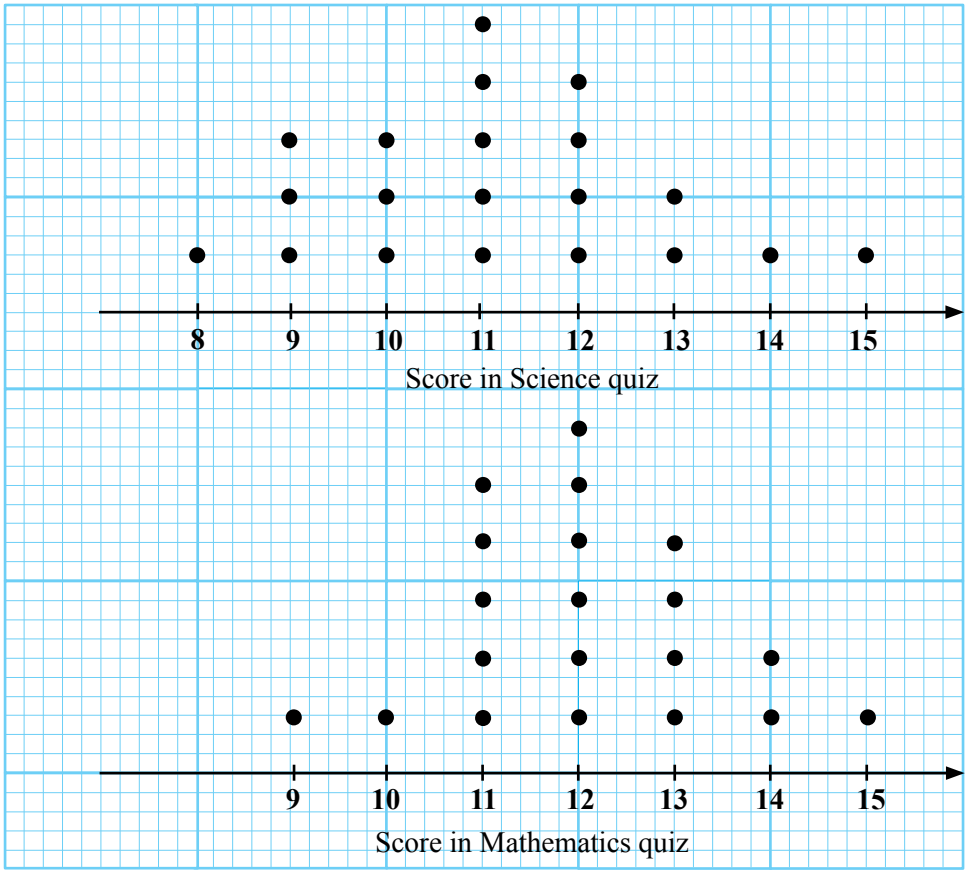
When two sets of data are plotted using stem-and-leaf plots, we can compare the patterns of the two plots.

How do you compare and interpret two sets of data based on dot plots?

From the dot plots, we can compare and interpret the shapes, the dispersion of points, the frequencies as well as the values concentrated on the left or on the right of the two sets of data.

Example 2

A Mathematics quiz and a Science quiz are held at SMK Bestari in conjunction of the Mathematics and Science Week. 20 pupils take part in the quizzes. The diagram below shows two dot plots of the scores obtained by the pupils.



(a) Which set of data shows a wider dispersion? Justify your answer.
(b) Which quiz has a higher difference in value?
(c) Between the scores of Science and Mathematics quizzes, which pupils' achievement is better?

Solution:

(a) The dispersion in Science quiz is larger because it has larger difference in value.
(b) Science quiz has a higher difference in value, that is 7.
(c) The pupils score better in Mathematics quiz because most of the values are concentrated on the right of the dot plot.

Self Practice 8.1b

1. The data below shows the masses, in kg, of two groups of pupils.

Group A

58	47	68	63	61
60	54	70	63	45
69	54	52	41	82
70	53	70	60	52
81	67	56	50	76
86	66	62	73	75
44	46	62	72	49
84	76	82	68	64

Group B

72	54	76	49	64
58	76	40	64	65
58	69	48	52	42
40	70	66	57	83
41	70	69	56	61
78	52	75	63	84
46	82	55	59	57
83	42	56	60	59

Draw a stem-and-leaf plot to show the distribution of masses of the two groups of pupils. Comment on the distribution of masses of the two groups of pupils.

2. The data below shows the marks obtained by a group of 25 pupils in two monthly Geography tests.

April monthly test

35	56	42	56	48
65	51	58	42	60
46	61	46	48	62
54	50	41	50	55
50	69	57	51	50

May monthly test

63	42	47	52	52
48	45	51	41	50
55	48	61	42	49
39	63	54	56	54
46	50	51	57	53

Draw two dot plots using the same scale to show the difference of the distribution of marks of the two monthly tests. Comment on the achievement of the group of pupils in the two monthly tests.

3. The data below shows the shoe size of 20 pupils in two different classes.

Class Rose

8.0	8.0	12.5	7.5	8.0
6.5	6.0	8.0	11.0	8.5
7.0	7.5	10.5	8.0	7.0
8.0	10.5	7.5	6.5	6.5

Class Lotus

8.5	9.0	9.5	10.5	9.0
9.5	11.0	9.0	9.5	9.5
10.5	9.5	8.5	9.0	9.0
9.0	9.0	10.5	9.5	11.0

(a) Construct two dot plots using the same scale.

(b) Compare the two dot plots obtained in (a). Which class has a greater difference in shoe size? Justify your answer.

8.2 Measures of Dispersion

How do you determine the range, interquartile range, variance and standard deviation as a measure to describe dispersion of an ungrouped data?

Learning Standard

Determine the range, interquartile range, variance and standard deviation as a measure to describe dispersion of an ungrouped data.

In statistics, the range of an ungrouped data is the difference between the largest and the smallest values. This measure shows how data is distributed. For example, in a Bahasa Melayu test, a class that has a larger range of marks means that pupils' achievement varies greatly.

Range = Largest value – Smallest value

Example 3

Given a set of data 34, 23, 14, 26, 40, 25, 20, determine the range of this set of data.

Solution:

34, 23, 14, 26, 40, 25, 20

Smallest value: 14

Largest value: 40

Range = 40 – 14
= 26

How do you determine the range from the frequency table?

Example 4

The table below shows the scores obtained by a group of pupils in a Chemistry test.

Score	2	3	4	5	6	7	8
Number of pupils	3	5	6	8	12	7	3

Determine the range of the above data.

Solution:

Smallest value: 2

Largest value: 8

Score	2	3	4	5	6	7	8
Number of pupils	3	5	6	8	12	7	3

Range = 8 – 2
= 6

How do you determine the interquartile range of a set of ungrouped data?

When the values of a set of data are arranged in an ascending order, the first quartile, Q_1 is the value of data that is at the first $\frac{1}{4}$ position while the third quartile, Q_3 is the value of data that is at the $\frac{3}{4}$ position from the data arranged in order.

Use the following steps to determine Q_1 and Q_3 from a set of ungrouped data.

1. Arrange the data in ascending order.
2. Determine the median of the data.
3. Divide the set of data into two parts, the part of data before the median and the part of data after the median.
4. Q_1 is the value of data in the middle position of the part of data before the median.
5. Q_3 is the value of data in the middle position of the part of data after the median.
6. If the number of data is odd, the median, Q_1 and Q_3 can be identified directly from the the data.
7. It the number of data is even, the median, Q_1 and Q_3 can be obtained by finding the average of the two values in the middle positions of the parts of data before or after the median.
8. Interquartile range = $Q_3 - Q_1$

Example 5

Determine the interquartile range of each of the following sets of data:

(a) 4, 4, 6, 8, 10, 13, 15, 16, 20, 22, 23

(b) 11, 14, 14, 17, 19, 20, 20, 21, 22

(c) 2.3, 2.5, 2.5, 2.6, 2.8, 2.9, 3.0, 3.2

Solution:

(a) 4 4 | 6 | 8 10 | 13 | 15 16 | 20 | 22 23

(2 values) (2 values) (2 values) (2 values)

Q_1 = 6, median = 13, Q_3 = 20

Interquartile range = $20 - 6$

$= 14$

Determine the position of the median first.

(b) 11 14 | 14 17 | 19 | 20 20 | 21 22

(2 values) (2 values) (2 values) (2 values)

$Q_1 = \frac{14 + 14}{2}$, median = 19, $Q_3 = \frac{20 + 21}{2}$

Interquartile range $= \frac{20 + 21}{2} - \frac{14 + 14}{2}$

$= 6.5$

(c) 2.3 2.5 | 2.5 2.6 | 2.8 2.9 | 3.0 3.2

(2 values) (2 values) (2 values) (2 values)

$Q_1 = \frac{2.5 + 2.5}{2}$, median = $\frac{2.6 + 2.8}{2}$, $Q_3 = \frac{2.9 + 3.0}{2}$

Interquartile range $= \frac{2.9 + 3.0}{2} - \frac{2.5 + 2.5}{2}$

$= 0.45$

INTERACTIVE ZONE

Data can also be arranged in a descending order. In this case, how do you obtain Q_1 and Q_3? Discuss.

MY MEMORY

Median is the value in the middle when a set of data is arranged in an ascending or descending order.

INFO ZONE

Second quartile, Q_2 is also the median.

INFO ZONE

First quartile, Q_1 is also known as lower quartile while third quartile, Q_3 is known as upper quartile.

Example 6

Determine the interquartile range of the following frequency table.

Score	2	3	4	5	6	7	8
Number of pupils	3	5	6	8	12	7	3

Solution:

11th value of score

33rd value of score

Score	2	3	4	5	6	7	8
Number of pupils	3	5	6	8	12	7	3
Cumulative frequency	3	8	14	22	34	41	44

Q_1 = the $\left(\frac{1}{4} \times 44\right)$ th value

= the 11th value

= 4

Q_3 = the $\left(\frac{3}{4} \times 44\right)$ th value

= the 33rd value

= 6

Interquartile range = 6 – 4

= 2

INFO ZONE

Cumulative frequency of a given data is determined by adding its frequency and all the frequencies before it.

What do you understand about variance and standard deviation?

Variance and standard deviation are the measures of dispersion commonly used in statistics. The variance is the average of the square of the difference between each data and the mean.

The standard deviation is the square root of the variance which also measures the dispersion of a data set relative to its mean; measured in the same units of the original data.

How do you determine the variance and standard deviation for a set of ungrouped data?

Variance of a set of ungrouped data can be obtained using the formula of variance.

$$\sigma^2 = \frac{\Sigma(x - \bar{x})^2}{N} \quad \text{or} \quad \sigma^2 = \frac{\Sigma x^2}{N} - \bar{x}^2$$

Standard deviation, $\sigma = \sqrt{\frac{\Sigma(x - \bar{x})^2}{N}}$ or $\sigma = \sqrt{\frac{\Sigma x^2}{N} - \bar{x}^2}$

MY MEMORY

$\bar{x}$ = mean of data

$\bar{x} = \frac{\Sigma x}{N}$

Example 7

Determine the variance of the set of data 2, 4, 5, 5, 6.

Solution:

Mean, $\bar{x} = \frac{2 + 4 + 5 + 5 + 6}{5}$

$= 4.4$

$$\text{Variance, } \sigma^2 = \frac{(2-4.4)^2 + (4-4.4)^2 + (5-4.4)^2 + (5-4.4)^2 + (6-4.4)^2}{5}$$
$$= 1.84$$

Alternative Method

$$\sigma^2 = \frac{2^2 + 4^2 + 5^2 + 5^2 + 6^2}{5} - 4.4^2$$
$$= 1.84$$

Example 8

Determine the standard deviation of the set of data 5, 7, 8, 8, 10, 13, 15, 16, 16, 20.

Solution:

$$\text{Mean, } \bar{x} = \frac{5 + 7 + 8 + 8 + 10 + 13 + 15 + 16 + 16 + 20}{10}$$
$$= 11.8$$

$$\text{Variance, } \sigma^2 = \frac{5^2 + 7^2 + 8^2 + 8^2 + 10^2 + 13^2 + 15^2 + 16^2 + 16^2 + 20^2}{10} - 11.8^2$$
$$= 21.56$$

$$\text{Standard deviation, } \sigma = \sqrt{21.56}$$
$$= 4.643$$

INFO ZONE

σ^2 = variance
σ = standard deviation

Example 9

The table below shows the number of books read by a group of pupils in a particular month.

Number of books	0	1	2	3	4
Number of pupils	3	5	8	2	2

Calculate the variance and standard deviation for the number of books read.

Solution:

$$\text{Mean, } \bar{x} = \frac{3(0) + 5(1) + 8(2) + 2(3) + 2(4)}{3 + 5 + 8 + 2 + 2}$$
$$= 1.75$$

$$\text{Variance, } \sigma^2 = \frac{3(0-1.75)^2 + 5(1-1.75)^2 + 8(2-1.75)^2 + 2(3-1.75)^2 + 2(4-1.75)^2}{20}$$
$$= 1.2875$$

Alternative Method 1

$$\sigma^2 = \frac{3(0)^2 + 5(1)^2 + 8(2)^2 + 2(3)^2 + 2(4)^2}{20} - 1.75^2$$
$$= 1.2875$$

INFO ZONE

If the ungrouped data given in a frequency table, then

$\bar{x} = \frac{\Sigma fx}{\Sigma f}$ and

$\sigma^2 = \frac{\Sigma f(x-\bar{x})^2}{\Sigma f}$ or

$\sigma^2 = \frac{\Sigma fx^2}{\Sigma f} - \bar{x}^2$

Alternative Method 2

x	f	fx	x^2	fx^2
0	3	0	0	0
1	5	5	1	5
2	8	16	4	32
3	2	6	9	18
4	2	8	16	32
	$\sum f = 20$	$\sum fx = 35$		$\sum fx^2 = 87$

Mean, $\bar{x} = \frac{35}{20}$
$= 1.75$

Variance, σ^2
$= \frac{87}{20} - 1.75^2$
$= 1.2875$

Standard deviation, $\sigma = \sqrt{1.2875}$
$= 1.1347$

Checking Answer

1. Press the Mode key 2 times
   Display: SD REG BASE (1 2 3)
2. Press 1 to select SD
3. Enter the data
   Press 0, followed by M+ 3 times
   Press 1, followed by M+ 5 times
   Press 2, followed by M+ 8 times
   Press 3, followed by M+ 2 times
   Press 4, followed by M+ 2 times
   Display: n = 20
4. Press shift 2
   Display: $\bar{x}$ (1) $x\sigma n$ (2) $x\sigma n\text{-}1$ (3)
5. Press 2, then =
   Display: 1.134680572
6. Press button x^2, then =
   Display: 1.2875

Self Practice 8.2a

1. Determine the range and interquartile range of each of the following sets of data.
(a) 3, 7, 5, 9, 4, 4, 8, 7, 6, 2, 5
(b) 13, 15, 19, 22, 17, 14, 15, 16, 18, 19, 11, 10, 20
(c) 2.3, 2.2, 3.1, 2.8, 2.7, 2.4, 2.5, 2.3

2. Determine the range and interquartile range of each of the following sets of data.

(a)

Pocket money (RM)	4	5	6	7	8	9
Number of pupils	5	10	7	6	3	1

(b)

Score	10	11	12	13	14	15
Number of pupils	3	10	13	4	8	6

3. For each of the following sets of data, determine the variance and standard deviation.
(a) 5, 7, 6, 9, 12, 10, 10, 13
(b) 32, 40, 35, 39, 44, 48, 42

4. The table below shows the number of goals scored by 20 football players in a match.

Number of goals	0	1	2	3	4
Number of players	3	5	6	5	1

Determine the variance and standard deviation of the distribution.

What are the advantages and disadvantages of various measures of dispersion?

Learning Standard

Explain the advantages and disadvantages of various measures of dispersion to describe ungrouped data.

Measures of dispersion measure the distribution of a set of data. The range is the measure of dispersion easiest to calculate. However, the range cannot provide a good overview of how data is distributed.

In cases where there is an outlier or extreme value, the interquartile range would be the more appropriate measure of dispersion to show the distribution of the data.

Standard deviation is usually used to compare two sets of data. In general, a low standard deviation indicates that the data is dispersed close to the mean while a high standard deviation indicates that the data is dispersed far from the mean.

Example 10

Calculate the range and interquartile range of the set of data 10, 11, 11, 14, 18, 20, 21, 21, 25 and 40. Determine the most appropriate measure of dispersion to be used, to measure the distribution of the data set.

Solution:

Range = 40 – 10
= 30

Interquartile range = 21 – 11
= 10

Interquartile range is the most appropriate measure of dispersion because the existence of the outlier, 40.

Example 11

The table below shows the achievements of two pupils in 5 Physics tests.

	Test 1	Test 2	Test 3	Test 4	Test 5
Aiman	40	70	90	85	64
John	80	65	73	58	73

One of the pupils will be selected to represent the school in a Physics quiz competition. Determine who is eligible to be selected and justify your answer.

Solution:

Aiman

$$\text{Mean marks} = \frac{40 + 70 + 90 + 85 + 64}{5} = 69.8$$

$$\text{Standard deviation, } \sigma = \sqrt{\frac{40^2 + 70^2 + 90^2 + 85^2 + 64^2}{5} - 69.8^2} = 17.67$$

John

$$\text{Mean marks} = \frac{80 + 65 + 73 + 58 + 73}{5} = 69.8$$

Standard deviation, $\sigma = \sqrt{\frac{80^2 + 65^2 + 73^2 + 58^2 + 73^2}{5} - 69.8^2}$

$= 7.574$

John will be selected to participate in the Physics quiz competition. The standard deviation of John's scores is lower because John's achievement is more consistent.

Checking Answer

1. Press the Mode key 2 times.
   Display: SD REG BASE
   1 2 3
2. Press 1 to select SD
3. Enter the data
   Press 80, followed by M+
   Display $n = 1$
   Press 65, followed by M+
   Display $n = 2$
   Press 73, followed by M+
   Display $n = 3$
   Press 58, followed by M+
   Display $n = 4$
   Press 73, followed by M+
   Display $n = 5$
4. Press Shift 2
   Display: $\bar{x}$ $x\sigma n$ $x\sigma n\text{-}1$
   1 2 3
5. Press 2
6. Press =
   Display: 7.573638492

Self Practice 8.2b

1. Calculate the range and interquartile range of the set of data 2, 16, 16, 19, 20, 20, 24, 26, 27, 27, 29. Determine which measure of dispersion is more appropriate to be used to measure the distribution of the set of data. Justify your answer.

2. Mr Rakesh wants to choose one of his two pupils to represent his school in a National Business Quiz competition. Mr Rakesh gave 5 tests to the two pupils. The following are the results of the 5 tests.

	Test 1	Test 2	Test 3	Test 4	Test 5
Pupil A	32	61	75	82	90
Pupil B	50	67	70	73	80

Determine the appropriate measure of dispersion to be used by Mr Rakesh to make the selection. State who Mr Rakesh should select based on the measures of dispersion calculated.

3. The table below shows the monthly salaries of seven employees of Texan Company.

Employee	1	2	3	4	5	6	7
Salary (RM)	900	920	950	1 000	1 100	1 230	3 000

(a) Calculate the range, interquartile range and standard deviation of the monthly salaries of the employees in Texan Company.

(b) State the most appropriate measure of dispersion to be used in order to show the distribution of the monthly salaries of the employees in Texan Company.

How do you construct and interpret the box plot for a set of ungrouped data?

Apart from the stem-and-leaf plot and dot plot, box plot is a useful method to show the dispersion of a set of data.

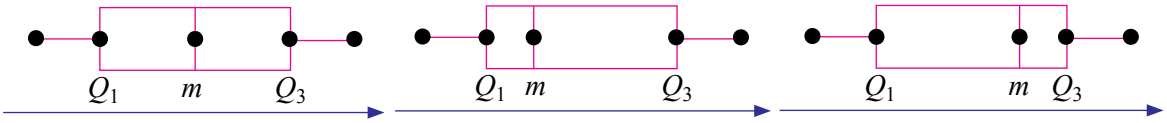
Learning Standard

Construct and interpret the box plot for a set of ungrouped data.

What is a box plot for a set of ungrouped data?

Box plot is a way of showing the distribution of a set of data based on five values, namely the minimum value, first quartile, median, third quartile and the maximum value of the set of data. The box plot can show whether the set of data is symmetric about the median. The box plot is often used to analyse a large number of data.

Some of the types of box plots are as follows.



How do you construct a box plot?

The box plot is constructed on a number line.
Consider the data below.

The data below shows the marks obtained by 15 pupils in a Chinese Language test.

35	40	42	45	52	52	53	57	62	62	66	68	73	73	75

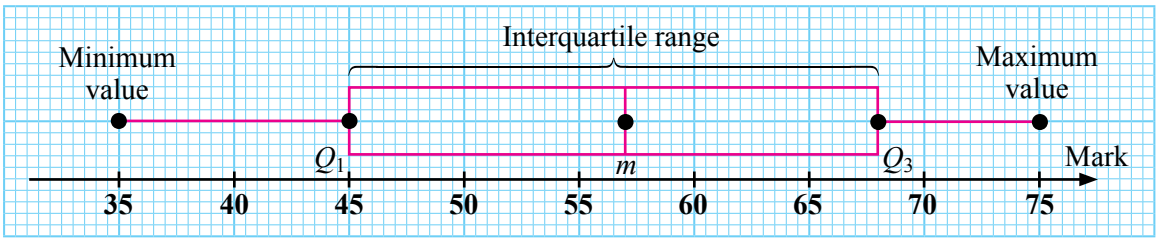
For the set of data, determine the values shown in the table below first.

Minimum value	35
First quartile	45
Median	57
Third quartile	68
Maximum value	75

INFO ZONE

The box plot can also be constructed vertically.

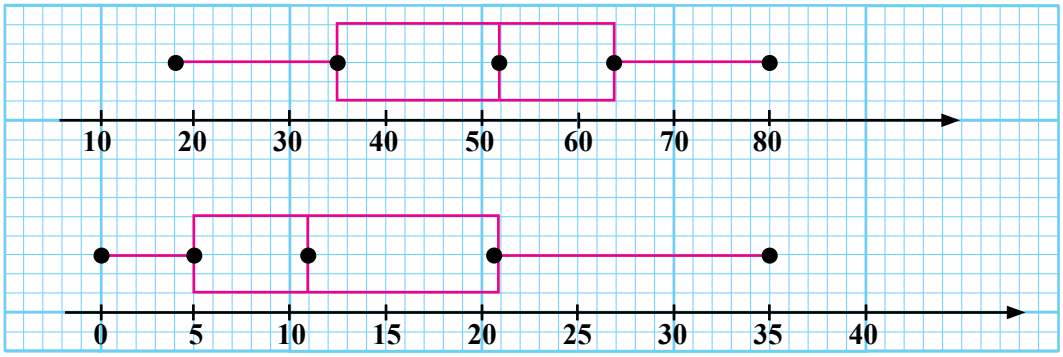
Thus, the box plot for the above set of data is as follows.



Example 12

Two box plots are given below.

(a)



(b)

For each box plot, state

(i) the median. (ii) the first quartile. (iii) the third quartile.
(iv) the interquartile range. (v) the minimum value. (vi) the maximum value.
(vii) the range.

Solution:

(a) (i) median = 52
(ii) first quartile = 35
(iii) third quartile = 64
(iv) interquartile range = 29
(v) minimum value = 18
(vi) maximum value = 80
(vii) range = 62

(b) (i) median = 11
(ii) first quartile = 5
(iii) third quartile = 21
(iv) interquartile range = 16
(v) minimum value = 0
(vi) maximum value = 35
(vii) range = 35

Self Practice 8.2c

1. For each of the following sets of data, construct a box plot.

(a)

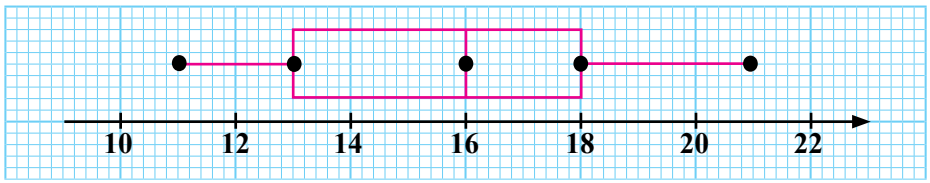
40	43	40	35	44	25
38	46	64	48	40	31
35	35	43	34	30	40

(b)

70	52	43	62	56	66	52
63	51	48	43	56	52	40
65	50	70	55	43	57	70

2. For the box plot below, state

(a) the minimum value. (b) the maximum value. (c) the first quartile.
(d) the third quartile. (e) the interquartile range. (f) the median.



What is the effect of data changes on dispersion?

Learning Standard

Determine the effect of data changes on dispersion based on:
- the value of measure of dispersion
- graphical representation

The data changes include the following cases.
(a) Each data is added or subtracted by a constant uniformly.
(b) Each data is multiplied or divided by a constant uniformly.
(c) The existence of outliers or extreme values in the set of data.
(d) Certain values are removed from the set of data or added into the set of data.

What are the effects on the measures of dispersion when each data is changed by adding or subtracting a constant?

Consider this case.
Initially, I have a set of data 5, 8, 3, 4, 1.
If each value of the set of data is added by 5, then the data above will be 10, 13, 8, 9, 6. What will happen to the measures of dispersion such as range, interquartile range, variance and standard deviation for the new set of data?

Mind Stimulation 3

Aim: To identify the effect on the range, interquartile range and standard deviation when each value of a set of data is added or subtracted by a constant.

Steps:
1. Divide the class into groups.
2. Each group is given cards A1, A2 and A3. (Refer to the QR Code)
3. Each group is required to complete the information stated on the card.

Discussion:
What is the effect on the measures of dispersion when each value of the set of data is added or subtracted by a constant?

Scan the QR Code to download card A1, card A2 and card A3.
http://bt.sasbadi.com/m4228

From the activity in Mind Stimulation 3, it is found that:

There are no changes to the measures of dispersion when each value of a set of data is added or subtracted by a constant.

What are the effects on the measures of dispersion when each value of a set of data is multiplied or divided by a constant?

Consider this case.
Initially, I have a set of data 5, 8, 3, 4, 1.
If each value of the set of data is multiplied by 2, then the data above will be 10, 16, 6, 8, 2. What will happen to the measures of dispersion such as range, interquartile range, variance and standard deviation for the new set of data?

Mind Stimulation 4

Aim: To identify the effect on the range, interquartile range and standard deviation when each value of a set of data is multiplied or divided by a number.

Steps:

1. Divide the class into groups.
2. Each group is given cards B1, B2 and B3. (Refer to the QR Code).
3. Each group is required to complete the information stated on the card.

Discussion:

What are the effects on the measures of dispersion when each value of the set of data is multiplied by 3? What are the effects on the measures of dispersion when each value of the set of data is divided by 2?

Scan the QR Code to download card B1, card B2 and card B3. *http://bt.sasbadi.com/m4229*

From the activity in Mind Stimulation 4, it is found that:

(a) When each value of a set of data is multiplied by a constant k, then
- (i) new range $= k \times$ original range
- (ii) new interquartile range $= k \times$ original interquartile range
- (iii) new standard deviation $= k \times$ original standard deviation
- (iv) new variance $= k^2 \times$ original variance

(b) When each value of a set of data is divided by a constant k, then
- (i) new range $= \dfrac{\text{original range}}{k}$
- (ii) new interquartile range $= \dfrac{\text{original interquartile range}}{k}$
- (iii) new standard deviation $= \dfrac{\text{original standard deviation}}{k}$
- (iv) new variance $= \dfrac{\text{original variance}}{k^2}$

What are the effects on the measures of dispersion when an outlier is added or removed from the set of data?

(a) **Range**

The range will change dramatically when an outlier is added or removed from the set of data.

(b) **Interquartile range**

The interquartile range is less affected when an outlier is added or removed from the set of data.

(c) **Variance and standard deviation**

The variance and the standard deviation will increase significantly when an outlier is added into the set of data.

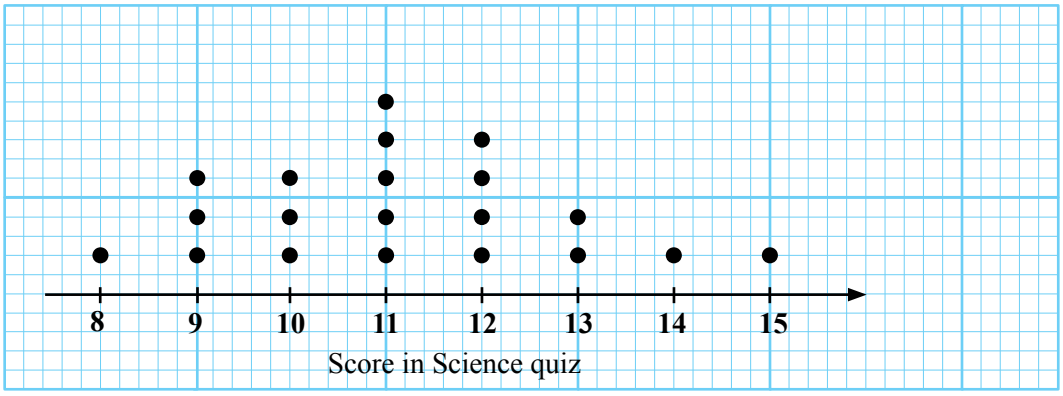
If the difference between the new value of the data and the mean is small, then the new standard deviation will be smaller and vice versa.

If the difference between the value that is removed and the mean is small, then the new standard deviation will be larger and vice versa.

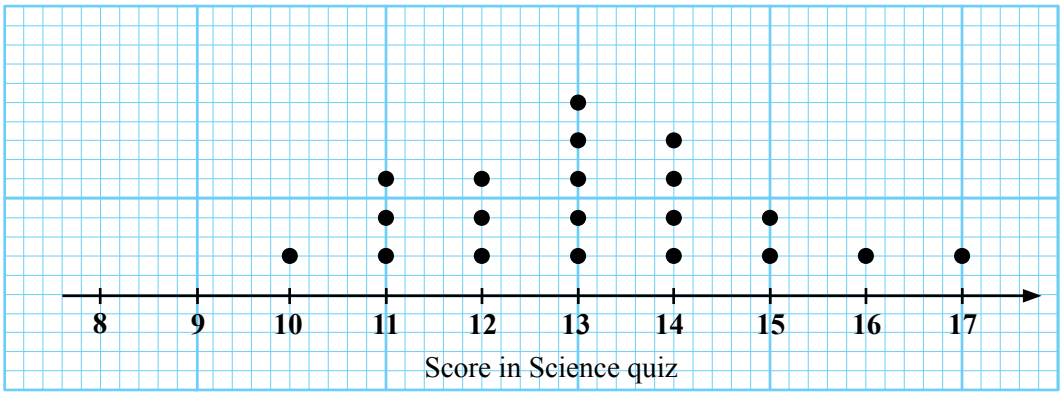
How do you determine the effect of data changes on dispersion based on graphical representation?

The effect of data changes can be graphically displayed by using dot plot, stem-and-leaf plot or box plot.

(a) The data is changed uniformly by adding or subtracting a constant.



When each value of a set of data is added by 2



In the above diagram, when each value of a set of data is added by 2, it is found that:

(i) The whole dot plot is shifted to the right by 2 units where the dispersion remains unchanged.

(ii) Original score range = 15 – 8 = 7

New score range = 17 – 10 = 7

The range of both sets of the data is the same.

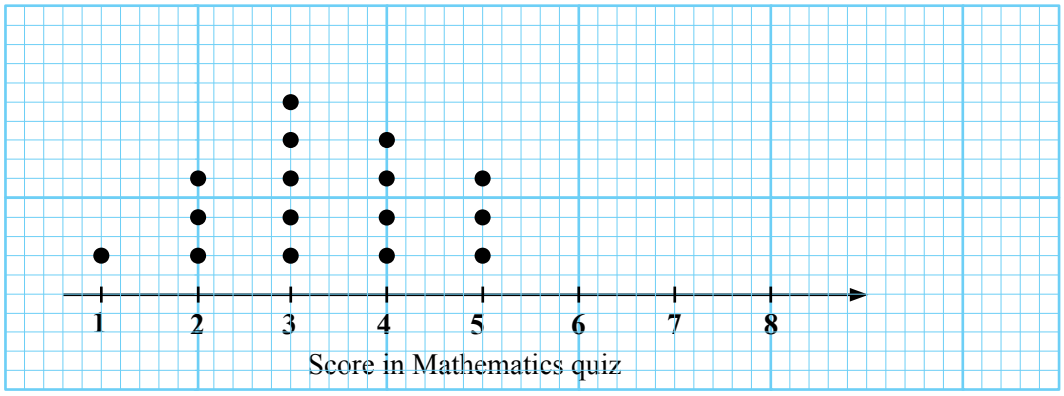
In general,

When each value of a set of data is added by a constant, the measures of dispersion remain unchanged.

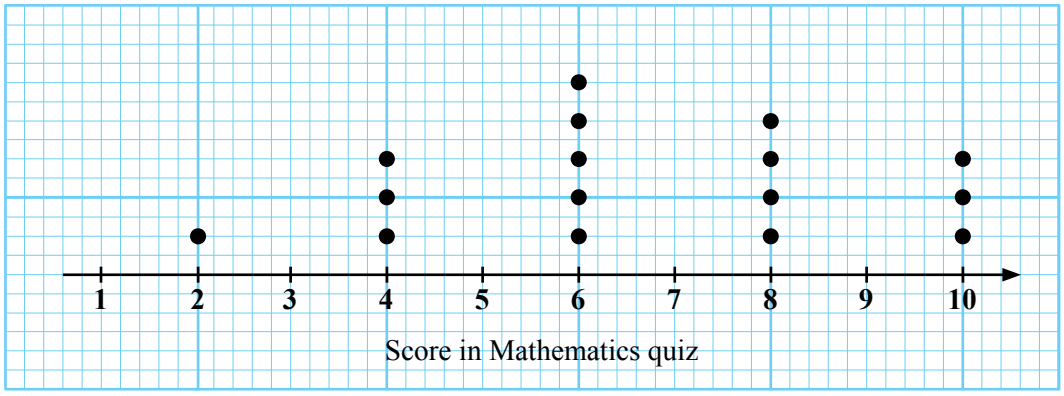
INTERACTIVE ZONE

Discuss what will happen to the measures of dispersion if each value of a set of data is subtracted by a constant.

(b) The data is changed uniformly by multiplying or dividing by a constant.



When each value of a set of data is multiplied by 2



In the above diagram, when each value of a set of data is multiplied by 2, it is found that:

(i) The dispersion of the whole dot plot becomes twice as wide.

(ii) Original score range = 5 – 1 = 4 New score range = 10 – 2 = 8

The range of the new set of data is two times the range of the original set of data.

When each value of a set of data is multiplied by a constant, k,

New range = k × original range

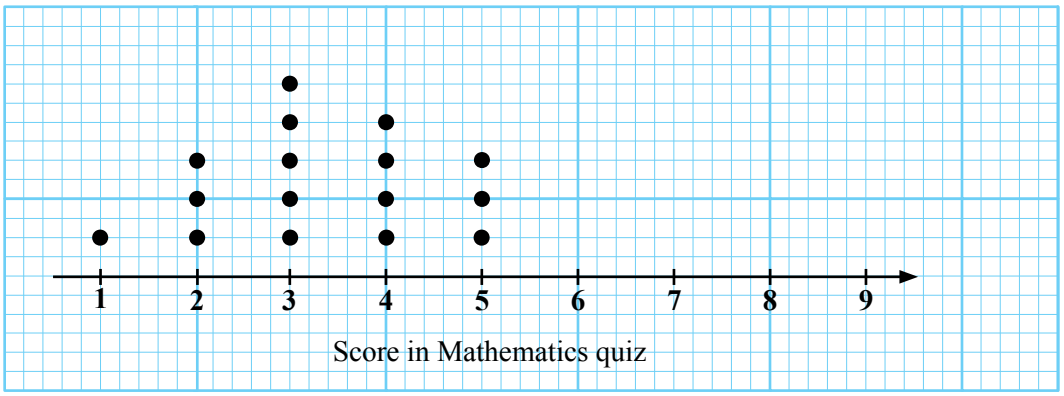
In general,

When each value of a set of data is multiplied by a constant, k, the measures of dispersion will change.

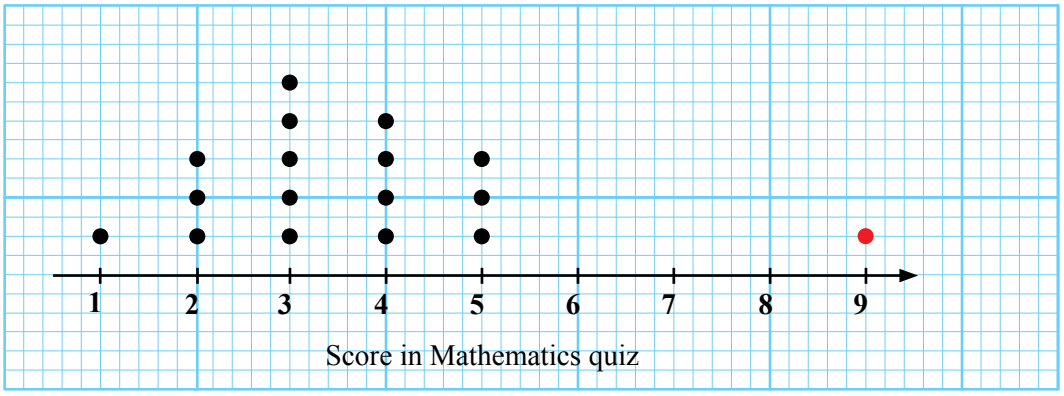
INTERACTIVE ZONE

Discuss what will happen to the measures of dispersion if each value of a set of data is divided by a constant.

(c) The value added or removed is an outlier.



When an outlier 9 is added



In the set above diagram, when an outlier is added to a set of data, it is found that:

(i) The dispersion of the whole dot plot becomes wider.

(ii) Original score range = 5 – 1 = 4 New score range = 9 – 1 = 8

The range of the new set of data becomes larger compared to the range of the original set of data when an outlier is added.

In general,

The measures of dispersion of the new set of data become larger compared to the measures of dispersion of the original set of data when an outlier is added.

INTERACTIVE ZONE

Discuss what will happen to the range when an outlier is removed from the set of data.

INTERACTIVE ZONE

Discuss what will happen to the measures of dispersion when an outlier is removed from the set of data.

Example 13

Given a set of data 4, 6, 7, 7, 9, 11, 12, 12, calculate the standard deviation of the set of data. Hence, calculate the new standard deviation when

(a) 4 is removed. (b) 7 is added. (c) 18 is added.

Solution:

$$\text{Mean} = \frac{4+6+7+7+9+11+12+12}{8}$$
$$= 8.5$$

$$\text{Standard deviation, } \sigma = \sqrt{\frac{4^2+6^2+7^2+7^2+9^2+11^2+12^2+12^2}{8} - 8.5^2}$$
$$= 2.784$$

(a) $$\text{New mean} = \frac{6+7+7+9+11+12+12}{7}$$
$$= 9.143$$

$$\text{New standard deviation, } \sigma = \sqrt{\frac{6^2+7^2+7^2+9^2+11^2+12^2+12^2}{7} - 9.143^2}$$
$$= 2.356$$

When the value that has a greater difference from mean is removed, the new standard deviation will be smaller.

(b) $$\text{New mean} = \frac{4+6+7+7+7+9+11+12+12}{9}$$
$$= 8.333$$

$$\text{New standard deviation, } \sigma = \sqrt{\frac{4^2+6^2+7^2+7^2+7^2+9^2+11^2+12^2+12^2}{9} - 8.333^2}$$
$$= 2.668$$

When the value that is close to the mean is added, the new standard deviation will be smaller.

(c) $$\text{New mean} = \frac{4+6+7+7+9+11+12+12+18}{9}$$
$$= 9.556$$

$$\text{New standard deviation, } \sigma = \sqrt{\frac{4^2+6^2+7^2+7^2+9^2+11^2+12^2+12^2+18^2}{9} - 9.556^2}$$
$$= 3.974$$

When the value that has a greater difference from mean is added, the new standard deviation will be larger.

Self Practice 8.2d

1. The interquartile range and the standard deviation of a set of data are 6 and 3.2 respectively. Calculate the new interquartile range and the new standard deviation if each value of the set of data is added by 4.

2. The interquartile range and the standard deviation of a set of data are 3 and 1.8 respectively. Calculate the new interquartile range and the new variance if each value of the set of data is multiplied by 3.

3. Given a set of data 4, 6, 7, 8, 10, 12, 16, calculate the standard deviation of the set of data. Explain the change on the standard deviation if 70 is added to the set of data.

4. (a) Determine the standard deviation of the set of data 3, 4, 7, 9, 10.
(b) Use the answer in (a) to determine the new standard deviation of each of the following sets of data.
(i) 6, 8, 14, 18, 20 (ii) 1.5, 2, 3.5, 4.5, 5

5. The range and the variance of a set of data are 25 and 2.7 respectively. If each value of the data is added by 2 then multiplied by 4, determine the new range and the new variance.

6. The interquartile range and the standard deviation of a set of data are 2.7 and 3.6 respectively. If each value of the data is divided by 3 then subtracted by 2, determine the new interquartile range and the new standard deviation.

How do you compare and interpret two or more sets of ungrouped data based on the appropriate measures of dispersion?

The measures of central tendency may not give us enough information to do comparisons. We need to determine the measures of dispersion of a set of data that can give an impression about the features of the data. Thus, the conclusion made is valid and acceptable.

Learning Standard

Compare and interpret two or more sets of ungrouped data, based on the appropriate measures of dispersion, and hence make conclusion.

Example 14

The table below shows the time taken by 20 employees in two factories to complete a job.

Time (hours)	5	6	7	8	9
Factory A	3	2	9	2	4
Factory B	1	5	7	5	2

Calculate the mean and the standard deviation for the time taken to complete the job. Hence, determine which factory's employees are more efficient to complete the job. Justify your answer.

Solution:

Factory A

$$\text{Mean, } \bar{x} = \frac{(5 \times 3) + (6 \times 2) + (7 \times 9) + (8 \times 2) + (9 \times 4)}{3 + 2 + 9 + 2 + 4}$$
$$= 7.1$$

$$\text{Standard deviation, } \sigma = \sqrt{\frac{(5^2 \times 3) + (6^2 \times 2) + (7^2 \times 9) + (8^2 \times 2) + (9^2 \times 4)}{3 + 2 + 9 + 2 + 4} - 7.1^2}$$
$$= 1.261$$

Factory B

$$\text{Mean, } x = \frac{(5 \times 1) + (6 \times 5) + (7 \times 7) + (8 \times 5) + (9 \times 2)}{1 + 5 + 7 + 5 + 2}$$
$$= 7.1$$

$$\text{Standard deviation, } \sigma = \sqrt{\frac{(5^2 \times 1) + (6^2 \times 5) + (7^2 \times 7) + (8^2 \times 5) + (9^2 \times 2)}{1 + 5 + 7 + 5 + 2} - 7.1^2}$$
$$= 1.044$$

The employees in Factory B are more efficient because its standard deviation is smaller.

Self Practice 8.2e

1. The table below shows the record of time in seconds, for a running event of 100 metres in 5 trials of two school athletes.

Athlete A	12.78	12.97	12.56	12.34	13
Athlete B	12.01	13.03	12.98	12.84	12.79

Using the appropriate measures of dispersion, determine which athlete is more consistent in his achievement.

2. The table below shows a study conducted on the effects of two types of fertilisers to the amount of tomatoes yield, in kg, for 10 tomato trees respectively.

Fertiliser A	**Fertiliser B**
12, 18, 25, 30, 36, 36, 40, 42, 50, 54	25, 28, 30, 32, 32, 38, 40, 40, 42, 45

Using the measures of dispersion, determine which fertiliser is more effective in improving the yield of tomatoes.

How do you solve problems involving measures of dispersion?

Learning Standard

Solve problems involving measures of dispersion.

Example 15

The table below shows the information of the masses of two groups of pupils.

Group	Number of pupils	Mean	Variance
A	18	52	2.5
B	12	56	1.8

All the pupils from group A and group B are combined. Determine the standard deviation of the mass of the combined groups of pupils.

Solution:

Understanding the problem

Determine the standard deviation of the mass of the combined groups of pupils.

Planning a strategy

Identify Σx and Σx^2 of the two groups of pupils and then calculate Σx and Σx^2 of the combined groups of pupils.

Implementing the strategy

Group A

$$\bar{x} = \frac{\Sigma x}{N}$$
$$52 = \frac{\Sigma x}{18}$$
$$\Sigma x = 936$$

$$\sigma^2 = \frac{\Sigma x^2}{N} - \bar{x}^2$$
$$2.5 = \frac{\Sigma x^2}{18} - 52^2$$
$$\Sigma x^2 = 48\,717$$

Group B

$$\bar{x} = \frac{\Sigma x}{N}$$
$$56 = \frac{\Sigma x}{12}$$
$$\Sigma x = 672$$

$$\sigma^2 = \frac{\Sigma x^2}{N} - \bar{x}^2$$
$$1.8 = \frac{\Sigma x^2}{12} - 56^2$$
$$\Sigma x^2 = 37\,653.6$$

Total of $\Sigma x = 936 + 672 = 1\,608$

Total of $\Sigma x^2 = 48\,717 + 37\,653.6 = 86\,370.6$

Mean, $\bar{x} = \frac{1\,608}{30} = 53.6$

Standard deviation, $\sigma = \sqrt{\frac{86\,370.6}{30} - 53.6^2}$

$= \sqrt{6.06}$

$= 2.462$

Conclusion

The new standard deviation is 2.462.

Self Practice 8.2f

1. The table below shows the scores obtained by seven participants in a contest. The scores are arranged in an ascending order.

Participant	A	B	C	D	E	F	G
Score	10	h	12	14	17	k	23

(a) The interquartile range and the mean score obtained are 7 and 15 respectively. Calculate the values of h and k.

(b) Calculate the standard deviation of the scores obtained by the participants.

2. A set of data contains 20 numbers. The mean and the standard deviation of the numbers are 9 and 2 respectively.

(a) Calculate the values of Σx and Σx^2.

(b) Some numbers in the set of data are removed. The sum of the numbers removed is 96 and the mean is 8. Given the sum of the squares of the numbers removed are 800, calculate the variance of the new set of data.

Comprehensive Practice

1. Calculate the range and the interquartile range of each set of data.

(a) 8, 25, 16, 11, 24, 18, 22

(b) 27, 33, 45, 18, 62, 50

(c) 3.4, 2.8, 2.7, 4.3, 3.8, 3.2, 3.0, 2.9

(d) 12, 19, 17, 18, 15, 12, 17, 20, 22, 30, 32, 16, 18

2. Calculate the range and the interquartile range of each set of data.

(a)

Diameter (cm)	6.0	6.2	6.4	6.6	6.8	7.0	7.2
Number of limes	6	9	12	18	20	10	5

(b)

Age (years)	13	14	15	16	17	18
Number of participants	12	18	21	20	21	8

3. Calculate the variance and the standard deviation of the following sets of data.

(a) 7, 9, 11, 8, 3, 7

(b) 50, 72, 63, 58, 55, 50, 70, 62, 66, 64

(c) 3.2, 4.4, 3.9, 4.1, 5.2, 4.8, 5.2

(d) 20, 27, 32, 47, 50, 38, 42, 40, 33, 37, 30

4. A set of data contains seven numbers. The sum of the seven numbers is 84 and the sum of the squares of the numbers is 1 920. Calculate the variance and the standard deviation of the set of data.

5. The range and the standard deviation of a set of numbers $x_1, x_2, x_3, ..., x_{10}$ are 10 and 5.2 respectively. Calculate

(a) the range and the standard deviation of the set of numbers $2x_1, 2x_2, 2x_3, ..., 2x_{10}$.

(b) the range and the standard deviation of the set of numbers $\frac{x_1 - 1}{4}, \frac{x_2 - 1}{4}, \frac{x_3 - 1}{4}, ..., \frac{x_{10} - 1}{4}$.

6. The masses of a group of eight pupils have a mean of 45 kg and a variance of 2.5 kg^2. Calculate

(a) the sum of the masses of the eight pupils.

(b) the sum of the squares of the masses of the pupils.

7. The mean of a set of numbers $(m - 4), m, (m + 2), 2m, (2m + 3)$ is 10.

(a) Calculate

(i) the value of m.

(ii) the standard deviation.

(b) Each number in the set of data is multiplied by 3 and then added by 2. Calculate the variance of the new set of data.

8. The table below shows the values of n, Σx and Σx^2 of a set of data.

n	Σx	Σx^2
12	66	1 452

(a) Calculate the variance.

(b) A number p is added to the set of data and it is found that the mean is increased by 0.5. Calculate

(i) the value of p.

(ii) the standard deviation of the new set of data.

9. Calculate the variance of the set of data $(p-4)$, $(p-2)$, $(p-1)$, p, $(p+4)$, $(p+9)$.

10. The table below shows the masses of players in two *sepak takraw* teams.

Team	Mass (kg)
A	48, 53, 65, 69, 70
B	45, 47, 68, 70, 75

(a) Calculate the mean, range, variance and standard deviation for the masses of the players in both teams.

(b) Is the range appropriate to be used as a measure of dispersion to represent the data above? Justify your answer.

(c) Determine the mass of which team has a greater dispersion from the mean.

11. The sum of a set of 10 numbers is 180 and the sum of the squares of the set of numbers is 3 800.

(a) Calculate the mean and the variance of the numbers.

(b) The number 19 is added to the set of numbers. Calculate the new mean and the new variance.

12. The table below shows the time taken, in hours, by 32 pupils to do revision in a week.

Time (hours)	1	2	3	4	5	6	7	10
Number of pupils	2	5	6	9	6	2	1	1

(a) Calculate the range, interquartile range, variance and standard deviation of the distribution.

(b) State the most appropriate measure of dispersion to show the time spent on revision by the pupils.

PROJECT

1. You will learn how to draw different graphs using a dynamic geometry software.
2. Scan the QR Code to carry out this project.
3. Print and display your drawing at the Mathematics Corner.

Scan the QR Code to carry out this activity.
https://www.geogebra.org/classic/h4frqvzj

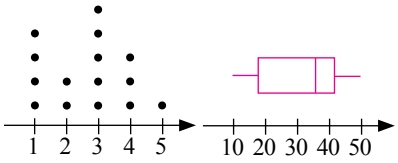
CONCEPT MAP

Measures of Dispersion

- Data Representation
  - Stem-and-Leaf Plot
  - Dot Plot
  - Box Plot

Stem	Leaf
2	0 1 3 4
3	1 2 2 2 3 4
4	0 2 3 5 7
5	1 3 4 6

Key:
2 | 1 means 21



- Range
  = Largest value – Smallest value
- Quartile
  - First quartile, Q_1
  - Third quartile, Q_3
  - Interquartile range $= Q_3 - Q_1$
- **Variance**

$$\sigma^2 = \frac{\Sigma(x-\bar{x})^2}{N}$$

$$\sigma^2 = \frac{\Sigma x^2}{N} - \bar{x}^2$$

$$\sigma^2 = \frac{\Sigma f(x-\bar{x})^2}{\Sigma f}$$

$$\sigma^2 = \frac{\Sigma f x^2}{\Sigma f} - \bar{x}^2$$

- **Standard deviation**

$$\sigma = \sqrt{\frac{\Sigma(x-\bar{x})^2}{N}}$$

$$\sigma = \sqrt{\frac{\Sigma x^2}{N} - \bar{x}^2}$$

$$\sigma = \sqrt{\frac{\Sigma f(x-\bar{x})^2}{\Sigma f}}$$

$$\sigma = \sqrt{\frac{\Sigma f x^2}{\Sigma f} - \bar{x}^2}$$

Self Reflection

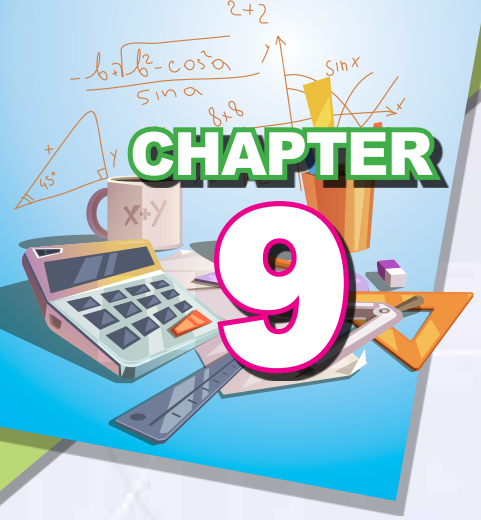
Circle your answers in the given word search.

1. _____ is the difference between the largest value and the smallest value.
2. The data that divides a set of data is known as quartile. The _____ quartile is the value of data at the $\frac{1}{4}$ position while the _____ quartile is the value of data at the $\frac{3}{4}$ position from the data arranged in order.
3. _____ is the measure of dispersion that refers to the difference between the third quartile and the first quartile.
4. The range, interquartile range and standard deviation are known as measures of _____.
5. _____ plot can show the distribution of a set of data.
6. The data with a smaller standard deviation is said to be more _____.
7. _____ is the measure of dispersion that measures how the data disperse around the mean of the set of data.

N	Q	D	Y	M	D	M	K	Z	D	Z	R	L	Q	Y	J	Y	J	E
O	G	R	R	J	Y	L	J	R	M	R	G	Y	K	L	W	L	G	R
I	U	A	R	T	I	L	T	H	I	R	D	A	K	R	Y	N	P	Q
T	J	D	N	B	M	D	M	N	L	D	L	Y	V	L	A	D	C	G
A	X	N	O	I	S	R	E	P	S	I	D	Z	X	R	J	O	V	N
I	L	U	D	D	W	V	T	D	T	L	G	M	E	J	N	Z	Y	T
V	G	L	A	X	K	Q	L	D	Q	D	J	L	M	S	M	J	B	Y
E	R	L	M	R	L	Y	Z	L	G	J	I	N	I	B	J	J	M	T
D	V	L	G	B	T	D	L	N	Q	T	X	S	B	Y	T	Y	D	W
D	D	J	T	N	B	I	B	V	R	J	T	K	L	G	M	T	R	T
R	B	M	K	L	L	M	L	A	R	E	L	N	R	L	P	G	L	L
A	L	Y	N	K	L	Z	U	F	N	A	Z	J	T	M	V	D	Z	B
D	Y	B	M	N	X	Q	B	T	I	N	N	T	Z	Q	L	T	Y	P
N	G	M	K	R	R	O	Q	R	T	R	R	G	D	W	R	T	B	N
A	B	V	R	E	V	N	B	Y	L	L	S	J	E	D	L	L	J	G
T	B	M	T	R	L	B	L	T	B	J	T	T	R	J	B	K	Z	T
S	L	N	Y	R	M	G	T	J	O	D	N	R	M	L	Z	D	P	B
K	I	V	G	R	D	N	D	R	M	I	N	R	M	A	R	W	B	D

Mathematics Exploration

1. Divide the class into groups.
2. Each group is required to obtain information about the months of birth of all their classmates.
3. Based on the information obtained, construct an appropriate data representation.
4. Using the data, determine
   (a) the range. (b) the first quartile. (c) the third quartile.
   (d) the interquartile range. (e) the variance. (f) the standard deviation.
5. Conduct Gallery Walk activity to see the work of other groups.



Probability of Combined Events

You will learn

- Combined Events
- Dependent Events and Independent Events
- Mutually Exclusive Events and Non-Mutually Exclusive Events
- Application of Probability of Combined Events

The Malaysia National Football team had qualified for the 1972 Olympic Games in Munich. In 1974, the team won the bronze medal in the Asian Games held in Tehran. The success of the National team continued thereafter when it was qualified consecutively for the Asian Cups in 1976 and 1980. Malaysia won its first Suzuki Cup in 2010.

Do you know how a coach selects players to be defenders, midfielders and strikers in a football team?

Why Study This Chapter?

Apart from sports, the knowledge of probability is applied in the insurance industry to determine the amount of insurance premium. The knowledge of probability is also used in the fields of production and business, especially for risk management.

WORD BANK

• tree diagram	• *gambar rajah pokok*
• probability	• *kebarangkalian*
• dependent event	• *peristiwa bersandar*
• mutually exclusive event	• *peristiwa saling eksklusif*
• independent event	• *peristiwa tak bersandar*
• non-mutually exclusive event	• *peristiwa tidak saling eksklusif*
• sample space	• *ruang sampel*

Walking Through Time

Blaise Pascal
(1623 – 1662)

Blaise Pascal was a French mathematician. He was also a physician, inventor and writer. Blaise Pascal invented the theory of probability together with Pierre de Fermat, a French mathematician. The theory of probability is an important mathematical foundation for statistics.

http://bt.sasbadi.com/m4243

9.1 Combined Events

What are combined events?

Learning Standard

Describe combined events and list out the possible combined events.

In our daily lives, we need to make a lot of decisions based on uncertainties. For example, choosing the science or the art stream class or selecting what products to sell on the school entrepreneurship day. These decisions incur risks and we should be able to assess the risks before making any decisions. Probability is used to evaluate the uncertainties that are associated in the process of decision making.

The combined events are the combination of two or more events in an outcome. For example, the possible outcomes for two pupils playing "Rock-Paper-Scissors" are (Scissors, Rock), (Scissors, Paper), (Scissors, Scissors), (Rock, Scissors), (Rock, Paper), (Rock, Rock), (Paper, Scissors), (Paper, Rock) and (Paper, Paper). The combined events can result from one or more experiments.

The outcomes of a combined event can be represented by ordered pairs.

Mind Stimulation 1

Aim: To list the possible outcomes of combined events

Material: Coins (10 sen, 20 sen and 50 sen), an empty box

Steps:

1. Pupils sit in pairs. Each pair is given a box containing three types of coins which are 10 sen, 20 sen and 50 sen coins.
2. Each pupil in the pair chooses a coin from the box at random. The values of the coins are recorded in the table below.
3. Return the coins to the box.
4. Repeat Steps 2 and 3 for 25 times.

(10, 20)				

MY MEMORY

A sample space is a set that contains all the possible outcomes of an experiment.

TIPS

(10, 20)

Coin chosen by the first pupil

Coin chosen by the second pupil

5. Write the sample space for the coin selection experiment for each pair of pupils.

$S = \{$ ____ $\}$

Discussion:

What is the possible number of outcomes in the activity?

From the activity in Mind Stimulation 1, it is found that:

The possible number of outcomes is $3 \times 2 = 6$.

In general,

$$n(S) = n(A) \times n(B)$$

$n(S)$ is all the possible number of outcomes, $n(A)$ and $n(B)$ represent the number of outcomes of event A and event B.

Example 1

Write the sample spaces for the combined events below.

(a) Five cards labelled with the letters "T, E, K, U, N" are put into a box. Two cards are taken out at random from the box one by one without replacement.

(b) Two coins are tossed (T and H representing tails and heads respectively).

Solution:

(a) {(T, E), (T, K), (T, U), (T, N), (E, T), (E, K), (E, U), (E, N), (K, T), (K, E), (K, U), (K, N), (U, T), (U, E), (U, K), (U, N), (N, T), (N, E), (N, K), (N, U)}

(b) {(T, T), (T, H), (H, T), (H, H)}

Self Practice 9.1a

Write the sample spaces for the combined events below.

1. Two books are chosen at random from a bookshelf that contains two history books (H), a geography book (G) and a mathematics book (M).

2. The children's gender for the family of two children.

3. A fair dice is rolled and a fair coin is tossed simultaneously.

4. Azhar (A) and Kai Meng (K) play a maximum of five badminton matches. The player that wins three sets is the winner.

9.2 Dependent Events and Independent Events

How do you differentiate between dependent and independent events?

Learning Standard

Differentiate between dependent and independent events.

Combined events can be categorised as dependent events and independent events.

Event A and event B are independent events if the occurrence of event A has no effect on the occurrence of event B and vice versa.

In other words, event A and event B are dependent events if the occurrence of event A affects the occurrence of event B.

Mind Stimulation 2

Aim: To differentiate dependent events and independent events

Steps:

1. Divide the class into groups.
2. Complete the table in the Activity Sheet below.

Activity Sheet:

Box P contains five cards labelled with the letters "R, U, A, N, G".

(a) Case I: Two cards are chosen at random from box P one by one without replacement. Write the probability of getting a consonant card on the first time and the second time in the table below.

(b) Case II: Two cards are chosen at random from box P one by one with replacement. The letter for the first card is recorded and it is returned to box P before the second card is chosen. Write the probability of getting a consonant card on the first time and the second time in the table below.

Case	Probability of getting a consonant card	
	First time	Second time
I		
II		

Discussion:

Why are the probabilities for the second time in case I and case II different? Discuss.

From the activity in Mind Stimulation 2, it is found that:

In case I, the first consonant card chosen is not returned to box *P*. The short of this first consonant card affected the probability of selecting the second consonant card.

Therefore,

The combined events of case I are dependent events.

In case II, the first consonant card chosen is returned to box *P* before the second card is chosen. The return of the first card results in the probability of selecting the second consonant card being the same as the probability of selecting the first consonant card. The probability of selecting the second consonant card is not affected by the probability of selecting the first consonant card.

Therefore,

The combined events of case II are independent events.

Example 2

Identify whether the following combined events are dependent events or independent events. Justify your answers.

(a) Obtain a tail twice when a fair coin is tossed twice.

(b) Obtain a tail in tossing a fair coin and obtained the number 4 in rolling a fair dice.

(c) Obtain two pens of the same colour when two pens are taken out one by one from a container that contains three red pens and two blue pens without replacement.

(d) Obtain two cards with the same letters when two cards are chosen at random from the cards labelled with the letters "B, A, I, K" one by one with replacement.

Solution:

(a) Independent events because the probability of getting a tail in the first toss does not affect the probability of getting a tail in the second toss.

(b) Independent events because the probability of getting a tail in tossing a fair coin does not affect the probability of getting the number 4 in rolling a fair dice.

(c) Dependent events because the probability of getting the first red pen affects the probability of getting the second red pen.

(d) Independent events because the probability of choosing the first card does not affect the probability of choosing the second card.

Self Practice 9.2a

Determine whether the following events are dependent events or independent events.

1. The pointer of a lucky wheel stops at the same sector twice consecutively.
2. The selection of two boys from a group of ten girls and fourteen boys at random.
3. Answer three objective questions with four options correctly if the answer of each question is chosen at random.
4. Box P contains two red cards and three black cards while box Q contains five red cards and six green cards. A card is chosen at random from box P and then put into box Q. After that, a card is chosen at random from box Q. Both cards chosen from box P and box Q are of the same colour.
5. Vincent and Bajat sit for a History test in school. Vincent and Bajat pass the History test.

How do you make and verify conjecture about the formula of probability of combined events?

Mind Stimulation 3

Aim: To make and verify conjecture about the formula of probability of combined events

Steps:

1. Divide the class into groups.
2. Roll a fair dice and toss a fair coin at the same time.
3. Complete the table below by recording all the possible outcomes.

Dice	Coin	
	Tail (T)	Head (H)
1		
2		
3		
4		
5		
6		

4. Based on the table above,
   (a) state the sample space for the above experiments.
   (b) state the probability, by listing all the possible outcomes of the combined events below.
      (i) Obtain an even number in rolling a dice and a tail in tossing a coin.

Learning Standard

Make and verify conjecture about the formula of probability of combined events.

Indicator

- Probability of event A, $P(A) = \frac{n(A)}{n(S)}$
- $0 \leq P(A) \leq 1$
- $P(A) = 0$ when event A will definitely not occur.
- $P(A) = 1$ when event A will definitely occur.

(ii) Obtain a prime number in rolling a dice and a head in tossing a coin.
(iii) Obtain a number less than 3 in rolling a dice and a tail in tossing a coin.

(c) Calculate the product of the probability of
(i) obtaining an even number in rolling a dice and a tail in tossing a coin.
(ii) obtaining a prime number in rolling a dice and a head in tossing a coin.
(iii) obtaining a number less than 3 in rolling a dice and a tail in tossing a coin.

Discussion:
Compare your answers in 4(b) and 4(c). What did you observe?

From the activity in Mind Stimulation 3, it is found that:

The probability of the intersection of two independent events A and B is equal to the product of the probability of event A and the probability of event B.

In general,

Multiplication rule of probability is $P(A \text{ and } B) = P(A) \times P(B)$

Example 3

Box F contains seven cards labelled with the letters "P, A, M, E, R, A, N" and box G contains five cards labelled with the numbers "3, 5, 6, 8, 11". A card is chosen at random from box F and box G respectively. Verify the conjecture about the formula of probability to get the letter "P" and an even number by listing all the possible outcomes.

Solution:

(a) Multiplication rule

$P(\text{getting a letter "P"}) = \frac{1}{7}$

$P(\text{getting an even number}) = \frac{2}{5}$

$P(\text{getting a letter "P" and an even number}) = \frac{1}{7} \times \frac{2}{5}$

$= \frac{2}{35}$

List all the possible outcomes.

Possible outcomes = {(P, 6), (P, 8)}

$n(S) = 7 \times 5$
$= 35$

$P(\text{getting a letter "P" and an even number}) = \frac{2}{35}$

Therefore, it is shown that both methods give the same answer.

Indicator

$P(A \cap B \cap C)$
$= P(A) \times P(B) \times P(C)$

Self Practice 9.2b

1. Two fair dice are rolled.

(a) Complete the following table by listing all the possible outcomes.

First dice	Second dice					
	1	2	3	4	5	6
1		(1, 2)	(1, 3)	(1, 4)		
2					(2, 5)	(2, 6)
3						
4						
5						
6						

(b) State $n(S)$ in this experiment.

(c) Verify the conjecture about the formula of probability of getting an odd number in the first dice and a prime number in the second dice by listing all the possible outcomes.

2. Kamal chooses two marbles randomly from a box which contains four red marbles, three yellow marbles and one green marble. The first marble is returned to the box before the second marble is chosen. Verify the conjecture about the formula of probability that two yellow marbles are chosen by listing all the possible outcomes.

3. Box *A* contains a red card and two yellow cards. Box *B* contains three red cards and a yellow card. Fauziah chooses a card from box *A* and box *B* respectively. Verify the conjecture about the formula of probability that Fauziah gets two yellow cards by listing all the possible outcomes.

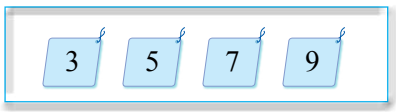
How do you determine the probability of combined events for dependent events and independent events?

Learning Standard

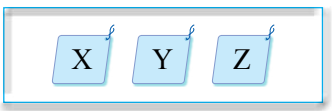
Determine the probability of combined events for dependent and independent events.

Example 4

Box *A* and box *B* contain cards labelled with the numbers "3, 5, 7, 9" and the letters "X, Y, Z" respectively. A card is chosen randomly from box *A* and box *B* respectively.



Box *A*



Box *B*

Calculate the probability of getting a factor of 9 and the letter "*Z*".

Solution:

$P(\text{A factor of 9}) = \frac{2}{4}$

$P(\text{The letter "}Z\text{"}) = \frac{1}{3}$

$P(\text{A factor of 9 and the letter "}Z\text{"}) = \frac{2}{4} \times \frac{1}{3}$

$= \frac{1}{6}$

Alternative Method

Factors of 9 and the letter "Z" = {(3, Z), (9, Z)}

$n(S) = 4 \times 3 = 12$

$P(\text{A factor of 9 and the letter "}Z\text{"}) = \frac{2}{12}$

$= \frac{1}{6}$

Example 5

A bag contains eight green marbles and a red marble. Two marbles are chosen randomly one by one from the bag without replacement. The colours of the marbles are recorded.

(a) Represent the above situation using a tree diagram.

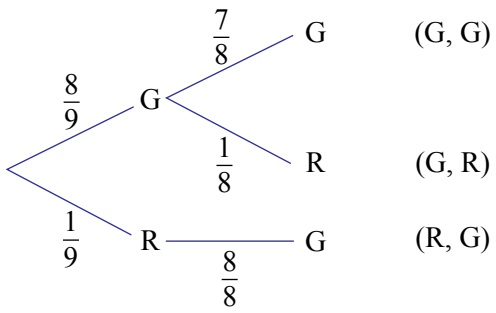
(b) Calculate the probability that

(i) the second marble is red.

(ii) both are green marbles.

Solution:

(a)



MY MEMORY

A tree diagram displays all the possible outcomes of an event. Each branch in the tree diagram represents a possible outcome.

(b) (i) $P(\text{The second marble is red}) = \frac{8}{9} \times \frac{1}{8}$

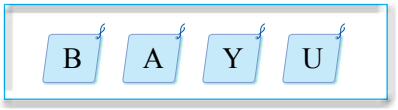
$= \frac{1}{9}$

(ii) $P(\text{Both are green marbles}) = \frac{8}{9} \times \frac{7}{8}$

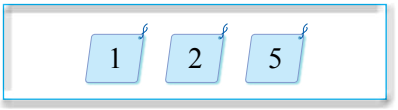
$= \frac{7}{9}$

Self Practice 9.2c

1. Box K and box L contain four cards labelled with the letters "B, A, Y, U" and three cards labelled with the numbers "1, 2, 5" respectively. A card is chosen at random from box K and box L respectively.



Box K



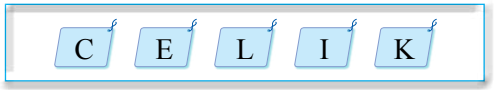
Box L

By listing all the possible outcomes, calculate the probability of getting a vowel and an even number.

2. A fair dice with four faces is labelled with "1, 2, 3, 4". The dice is rolled twice and the numbers that the dice lands on are recorded. By listing all the possible outcomes, calculate the probability of getting two odd numbers.

3. According to an investigation, the probability of rainfall on Mountain X in May is 0.45. Calculate the probability that Mountain X will have two consecutive rainy days in May.

4. Box T contains five cards labelled with the letters "C, E, L, I, K". Two cards are taken out randomly one by one from box T without replacement.



Box T

Calculate the probability of getting the first card labelled with a consonant and the second card labelled with a vowel.

5. A box contains twelve bulbs where two of the bulbs are burnt. Two bulbs are selected at random from the box. By sketching a tree diagram, calculate the probability of getting two burnt bulbs.

6. The following table shows the number of the Science and Mathematics Society members in SMK Didik Jaya.

Session	Number of members	
	Female	Male
Morning	146	124
Afternoon	82	96

Two members are selected randomly.

(a) from the male members. Calculate the probability that both members chosen are from the morning session. Give your answer correct to four significant figures.

(b) from the afternoon session. Calculate the probability that both members chosen are female. Give your answer correct to four significant figures.

9.3 Mutually Exclusive Events and Non-Mutually Exclusive Events

How do you differentiate between mutually exclusive and non-mutually exclusive events?

Learning Standard

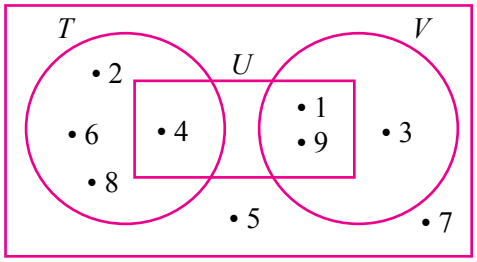
Differentiate between mutually exclusive and non-mutually exclusive events.

Some table tennis balls labelled from 1 to 9 are put in an empty basket. A pupil chooses a table tennis ball from the basket at random.

Let T is the event of getting an even number.
U is the event of getting a perfect square.
V is the event of getting a factor of 9.

The relationship between the three events, T, U and V, can be illustrated using a Venn diagram.

Based on the Venn diagram on the right, it is observed that event T and event V cannot happen at the same time. Thus, event T and event V are said to be mutually exclusive events. Event T and event U are non-mutually exclusive events as the table tennis ball labelled with 4 is the common outcome for both event T and event U. Are event U and event V mutually exclusive events? Discuss.



Mind Stimulation 4

Aim: To differentiate mutually exclusive events and non-mutually exclusive events

Steps:

1. Divide the class into groups.
2. Complete the following Activity Sheet.

Activity Sheet:

A pupil is chosen at random from your class. The following are event A to event F.

Event A: Pupils who are wearing spectacles.
Event B: Members of Girl Guides.
Event C: Boys.
Event D: Pupils who obtained Grade A in Mathematics test.
Event E: Pupils who love Mathematics.
Event F: Pupils who obtained Grade D in Mathematics test.

Mark ✓ for the mutually exclusive events or non-mutually exclusive events for the following combined events.

Combined event	Mutually exclusive events	Non-mutually exclusive events
Events A and B		
Events B and C		
Events B and D		
Events D and E		
Events E and F		
Events D and F		

Discussion:

Are all the mutually exclusive events stated in the table? If they are not, please state the events.

From the activity in Mind Stimulation 4, it is found that:

For each combined event above, whether it is a mutually exclusive event or non-mutually exclusive event depends on the group of pupils in the respective classes. If there are only boys wearing spectacles, then event A and event B are mutually exclusive. However, for classes that have girls wearing spectacles, then event A and event B are non-mutually exclusive events.

In general,

A combined event A and B is known as a mutually exclusive event if there is no intersection between events A and B, $A \cap B = \phi$.

Example 6

A worker in a factory is chosen at random.
Given

A = A worker with wages less than RM2 500.
B = A worker that needs to pay income tax.
C = A worker who goes to work by car.

Determine whether the following pairs of events are mutually exclusive events or non-mutually exclusive events.

(a) A and B (b) A and C (c) B and C

INFO ZONE

Starting 2015, a person with an annual income of more than RM34 000 (after the deduction of EPF) will need to register an income tax file.

Lembaga Hasil Dalam Negeri, Updated: 16 March 2017

Solution:

(a) Event A and event B cannot occur together. Therefore, event A and event B are mutually exclusive.

(b) Event A and event C can occur together. Therefore, event A and event C are non-mutually exclusive.

(c) Event B and event C can occur together. Therefore, event B and event C are non-mutually exclusive.

Self Practice 9.3a

1. A fair dice is rolled. A list of events is given as follows.

P is the event of getting a number that is greater than 4.
Q is the event of getting an even number.
R is the event of getting a perfect square.

Determine whether the following pairs of events are mutually exclusive events or non-mutually exclusive events.

(a) P and Q (b) P and R (c) Q and R

2. An egg is chosen at random from a farm.

K is the event of selecting a cracked egg.

L is the event of selecting a Grade A egg.

M is the event of selecting a Grade C egg.

Determine whether the following pairs of events are mutually exclusive events or non-mutually exclusive events.

(a) K and L (b) K and M (c) L and M

3. A tourist is selected randomly at Kuala Lumpur International Airport.

R is the event of selecting a tourist from an European country.

S is the event of selecting a tourist from an ASEAN country.

T is the event of selecting a tourist from a Commonwealth country.

Determine whether the following pairs of events are mutually exclusive events or non-mutually exclusive events.

(a) R and S (b) R and T (c) S and T

INFO ZONE

ASEAN countries: Malaysia, Brunei, Singapore, Cambodia, Indonesia, Vietnam, Myanmar, Philippines, Thailand, Laos.
Commonwealth countries: The countries that had been colonised by the British.

Learning Standard

Verify the formula of probability of combined events for mutually exclusive and non-mutually exclusive events.

How do you verify the formula of probability of combined events for mutually exclusive and non-mutually exclusive events?

Mind Stimulation 5

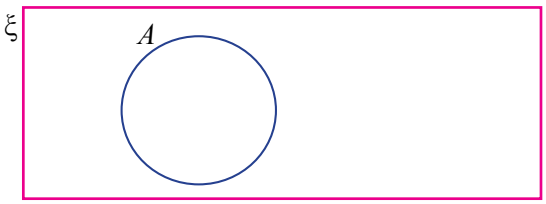
Aim: To verify the formula of probability of combined events for mutually exclusive and non-mutually exclusive events.

Steps:

1. Divide the class into groups.
2. Study the following case.

Fahmi holds an open house in conjunction of the Aidilfitri celebration.
80 guests come to the open house.
$\frac{2}{5}$ of the guests who come are Fahmi's colleagues.
There are 55 guests who come together with their family members. 18 of them are Fahmi's colleagues.
$\frac{1}{10}$ of the guests who come are Fahmi's childrens schoolmates.
All of Fahmi's children's schoolmates do not come with their family members.
A guest who comes to Fahmi's open house is selected at random.
A is the event that a guest chosen comes with family members.
B is the event that a guest chosen is Fahmi's colleague.
C is the event that a guest chosen is Fahmi's children's schoolmate.

3. Complete the following Venn diagram to show the relationship between events A, B and C.



4. Based on the Venn diagram drawn, complete the following table.

	Probability			
(a)	$P(A) =$	$P(B) =$	$P(A \text{ and } B) =$	$P(A \text{ or } B) =$
(b)	$P(A) =$	$P(C) =$	$P(A \text{ and } C) =$	$P(A \text{ or } C) =$
(c)	$P(B) =$	$P(C) =$	$P(B \text{ and } C) =$	$P(B \text{ or } C) =$

Discussion:

1. Why do $P(A \text{ and } B)$, $P(A \text{ and } C)$ and $P(B \text{ and } C)$ need to be determined before calculating $P(A \text{ or } B)$, $P(A \text{ or } C)$ and $P(B \text{ or } C)$?
2. Based on the answers in (a), (b) and (c) in the above table, form an equation to relate all four probabilities for each (a), (b) and (c). Justify your answers.

From the activity in Mind Stimulation 5, it is found that:

1. $P(A \text{ and } B)$, $P(A \text{ and } C)$ and $P(B \text{ and } C)$ are identified first so that we can determine whether the combined events are mutually exclusive or non-mutually exclusive.
2. (a) The combined event A and B is non-mutually exclusive because $P(A \cap B) \neq 0$, then $P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)$.
   (b) The combined event A and C and combined event B and C are both mutually exclusive because $P(A \cap C) = 0$ and $P(B \cap C) = 0$. Then, $P(A \text{ or } C) = P(A) + P(C)$ and $P(B \text{ or } C) = P(B) + P(C)$.

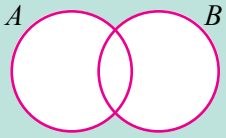
In general,

The **addition rule of probability** is
$P(A \cup B) = P(A) + P(B)$ or $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

MY MEMORY

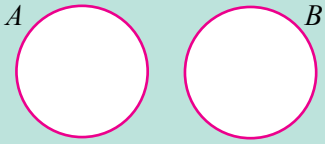
$P(A \text{ and } B) = P(A \cap B)$
$P(A \text{ or } B) = P(A \cup B)$

Events A and B are **non-mutually exclusive** events



$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

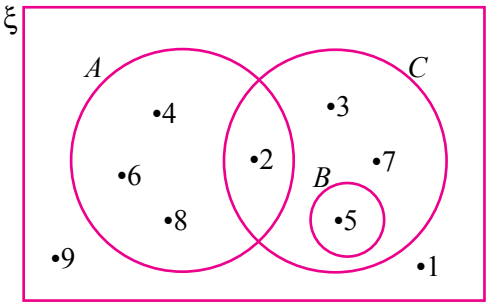
Events A and B are **mutually exclusive** events



$P(A \cup B) = P(A) + P(B)$

Example 7

The Venn diagram on the right shows the relationship between the universal set, ξ, A, B and C.



A number is chosen at random from the universal set, ξ. Verify the addition rule of probability for each of the following combined events.

(a) Obtaining an even number or a multiple of 5.
(b) Obtaining an even number or a prime number.

Solution:

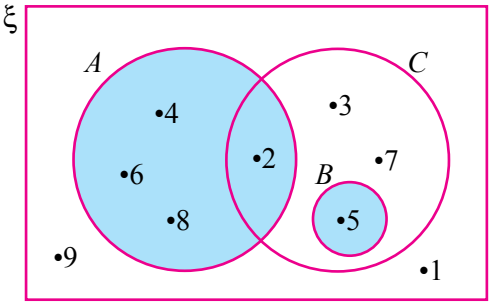
(a) $P(A \cup B) = \dfrac{n(A \cup B)}{n(S)}$

$= \dfrac{5}{9}$

$P(A) + P(B) = \dfrac{4}{9} + \dfrac{1}{9}$

$= \dfrac{5}{9}$

Hence, it is proven that $P(A \cup B) = P(A) + P(B)$.



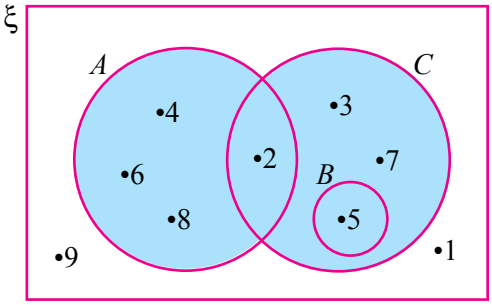
(b) $P(A \cup C) = \dfrac{n(A \cup C)}{n(S)}$

$= \dfrac{7}{9}$

$P(A) + P(C) - P(A \cap C) = \dfrac{4}{9} + \dfrac{4}{9} - \dfrac{1}{9}$

$= \dfrac{7}{9}$

Hence, it is proven that
$P(A \cup C) = P(A) + P(C) - P(A \cap C)$.



Example 8

Eight cards labelled with the numbers "4, 5, 6, 7, 8, 9, 10, 11" are put in a box. A card is chosen at random from the box.

A is the event of getting a number greater than 8.
B is the event of getting a prime number.
C is the event of getting an even number.

Verify the addition rule of probability for each of the following combined events by listing all the possible outcomes.

(a) $P(A \text{ or } B)$ (b) $P(A \text{ or } C)$ (c) $P(B \text{ or } C)$

Solution:

(a) $A \cap B = \{11\}$

$P(A) + P(B) - P(A \cap B) = \frac{3}{8} + \frac{3}{8} - \frac{1}{8}$

$= \frac{5}{8}$

$A = \{9, 10, 11\}, B = \{5, 7, 11\}$

$A \cup B = \{5, 7, 9, 10, 11\}$

$P(A \cup B) = \frac{5}{8}$

Hence, it is proven that $P(A) + P(B) - P(A \cap B) = P(A \cup B)$.

(b) $A \cap C = \{10\}$

$P(A) + P(C) - P(A \cap C) = \frac{3}{8} + \frac{4}{8} - \frac{1}{8}$

$= \frac{6}{8}$

$= \frac{3}{4}$

$A = \{9, 10, 11\}, C = \{4, 6, 8, 10\}$

$A \cup C = \{4, 6, 8, 9, 10, 11\}$

$P(A \cup C) = \frac{6}{8}$

$= \frac{3}{4}$

Hence, it is proven that $P(A) + P(C) - P(A \cap C) = P(A \cup C)$.

(c) $B \cap C = \{\ \}$

$P(B) + P(C) = \frac{3}{8} + \frac{4}{8}$

$= \frac{7}{8}$

$B = \{5, 7, 11\}, C = \{4, 6, 8, 10\}$

$B \cup C = \{4, 5, 6, 7, 8, 10, 11\}$

$P(B \cup C) = \frac{7}{8}$

Hence, it is proven that $P(B) + P(C) = P(B \cup C)$.

Self Practice 9.3b

1. Two fair dice are rolled simultaneously.

Q is the event that the total point obtained from the two dice is greater than 9.

R is the event that the product of the points obtained from the two dice is a multiple of 5.

S is the event that two equal points on the two dice are obtained.

Verify the addition rule of probability for each of the following combined events by listing all the possible outcomes.

(a) $P(Q \text{ or } R)$ (b) $P(Q \text{ or } S)$ (c) $P(R \text{ or } S)$

2. Two fair coins are tossed simultaneously.

J is the event of getting two tails.

K is the event of getting two heads.

L is the event of getting at least one tail.

Verify the addition rule of probability for each of the following combined events by listing all the possible outcomes.

(a) $P(J \text{ or } K)$ (b) $P(J \text{ or } L)$ (c) $P(K \text{ or } L)$

3. Seven cards labelled with the letters "B, A, H, A, G, I, A" are put in a box. A card is chosen at random.
L is the event of getting a vowel.
M is the event of getting a consonant.
N is the event of getting a letter "B".

(a) Draw a Venn diagram to represent the relationship between the events L, M and N.

(b) Verify the addition rule of probability for each of the following combined events by listing all the possible outcomes.

(i) $P(L \text{ or } M)$ (ii) $P(L \text{ or } N)$ (iii) $P(M \text{ or } N)$

INTERACTIVE ZONE

Is $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$? Discuss by using a Venn diagram.

How do you determine the probability of combined events for mutually exclusive and non-mutually exclusive events?

Learning Standard

Determine the probability of combined events for mutually exclusive and non-mutually exclusive events.

Example 9

Five cards labelled with the letters "C, I, N, T, A" are put in a box. A card is chosen at random. Calculate the probability that the card chosen is labelled with a consonant or letter "A".

Solution:

A card labelled with a consonant = {C, N, T}

A card labelled with letter "A" = {A}

$$P(\text{A card labelled with a consonant or letter "A"}) = \frac{3}{5} + \frac{1}{5}$$
$$= \frac{4}{5}$$

TIPS

For two mutually exclusive events,
$P(A \cup B) = P(A) + P(B)$

Example 10

In a banquet, the probabilities that Zalifah and Maran eat *cendol* are $\frac{5}{7}$ and $\frac{3}{5}$ respectively.

(a) Represent the probability that Zalifah and Maran eat *cendol* at the banquet using a Venn diagram.

(b) Calculate the probability that Zalifah or Maran eats *cendol* at the banquet.

Solution:

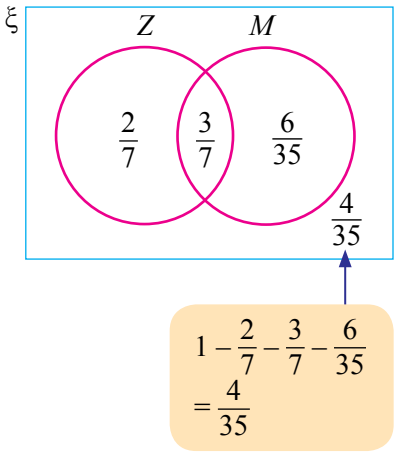
(a) $P(\text{Both Zalifah and Maran eat } cendol \text{ at the banquet})$
$$= \frac{5}{7} \times \frac{3}{5}$$
$$= \frac{3}{7}$$

$$P(\text{Only Zalifah eats } cendol \text{ at the banquet}) = \frac{5}{7} - \frac{3}{7}$$
$$= \frac{2}{7}$$

Malaysiaku

Cendol is a very popular dessert in Malaysia. The ingredients contain green droplets made of rice flour and *pandan* juice, and mixed together with ice, coconut milk and brown sugar.

P(Only Maran eats *cendol* at the banquet)
$= \frac{3}{5} - \frac{3}{7}$
$= \frac{6}{35}$



(b) P(Zalifah or Maran eats *cendol* at the banquet)
$= P(Z) + P(M) - P(Z \cap M)$
$= \frac{5}{7} + \frac{3}{5} - \frac{3}{7}$
$= \frac{31}{35}$

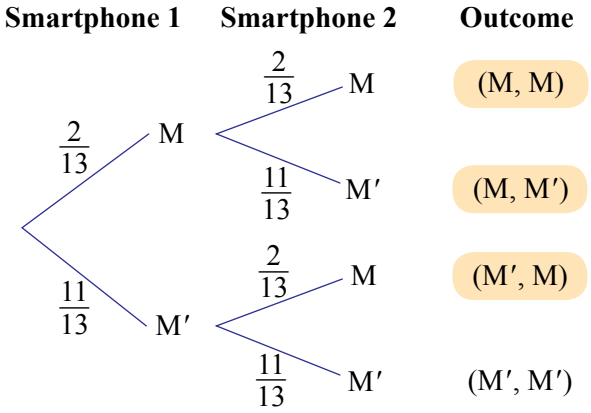
Alternative Method

P(Zalifah or Maran eats *cendol* at the banquet) $= \frac{2}{7} + \frac{3}{7} + \frac{6}{35}$
$= \frac{31}{35}$

Example 11

The probability that a smartphone manufactured by Jaya Factory has a display problem is $\frac{2}{13}$. Two smartphones are chosen at random. Draw a tree diagram to show all the possible outcomes. Hence, calculate the probability that at least one smartphone chosen has a display problem.

Solution:



M = Has a display problem
M′ = No display problem

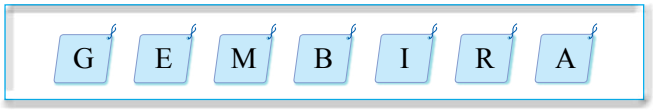
INTERACTIVE ZONE

Beside the given solution, what other methods can be used to solve Example 11? Discuss.

P(At least one smartphone has a display problem)
$= P(\text{M}, \text{M}) + P(\text{M}, \text{M}') + P(\text{M}', \text{M})$
$= \left(\frac{2}{13} \times \frac{2}{13}\right) + \left(\frac{2}{13} \times \frac{11}{13}\right) + \left(\frac{11}{13} \times \frac{2}{13}\right)$
$= \frac{48}{169}$

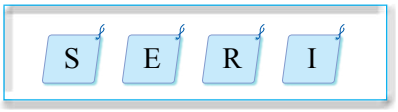
Self Practice 9.3c

1. Seven cards labelled with the letters "G, E, M, B, I, R, A" are put in a box. A card is chosen at random from the box.

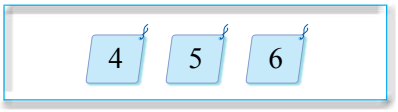


By listing all the possible outcomes, calculate the probability that the card chosen is labelled with a vowel or letter "R".

2. Two boxes labelled as *K* and *L* contain four cards labelled with the letters "S, E, R, I" and three cards labelled with the numbers "4, 5, 6" respectively. A card is chosen at random from each box.



Box *K*



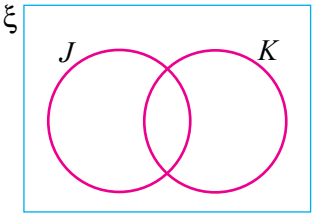
Box *L*

By listing all the possible outcomes, calculate the probability of getting a letter "S" from box *K* or a multiple of 3 from box *L*.

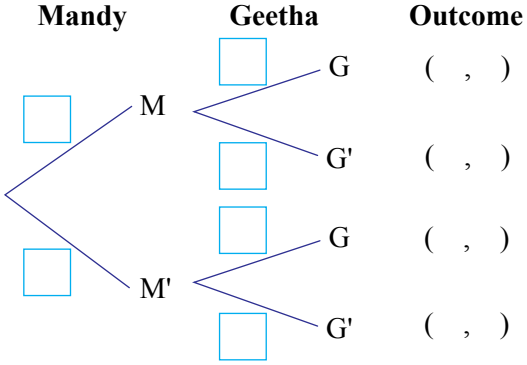
3. The probability of appointing Jessie as the chairman of the Finance Club (J) and the head of the sports house (K) are $\frac{3}{8}$ and $\frac{2}{9}$ respectively.

(a) Complete the Venn diagram on the right to represent the relationship between the probabilities of appointing Jessie as the chairman of the Finance Club and the head of the sports house.

(b) Calculate the probability of not appointing Jessie as the chairman of the Finance Club or the head of the sports house.



4. The Geography Club of SMK Cerdik organises a study group tour to Kota Kinabalu. The probability of Mandy and Geetha joining this tour are $\frac{4}{7}$ and $\frac{9}{14}$ respectively. Complete the following tree diagram. Hence, calculate the probability of either Mandy or Geetha joining this tour.



9.4 Application of Probability of Combined Events

How do you solve problems involving probability of combined events?

Learning Standard

Solve problems involving probability of combined events.

Example 12

A fair dice is rolled twice consecutively. If this experiment is carried out 540 times, how many times will at least one perfect square be obtained?

Solution:

Understanding the problem

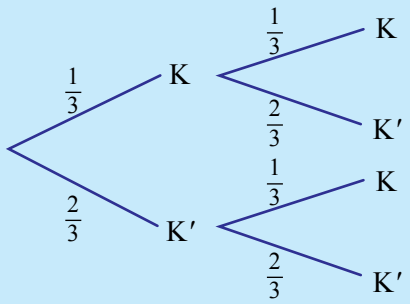
- Independent combined events
- Perfect squares = 1, 4
- At least one perfect square = (K, K), (K, K′) or (K′, K)

Planning a strategy

- Draw a tree diagram.
- $P(\text{perfect squares}) = \frac{2}{6} = \frac{1}{3}$
- Calculate $P[(K, K) \cup (K, K') \cup (K', K)]$
- $P[(K, K) \cup (K, K') \cup (K', K)] \times 540$ times

Implementing the strategy

K = Event of getting a perfect square
K' = Event of not getting a perfect square



$$P[(K, K) \cup (K, K') \cup (K', K)] = \left(\frac{1}{3} \times \frac{1}{3}\right) + \left(\frac{1}{3} \times \frac{2}{3}\right) + \left(\frac{2}{3} \times \frac{1}{3}\right) = \frac{5}{9}$$

$$n(\text{at least one perfect square}) = \frac{5}{9} \times 540 = 300 \text{ times}$$

Checking Answer

$$n(\text{at least one perfect square}) = \left(1 - \frac{2}{3} \times \frac{2}{3}\right) \times 540 = 300$$

Conclusion

There are 300 times to obtain at least one perfect square.

Example 13

A box contains seven red marbles, five yellow marbles and three blue marbles. Two marbles are chosen randomly one by one from the box. If the first marble is blue, the blue marble is then returned to the box before a second marble is chosen. If the first marble is not blue, the marble is not returned to the box and then a second marble is chosen. Calculate the probability of getting two marbles of different colours.

Solution:

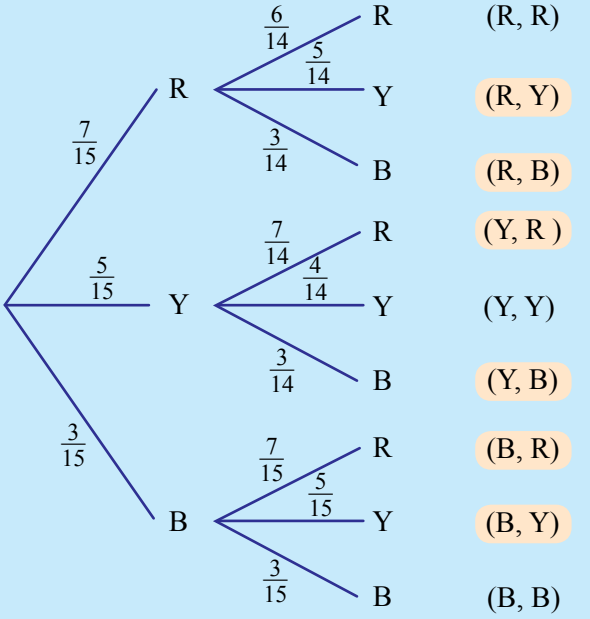
Understanding the problem

- Dependent combined events
- Total number of marbles = 15
- Two marbles of different colours
  = {(R, Y), (R, B), (Y, R), (Y, B), (B, R), (B, Y)}

Planning a strategy

- Draw a tree diagram.
- Calculate $P[(R, Y) \cup (R, B) \cup (Y, R) \cup (Y, B) \cup (B, R) \cup (B, Y)]$

Implementing the strategy



P(Marbles of different colours)
$= P(R, Y) + P(R, B) + P(Y, R) + P(Y, B) + P(B, R) + P(B, Y)$

$$= \left(\frac{7}{15} \times \frac{5}{14}\right) + \left(\frac{7}{15} \times \frac{3}{14}\right) + \left(\frac{5}{15} \times \frac{7}{14}\right) + \left(\frac{5}{15} \times \frac{3}{14}\right) + \left(\frac{3}{15} \times \frac{7}{15}\right) + \left(\frac{3}{15} \times \frac{5}{15}\right)$$

$$= \frac{349}{525}$$

Conclusion

The probability of getting two marbles of different colours is $\frac{349}{525}$.

Checking Answer

Complement method:
P(two marbles of different colours)
$= 1 - [P(R, R) + P(Y, Y) + P(B, B)]$

$$= 1 - \left[\left(\frac{7}{15} \times \frac{6}{14}\right) + \left(\frac{5}{15} \times \frac{4}{14}\right) + \left(\frac{3}{15} \times \frac{3}{15}\right)\right]$$

$$= \frac{349}{525}$$

Self Practice 9.4a

1. A study is carried out on the gender of the children from 16 000 families with two children. Estimate the number of families with at least one son in that study.

2. A box contains three yellow pens, five red pens and a black pen. Two pens are chosen at random from the box. Calculate the probability that both pens chosen are of the same colour.

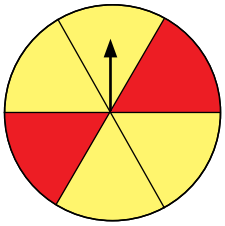
3. Jonathan enjoys watching the sunset on the beach. Jonathan has two options for either going to Pantai Jati or Pantai Cengal for two days.

The probability that it will rain every evening at Pantai Jati is $\frac{19}{25}$.

The probability that it will rain at Pantai Cengal depends on the previous day. If it does not rain on the previous day, the probability that it will rain in the evening is $\frac{5}{7}$. If it rains on the previous day, the probability it will rain in the evening is $\frac{2}{5}$.

If the weather is good on both beaches on the previous day Jonathan departed, which beach should Jonathan choose so that he can enjoy watching the sunset on the beach for both evenings? Justify your answers.

4. Each customer of Naga Shop who spends more than RM200 will be given a chance to spin a lucky wheel that has six similar sectors. Two of the sectors are red and the rest are yellow.



Box	Number of cash vouchers	
	RM50	RM10
A	20	5
B	10	20

If the pointer of the lucky wheel stops in the red sector, the customer has a chance to choose a cash voucher from box *A*. If the pointer of the lucky wheel stops in the yellow sector, the customer has a chance to choose a cash voucher from box *B*. The number of cash vouchers in box *A* and box *B* are shown in the table above.

It is estimated that 450 customers of Naga Shop will spend more than RM200. Calculate the number of RM10 vouchers the Naga Shop needs to prepare.
(The selected cash vouchers will be replaced with new vouchers by the shop so that the number of cash vouchers in both boxes remains the same.)

Comprehensive Practice

1. There are three blue coloured pencils and two green coloured pencils in a box. Two colour pencils are randomly selected one by one from the box without replacement. Write the sample space for the selected colour pencils.

2. A number is chosen at random from set $S = \{x : x \text{ is an integer}, 1 \leqslant x \leqslant 30\}$. Calculate the probability of

(a) getting a multiple of 3 and a multiple of 12.

(b) getting a factor of 20 or a factor of 8.

3. Two cards labelled with the numbers "77, 91" are put in box M and three cards labelled with the letters "R, I, A" are put in box N. A card is chosen at random from box M and box N respectively.

(a) Complete the following table with all the possible outcomes.

Box *M*	**Box *N***		
	R	I	A
77	(77, R)		
91		(91, I)	

(b) By listing all the possible outcomes, calculate the probability that

(i) a number card with the sum of its digits is greater than 10 and a consonant card are chosen.

(ii) a number card with the sum of its digits is greater than 10 or a consonant card are chosen.

4. Two prefects are chosen at random from five prefects, where three of them are in Form Four and two are in Form Five. Draw a tree diagram to show all the possible outcomes. Then, calculate the probability that both prefects chosen are in the same form.

5. The probability of Kam Seng passing his Physics and Chemistry tests are 0.58 and 0.42 respectively. Calculate the probability that

(a) Kam Seng passes both tests.

(b) Kam Seng passes only one test.

6. Fatimah sends her resume for job application to three companies. The probabilities of Fatimah getting a job offer from companies X, Y and Z are $\frac{3}{5}$, $\frac{4}{9}$ and $\frac{5}{12}$ respectively. Calculate the probability of Fatimah getting a job offer from

(a) any two companies.

(b) at least one company.

7. It is given that event A and event B are two mutually exclusive events and $P(A) = \frac{1}{3}$.

(a) State the maximum value of $P(B)$.

(b) If $P(A \cup B) = \frac{7}{9}$, identify $P(B)$.

8. Box R contains five red marbles and seven green marbles while box T contains four red marbles and eight green marbles. A marble is randomly selected from box R. If the marble is red, that marble will be put into box T. If the marble is green, that marble will be returned to box R. Then, a marble will be randomly selected from box T. The colours of the selected marbles will be recorded.

(a) Calculate the probability of

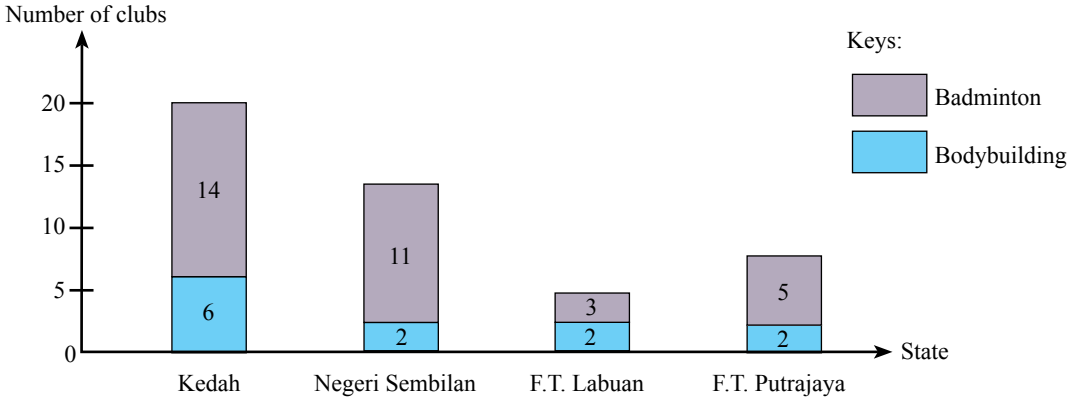
(i) selecting two red marbles.

(ii) selecting two marbles of different colours.

(b) Class 4 Amanah has 36 pupils. Each pupil is given an opportunity to select two marbles with the conditions stated above and a pupil who successfully selected two green marbles will be given a gift worth RM5. Estimate the cost of gifts needed.

9. Jacky has eight shirts and three of them are blue shirts. 40% of the shirts that Halim has are blue. $\frac{1}{5}$ of the shirts that Kumar has are blue. Jacky, Halim and Kumar each chooses a shirt at random to attend a meeting. Calculate the probability that two of them wear blue shirts.

10. The following bar chart shows the number of badminton clubs and bodybuilding clubs in Kedah, Negeri Sembilan, Federal Territory of Labuan and Federal Territory of Putrajaya.



A badminton club and a bodybuilding club in the four states are selected at random. Calculate the probability that

(a) both clubs selected are from Kedah.

(b) a club is selected each from the Federal Territories and Negeri Sembilan respectively.

11. The probability that Khaizan is involved in an accident in each round of a motorcycle race is 0.4. Khaizan has to quit the race if he is involved in an accident. The probability of Khaizan winning each round of the motorcycle race is 0.96 provided that he is not involved in any accident. Khaizan needs to complete three laps of the race track.

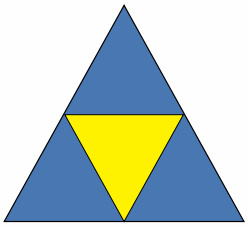
(a) Calculate the probability, correct to three decimal places, that

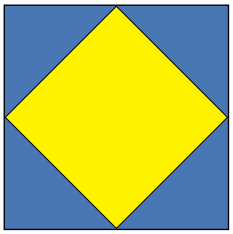
(i) Khaizan is the champion of the race.

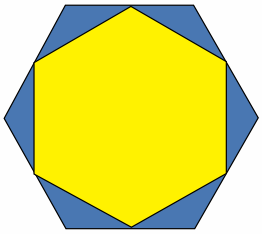
(ii) Khaizan is unable to finish the race.

(b) Based on the answer in (a), is it advisable for Khaizan to encourage his younger brother to participate in a motorcycle race? State a moral value that you have learned to support your answer.

PROJECT


10 cm


10 cm


10 cm

1. Construct three regular polygons as above. The polygons constructed have sides with equal lengths.

2. Carry out the activity stated below.

A dart is thrown towards each of the above regular polygons. The experiment is carried out twenty times and the results of the dart that lands on the yellow and blue regions are recorded as (Y, Y, Y), (Y, B, Y).

3. Based on the recorded results, what is your conclusion? Elaborate the reasons to support your conclusion.

4. Further exploration: If you are the owner of a game booth on your school's Entrepreneurship Day, which of the regular polygons will you choose? State the reasons to support your choice.

CONCEPT MAP

Probability of Combined Events

Dependent Events and Independent Events

A and B are Dependent Events
- Event A affects the occurrence of event B.

Example:
Choose two cards from a box that contains cards labelled with the letters "B, A, I, K" without replacement.

A and B are Independent Events
- Event A does not affect the occurrence of event B.

Example:
A fair dice is rolled twice and "6" is obtained for two times.

Multiplication Rule of Probability
$P(A \cap B) = P(A) \times P(B)$

Example:
A fair dice is rolled twice and "6" is obtained for two times.

$P(\text{Two times of "6"}) = \frac{1}{6} \times \frac{1}{6}$
$= \frac{1}{36}$

Addition Rule of Probability

A and B are Mutually Exclusive Events if $A \cap B = \phi$,
$P(A \cup B) = P(A) + P(B)$

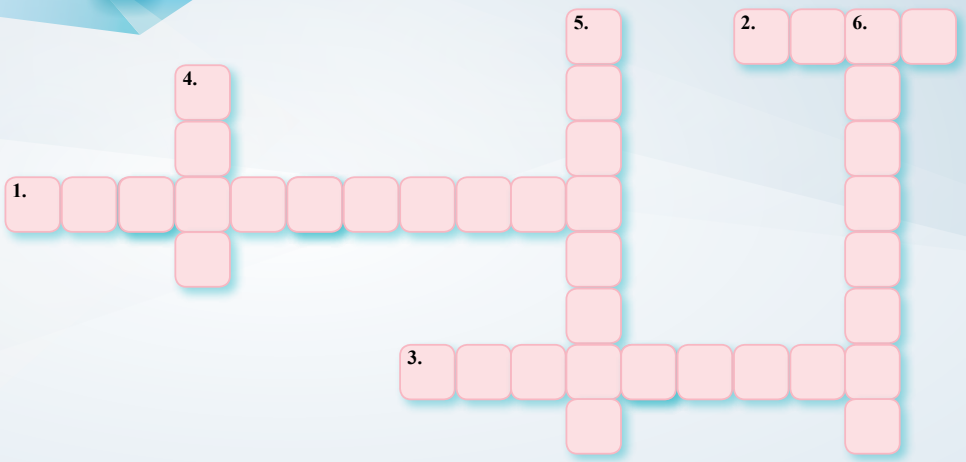
Example: $X = \{x : 1 \leqslant x \leqslant 10, x \in \mathbf{W}\}$
A number is chosen at random from set X.
Probability of choosing the number 2 or an odd number
$= P(\text{Number 2 or an odd number})$
$= \frac{1}{10} + \frac{5}{10}$
$= \frac{3}{5}$

A and B are Non-Mutually Exclusive Events if $A \cup B \neq \phi$,
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Example: $X = \{x : 1 \leqslant x \leqslant 10, x \in \mathbf{W}\}$
A number is chosen at random from set X.
Probability of choosing a prime number or an odd number
$= P(\text{A prime number or an odd number})$
$= \frac{4}{10} + \frac{5}{10} - \frac{3}{10}$
$= \frac{3}{5}$

Applications of Probability of Combined Events

Self Reflection



Across

1. A and B are two _____ events if the occurrence of event A does not affect the occurrence of event B and vice versa.
2. A _____ is rolled and a coin is tossed. The number of the outcomes is 12.
3. A and B are mutually _____ events meaning there is no intersection between events A and B.

Down

4. A number is chosen from $\{x : x \text{ is an integer and } 0 < x < 50\}$.
   K is the event of getting an _____ number and K is the event of getting an odd number.
   $P(K \cup L) = P(K) + P(L)$
5. $P(A \text{ and } B) = P(A)$ _____ $P(B)$
6. A _____ event is an outcome of the union or intersection of two or more events.

Mathematics Exploration

Bottle flipping is a game which involves throwing a plastic water bottle, either filled or partially filled with water so that the bottle rotates and then stands upright again.

Try to explore the factors that influence the likelihood of a successful bottle flipping.

CHAPTER

10 Consumer Mathematics: Financial Management

You will learn

- Financial Planning and Management

Financial planning and management need to be practised by each individual. Financial decisions need to be planned like a game of chess. Every financial decision must be considered carefully before taking the next steps. Prudent financial management can help us stay debt-free.

To what extent can a monthly budget help us achieve our financial goals?

Why Study This Chapter?

Effective financial planning is needed in our daily life. Financial management can be applied in accounting, stock exchange, business and others.

WORD BANK

• cash flow	• *aliran tunai*
• asset	• *aset*
• debt freedom	• *bebas hutang*
• long term	• *jangka panjang*
• short term	• *jangka pendek*
• financial goal	• *matlamat kewangan*
• income	• *pendapatan*
• investment	• *pelaburan*
• expense	• *perbelanjaan*

Walking Through Time

Tan Sri Dr Zeti Akhtar Aziz

According to the former governor of Bank Negara Malaysia, Tan Sri Dr Zeti Akhtar Aziz, financial education must start at a young age. Financial literacy must be cultivated at a young age. Bank Negara Malaysia has introduced *Buku Wang* to school children since 1999.

http://bt.sasbadi.com/m4271

10.1 Financial Planning and Management

What does financial management process mean?

Learning Standard

Describe effective financial management process.

Financial management is a process that involves managing money from sources of income into savings, expenses, protection and investment. There are five steps in financial management process.

Setting goals

Setting goals is the first step in the financial management process. The financial goals set must be prioritised and specific. For example, an individual would like to have RM15 000 of savings in the bank three years before marriage.

TIPS

Being disciplined and consistent in following the financial plan will help us achieve our financial goals.

(a) Short-term financial goals

Short-term financial goals can be achieved in less than a year. Short-term financial goals do not involve a large amount of money, for example, purchasing a laptop, furniture, a cell phone and others.

(b) Long-term financial goals

Long-term financial goals usually take more than five years to achieve. Long-term financial goals involve a large amount of money, for example, an individual needs to have savings for retirement, children's education, medical expenses and others.

INFO ZONE

Total Daily Expenses	
Breakfast	RM5
Lunch	
Photostat	
Bus fare	
Others	

A habit of tracking our daily expenses can help us control our finances (money given by parents).

The duration needed to achieve a financial goal depends on the percentage of the monthly savings from the person's income. However, this depends on the financial planning of the person.

The most important step is to set financial goals. We should prioritise to meet our needs rather than wants. For example, the basic needs such as food, accommodation and clothing should be given priority.

Having the desire to live a luxurious life is a want. Spending money on our wants will lead us to spend more than we earn. Effective financial planning will help us to avoid financial problems as well as to have an adequate savings in the event of an emergency.

SMART financial goals

Setting clear financial goals is the most important aspect in financial planning. Financial goals set based on the SMART concept will help us control our spending in order to achieve the desired financial goals.

The five key components in this strategy are:

Example 1

Encik Yusuf will be celebrating Family Day in Port Dickson in 6 months. This celebration will cost him RM1 500. Encik Yusuf needs to save RM250 a month from his income of RM2 500 in order to achieve his financial goal. Is Encik Yusuf's financial goal a SMART approach?

Solution:

Specific

Encik Yusuf's goal is to celebrate Family Day in Port Dickson. In this case, Encik Yusuf's goal is specific because he aims to go on vacation at a specific location.

Measurable

Encik Yusuf's specific goal can be calculated as he knows the amount of money needed to celebrate Family Day. In this case, Encik Yusuf's goal can be measured because he knows how much money is needed to celebrate Family Day.

Attainable

Encik Yusuf's financial goal is attainable because it can be achieved by saving RM250 a month from his monthly income of RM2 500.

Realistic

Encik Yusuf's financial goal of celebrating Family Day in Port Dickson in 6 months is realistic. He will save RM250 a month to achieve the goal.

Time-bound

Encik Yusuf's financial goal is time-bound because it has a time limit of six months.

Therefore, Encik Yusuf applied the SMART concept in his goal setting.

Evaluating financial status

Assets and liabilities are the benchmarks for evaluating our financial status. Examples of assets are cash, savings, real estate investments, fixed deposits, unit trusts or company shares.

TIPS

As a general guide, save money for basic needs at least for six months.

Liabilities are bank loans, credit card debts and other financial obligations. Besides that, we should know the outstanding payments such as unsettled rent, utility bills, credit card bills and others.

Evaluating our financial status helps us measure our performance in the effort of achieving our short-term and long-term financial goals.

Creating financial plan

Life management based on financial planning helps us monitor our cash flow. There are two important components in constructing a financial plan, namely the sources of income and expenses.

Income consists of **active income** and **passive income**. Active income is the main income such as salaries, allowances, commissions and other financial resources. Passive income consists of rent received, interest received, dividends and other investments. Both of these incomes are cash inflows.

Expenses consist of **fixed expenses** and **variable expenses**. Fixed expenses are expenses that must be paid or spent such as rent, insurance payments, housing loan instalments, car instalments and credit card payments.

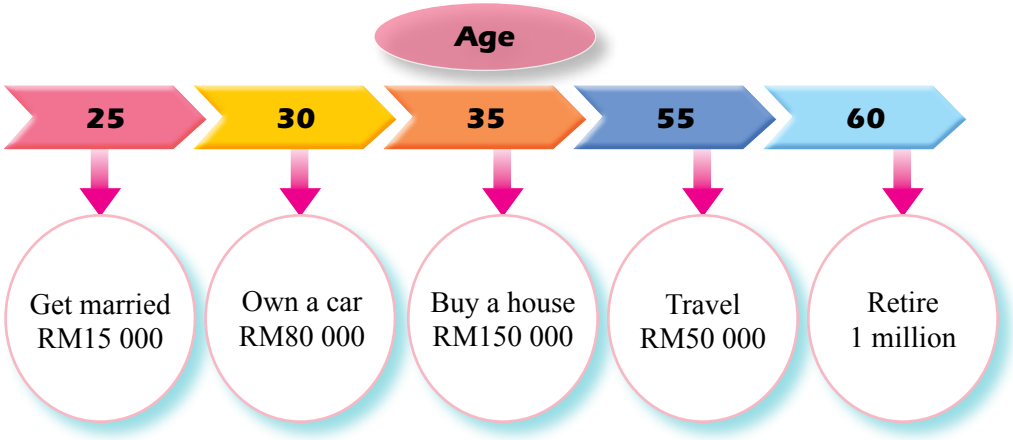
Variable expenses will change every month according to our spending behaviour. For example, petrol expenses, groceries, electricity bill payments and water bill payments. Both of these expenses are cash outflows.

A positive cash flow will exist if the total income exceeds the total expenses.

A negative cash flow will exist if the total income is less than the total expenses.

These are the steps to consider before creating a financial plan.

Example of an individual's financial goal:



Effective financial management helps us develop strategies to manage our spending behaviour to accumulate monthly savings.

Goals	Starting Year	Target Year	Duration (years)	Total (RM)	Monthly Savings Commitment (RM)
Get married	2019	2022	3	15 000	
Own a car	2023	2027	4	20 000 (down payment)	

Example 2

Mr Derrick receives an active income of RM3 000 and a passive income of RM1 000 in a month. Mr Derrick also has fixed expenses of RM1 350 and variable expenses of RM650 in a month.

(a) Calculate Mr Derrick's monthly cash flow. Explain your answer.

(b) Explain Mr Derrick's cash flow if Mr Derrick does not have any passive income and the total expenses increase by 60%.

Solution:

(a) Cash flow = Total income – Total expenses
= RM3 000 + RM1 000 – RM1 350 – RM650
= RM2 000 (Positive cash flow)

A positive cash flow of RM2 000 is good because Mr Derrick has a surplus of income after deducting the expenses. This will allow him to invest or deal with emergency situations.

(b) Total expenses = 160% × RM2 000
= RM3 200

Cash flow = Total income – Total expenses
= RM3 000 – RM3 200
= –RM200 (Negative cash flow)

A negative cash flow will burden Mr Derrick and may cause him to use the credit card facilities to solve financial problems.

Example 3

Cik Janani works as a private school teacher. She earns an income of RM3 500. She also gives tuition classes and earns an income of RM1 000. Cik Janani also rents out her house for RM850 a month. She has fixed expenses of RM1 200 and variable expenses of RM600 in a month.

Calculate Cik Janani's monthly cash flow. Explain your answer.

Solution:

Cash flow = RM3 500 + RM1 000 + RM850 – RM1 200 – RM600
= RM3 550 (Positive cash flow)

A positive cash flow of RM3 550 is good because Cik Janani can save money in the bank and benefit from the interest. This interest is a passive income for Cik Janani.

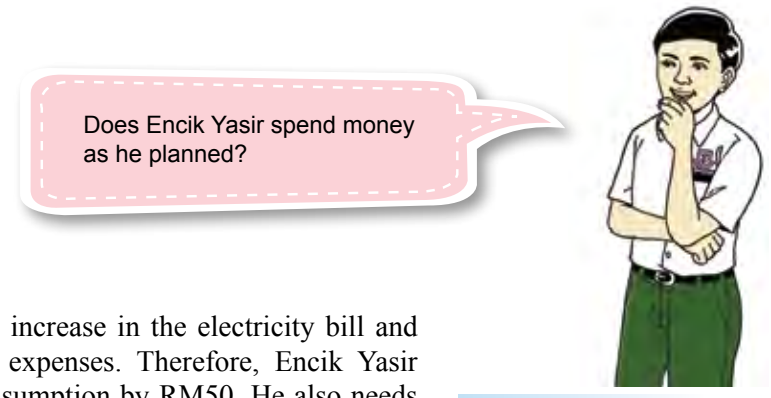
Carrying out financial plan

Carrying out a financial plan turns the financial planning into action that can be implemented.

When carrying out a financial plan, we must follow the plan at an early stage. We must be ready to change and compare the planned monthly expenses and actual expenses. This gives us an opportunity to identify any wastage and hence reduce the actual expenses in order to meet the monthly expenses as planned.

For example, Encik Yasir's planned monthly expenses and actual expenses for the first month are shown as follows.

Expenses	Planned monthly expenses (RM)	Actual expenses (RM)
Electricity bill	150	200
Food	600	850



Encik Yasir's expenses show an increase in the electricity bill and food compared to the planned expenses. Therefore, Encik Yasir should reduce the electricity consumption by RM50. He also needs to reduce the total expenses on food by RM250.

When carrying out a financial plan, we have to prioritise payments for fixed expenses, for example, housing loan instalments, credit card payments and car instalments. Late instalment payment will lead to additional interest charges and late payment penalty.

In addition, when carrying out a financial plan, we must ensure that the monthly cash flow is always positive, where the income exceeds the expenses. There is a negative monthly cash flow (deficit) when expenses are more than income. If there is a negative monthly cash flow, prompt action should be taken to change the spending behaviour. If this problem is not resolved, then we will fail in achieving our financial goals within the specific time frame.

INFO ZONE

Malaysians go bankrupt due to the failure in paying vehicle instalments, housing loans and credit card debts.

Example 4

Puan Aminah plans to buy a car worth RM50 000 within a year. She plans to pay a down payment of RM7 500 with her savings. Puan Aminah does not have any savings. She wants to get a car loan from a bank.

Bank *X* offers several options. After evaluating her financial status, Puan Aminah chooses to pay the monthly instalment for seven years as that is what she can only afford.

Puan Aminah's Family Financial Plan

Income and Expenditure	RM	
Net income		
Husband	3 500	
Wife	3 000	
(*Net salary*)		
Passive income	0	
Total monthly income	6 500	
Minus fixed monthly savings (*10% of monthly income*)	650	
Minus savings for emergency fund	100	
Income balance		5 750
Minus monthly fixed expenses		
Housing loan	1 500	
Husband's car instalment	600	
Insurance premiums	400	
Total monthly fixed expenses		2 500
Minus monthly variable expenses		
Nursery	400	
Children's needs	450	
Home utilities	400	
Petrol expenses	380	
Toll payments	100	
Groceries	1 000	
Allowances for parents	400	
Total monthly variable expenses		3 130
Surplus of income		120

Income balance – Total monthly fixed expenses – Total monthly variable expenses

10% of income should keep as savings before spending.

Puan Aminah's Family Financial Plan and Actual Cash Flow

Income and Expenditure	Financial Plan (RM)	Actual Cash Flow (RM)
Net income		
Husband	3 500	3 500
Wife	3 000	3 000
(*Net salary*)		
Passive income	0	0
Total monthly income	6 500	6 500
Minus fixed monthly savings (*10% of monthly income*)	650	650
Minus savings for emergency fund	100	100
Income balance	5 750	5 750
Minus monthly fixed expenses		
Housing loan	1 500	1 500
Husband's car instalment	600	600
Insurance premiums	400	400
Total monthly fixed expenses	2 500	2 500
Minus monthly variable expenses		
Nursery	400	400
Children's needs	450	450
Home utilities	400	400
Petrol expenses	380	380
Toll payments	100	100
Groceries	1 000	900
Allowances for parents	400	400
Total monthly variable expenses	3 130	3 030
Surplus of income	120	220

Based on Puan Aminah's financial plan, the expenses for groceries can be reduced by RM100. Being careful with money has reduced her spending on groceries.

Positive cash flow is an additional savings.

(a) Based on Puan Aminah's family financial planning, can she achieve her financial goal?

(b) Is Puan Aminah's financial management wise? Justify your answer.

(c) Does Puan Aminah use the SMART approach to achieve her financial goals? Justify your answer.

INTERACTIVE ZONE

What should Puan Aminah do if the cash flow is negative? Discuss.

Solution:

(a) Puan Aminah can achieve her short-term financial goal of buying a car with a down payment of RM7 500.

Total of monthly savings = RM650 × 12 (months)
= RM7 800

She still has a surplus of RM300 from her needs of RM7 500. She also has an excess of RM220 per month in case of emergency or unexpected expenses.

(b) It is a wise financial management because Puan Aminah is spending on basic needs. She also has monthly savings, prioritises protection by buying insurance and she does not have any debts like credit card debt.

(c) Puan Aminah uses the SMART approach to achieve her financial goals, which are:

- Specific – Buy a car worth RM50 000
- Measurable – Pay a down payment of RM7 500
- Attainable – Save RM7 500 for the down payment (Within a year)
- Realistic – Monthly savings of RM650 is only 10% of the total income of RM6 500
- Time-bound – Within 1 year

Reviewing and revising the progress

Reviewing and revising the progress of a financial plan from time to time is important to make sure the cash flow is always positive. This indirectly helps us to achieve our financial goals as planned. We should also be prepared to change our financial goals if they are not realistic for our monthly income.

Example 5

Puan Aminah's husband is promoted at the end of June in the same year with a 10% salary increase. The amount of their other expenses is fixed. What is the effect of Puan Aminah's husband's salary increment on her financial goals? You can refer to Example 4.

Solution:

$$\text{Puan Aminah's husband's salary increment} = \frac{10}{100} \times \text{RM3 500}$$
$$= \text{RM350}$$

Puan Aminah's husband's new salary = RM3 500 + RM350
= RM3 850

When the salary of Puan Aminah's husband is increased by 10%, the savings in the bank can be increased. This can help them in purchasing the car in less than one year as planned, since other expenses are fixed.

TIPS

Financial plans need to be reviewed from time to time. If you do not manage your financial strategy according to the current situation, then it is difficult to achieve the financial goals as planned.

Self Practice 10.1a

1. What is the meaning of financial management?

2. Explain the financial management process.

3. What is the importance of setting financial goals?

4. Differentiate between short-term and long-term financial goals.

5. Among the following individuals, who sets the SMART goals? Justify your answer.

Puan Salmah intends to buy a laptop which costs RM3 000 as a birthday gift for her son. Therefore, she plans to save RM300 a month for ten months.

Miss Tamarai is a teacher with an income of RM3 000 and plans to get married in two years' time. The wedding expenses are estimated to be RM30 000. Therefore, she wants to save RM1 000 a month for a period of two years. Besides that, she also plans to get a loan for the wedding expenses.

Mr Brian Lee wants to buy a house worth RM200 000 if he has sufficient amount of money.

How do you evaluate the feasibility of the short-term and long-term financial plans?

Learning Standard

Construct and present personal financial plans to achieve short-term and long-term financial goals, and hence evaluate the feasibility of the financial plans.

A financial plan is developed to achieve our short-term and long-term financial goals. We have to identify our sources of income and expenses.

An effective financial plan should set aside 10% savings of the total income prior to engaging any fixed expenses and variable expenses. In addition, the financial plan should prioritise the fixed expenses payments such as monthly instalments of cars, houses and credit card bills.

If there is a negative cash flow, we should adjust the financial plan by reducing the variable expenses. At the same time, we can add a source of income with our skills to keep the fixed expenses unaffected.

The information below shows Encik Yusuf's income and expenses for December 2019. Encik Yusuf works as an insurance agent while his wife is a housewife. They have three children who are still studying.

Encik Yusuf's Financial Plan

Income and Expenditure	Financial Plan (RM)	
Net income		
Husband	5 000	
(*Net salary*)		
Passive income	0	
Total monthly income	5 000	
Minus fixed monthly savings	0	
(*10% of monthly income*)		
Minus savings for emergency fund	200	
Income balance		4 800
Minus monthly fixed expenses		
Housing loan instalment	1 000	
Insurance premiums	500	
Total monthly fixed expenses		1 500
Minus monthly variable expenses		
Food and drinks	900	
Children's education	600	
Petrol expenses	420	
Telephone bill	480	
Utility bills	500	
Travel	400	
Total monthly variable expenses		3 300
Surplus of income / Deficit		0

Encik Yusuf wants to buy a Fast brand computer which costs RM6 000 to improve his insurance sales within a year.

Solution:

Cash flow = Income balance – Total expenses
= RM4 800 – RM1 500 – RM3 300
= RM0

Based on the financial plan, Encik Yusuf does not have any savings. So, it is difficult for him to achieve his short-term financial goal. Help Encik Yusuf to solve his financial problem without using the emergency fund.

(a) Does Encik Yusuf manage his financial effectively?

(b) How much monthly savings does Encik Yusuf need to save in order to achieve his goals?

(c) How can an additional income be generated to increase the total income?

(d) Create a new financial plan based on the SMART concept.

Solution:

(a) He does not manage his financial effectively because there are expenses that can be reduced such as the spending on telephone, food and drinks. Furthermore, he does not have any investment plan for his future.

(b) Monthly savings needed $= \frac{\text{RM6 000}}{12}$

$= \text{RM500}$

(c) Encik Yusuf can increase his income by selling more insurance products and recruiting more new agents.

(d) The SMART concept in the new financial plan.

Specific – Buy a computer that costs RM6 000.

Measurable – Save RM500 every month to achieve the goals.

Attainable – Can save RM500 from the income of RM5 000.

Realistic – RM500 is only 10% of the total income of RM5 000.

Time-bound – One year is enough to save RM6 000 with monthly savings of RM500.

Encik Yusuf's new financial plan after considering some measures in terms of variable expenses.

Encik Yusuf's New Financial Plan

Income and Expenditure	Financial Plan (RM)	
Net income		
Husband	5 000	
(*Net salary*)		
Passive income	0	
Total monthly income	5 000	
Minus fixed monthly savings	500	
(*10% of monthly income*)		
Minus savings for emergency fund	200	
Income balance		4 300
Minus monthly fixed expenses		
Housing loan instalment	1 000	
Insurance premiums	500	
Total monthly fixed expenses		1 500
Minus monthly variable expenses		
Food and drinks	900	
Children's education	600	
Petrol expenses	300	
Telephone bill	300	
Utility bills	350	
Travel	200	
Investments		
Unit trusts investments	150	
Total monthly variable expenses		2 800
Surplus of income	0	0

These expenses can be reduced if he spends money carefully.

Encik Yusuf's savings of RM500 a month can help him save RM6 000 by end of the year 2020 to achieve his short-term goal.

Evaluate the feasibility of Encik Yusuf's financial plan

Each financial plan should be evaluated from time to time based on several factors. One of the factors that Encik Yusuf should focus on is the current inflation rates that can lead to an increase in the cost of living. This can indirectly increase the total expenses. If this happens, Encik Yusuf should take action to increase his income. However, Encik Yusuf's financial plan can be achieved as he has invested in unit trusts. The additional expenses can be covered by the dividends received.

Long-term financial goals

Long-term financial goals are as important as short-term financial goals. The purpose of the long-term financial plan is to make sure the goals can be achieved as planned in the initial stage. Long-term financial plans usually exceed five years such as children's education, retirement and buying a house. To develop a long-term financial plan, the key aspects to be considered are as follows:

- Inflation rate
- Interest rate
- Personal health

In developing a long-term financial plan, it would be better to start saving early because this practice can help us in achieving our financial goals faster. For example, we should prepare for retirement, buy a property and save for children's education.

Long-term financial plans developed vary for each individual. The income of an individual or joint income of husband and wife allows an individual to have sufficient monthly savings in a shorter period of time.

INFO ZONE

Inflation – Situation of a continuing increase in the general price level.

INTERACTIVE ZONE

Discuss other aspects that need to be considered in developing a long-term financial plan.

Example 6

Encik Syed and his wife intend to buy a house in six years after getting married. Their total income is RM8 000 and their total fixed and variable expenses is RM6 500. They plan to buy a double-storey house priced at RM720 000 with a down payment of RM72 000.

(a) How much is the monthly savings that Encik Syed and his wife must save in order to achieve their financial goal?

(b) Is it wise for Encik Syed to buy a house priced at RM720 000 with his current financial planning? Justify your answer.

Solution:

(a) Annual savings needed by Encik Syed $= \frac{\text{RM72 000}}{\text{6 years}}$

$= \text{RM12 000}$

Monthly savings needed by Encik Syed $= \frac{\text{RM12 000}}{\text{12 months}}$

$= \text{RM1 000}$

For Encik Syed's family, saving RM1 000 every month to achieve their long-term goal is not difficult with the total income of RM8 000.

(b) No. Although Encik Syed is able to pay the down payment of RM72 000, his monthly housing loan instalments can burden him due to high expenses.

Case Study

Assume you are a financial consultant. Mr Wong as the head of his family has come to see you with the information of his monthly income and expenses as shown below. He seeks your consultancy to create a financial plan to buy a house.

Mr Wong's Financial Plan

Income and Expenditure	Financial Plan (RM)	
Net income		
Net salary	6 500	
Passive income (monthly rental)	500	
Total monthly income	7 000	
Minus fixed monthly savings (*10% of monthly income*)	650	
Minus savings for emergency fund	100	
Income balance		6 250
Minus monthly fixed expenses		
Housing loan instalment	1 500	
Car loan instalment	800	
Insurance premiums	600	
Total monthly fixed expenses		2 900
Minus monthly variable expenses		
Groceries	1 300	
Travel	500	
Home utilities	350	
Telephone bill	300	
Petrol expenses	800	
Gym	100	
Total monthly variable expenses		3 350
Surplus of income	0	0

Mr Wong works as a marketing officer in a company while his wife is a housewife. They have two children who are one and two years old. Mr Wong would like to save an amount of RM150 000 for his children's education in 15 years from now. Help Mr Wong to create a financial plan to achieve his financial goals.

Solution:

$$\text{Annual savings} = \frac{\text{RM150 000}}{\text{15 years}}$$

$$= \text{RM10 000}$$

$$\text{Monthly savings} = \frac{\text{RM10 000}}{\text{12 months}}$$

$$= \text{RM833.33}$$

$$\text{Additional savings needed} = \text{RM833.33} - \text{RM650}$$
$$= \text{RM183.33}$$

Suggestion:

Mr Wong needs to increase his monthly savings by RM183.33.

1. He can reduce the amount allocated for travelling by 25% in order to achieve his financial goal of saving money for his children's education.

$$\text{Reduced travel expenses} = \frac{25}{100} \times \text{RM500}$$

$$= \text{RM125}$$

$$\text{New travel expenses} = \text{RM500} - \text{RM125}$$
$$= \text{RM375}$$

2. He can also cut down expenses on petrol by RM100 by car-pooling with his colleagues. By practising the above two suggestions,

$$\text{additional savings} = \text{RM125} + \text{RM100}$$
$$= \text{RM225}$$

3. Mr Wong can also consider reducing the variable expenses to achieve his financial goals. Mr Wong can do some part-time jobs to generate additional income. Besides that, he can invest the amount of money saved each year to earn passive income, such as dividends, bonus shares and interest as an addition to the total income.

The new financial plan after taking into account the suggestions:

Mr Wong's New Financial Plan

Income and Expenditure	Financial Plan (RM)	
Net income		
Net salary	6 500	
Passive income (monthly rental)	500	
Total monthly income	7 000	
Minus fixed monthly savings (*10% of monthly income*)	650	
Minus savings for emergency fund	100	
Income balance		6 250
Minus monthly fixed expenses		
Housing loan instalment	1 500	
Car loan instalment	800	
Insurance premiums	600	
Total monthly fixed expenses		2 900
Minus monthly variable expenses		
Groceries	1 300	
Travel	375	
Home utilities	350	
Telephone bill	300	
Petrol expenses	700	
Gym	100	
Total monthly variable expenses		3 125
Surplus of income		225

Additional savings

Total savings for 1 month = Monthly savings + Additional savings
= RM650 + RM225
= RM875

Total savings for 15 years = RM875 × 12 × 15 years
= RM157 500

In fact, the amount of money saved is more than RM157 500 as the savings in banks offer interest annually.

The feasibility of Mr Wong's financial plan depends on following factors:

1. The inflation rates should not exceed pay rise.
2. The rental received is fixed.
3. Mr Wong is healthy and can continue to work.
4. The increase of expenses can be offset by pay rise, rentals and part-time jobs.
5. Nothing unexpected happens to Mr Wong's family that involves high costs.

Self Practice 10.1b

1. What are the factors that can influence a long-term financial plan?
2. The information below is Mrs Thong's financial plan.

Net Income	Financial Plan (RM)
Mrs Thong's salary	6 000
Expenses Budget	
House/apartment loan, including maintenance	500
Car instalment	700
Travel	500
Rental	1 500
Transport fares	900
Utility bills	250
Toll payments	100
Groceries	400
Allowances for parents	600
Savings	100
Insurance	200
Total expenses	5 750
Mrs Thong's income balance	250

(a) Does Mrs Thong spend her money wisely?

(b) Will Mrs Thong be able to achieve her long-term financial goal if she wishes to buy a property worth RM500 000 within 6 years with her current spending behaviour?

Comprehensive Practice

1. Why are more young people in Malaysia being declared bankruptcy nowadays?
2. What is the effect if the cash inflows are less than the cash outflows?
3. What is the purpose of creating a financial plan?
4. Give two advantages of reviewing and revising the progress of a financial plan.
5. Describe two factors that make a financial plan difficult to implement.
6. How should we review and revise the progress of a financial plan?
7. "The earlier we start saving, the better it is" to achieve our financial goals. Explain.

8. Encik Nabil works as an engineer in a factory with a monthly net salary of RM3 800. He is also a part-time sales agent of product X. The monthly commission earned is estimated to be RM450. The rental collected from the second house is RM600 per month. His estimated monthly expenses are shown below.

Monthly Expenses	RM
Housing loan instalment (1)	800
Housing loan instalment (2)	500
Food expenses	900
Utility payments	150
Toll and petrol expenses	200
Internet service subscription	100
Eat at a luxury restaurant	400
Insurance expenses	350

Encik Nabil sets aside 10% of his salary as fixed monthly savings in order to achieve his financial goals.

(a) You are required to create a monthly personal financial plan for Encik Nabil.

(b) Give comments on the surplus or deficit that will be experienced by Encik Nabil based on this financial plan.

9. What is the effect if there is

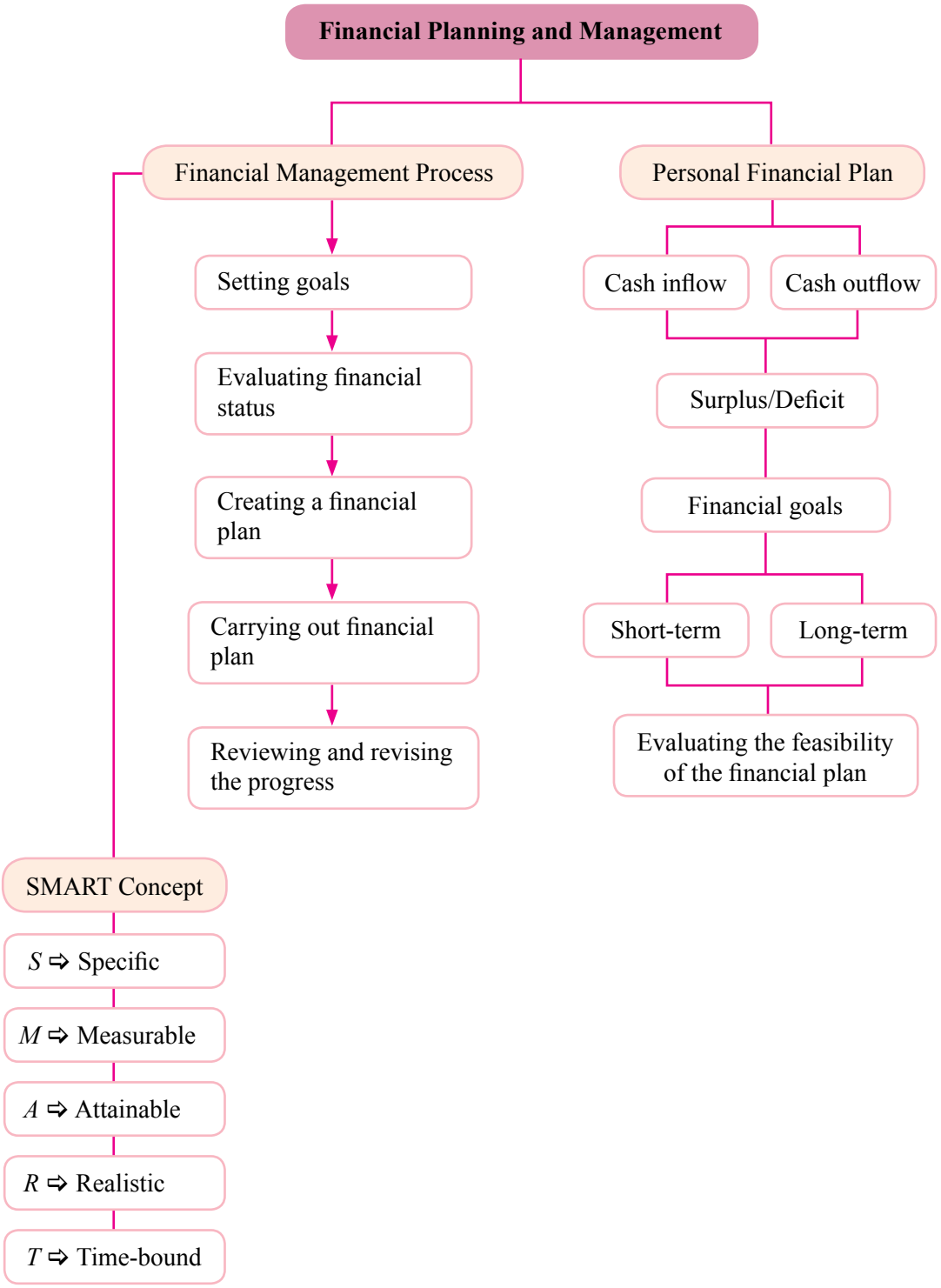
(a) a positive cash flow?

(b) a negative cash flow?

PROJECT

The above illustration shows the financial goal of each of your family members. Help them by developing short-term and long-term financial plans based on the concept of SMART approach. You should get the information on the income and expenses of your family. In future, you may act as a financial consultant for your family.

CONCEPT MAP


Financial Planning and Management
Financial Management Process
Setting goals
Evaluating financial status
Creating a financial plan
Carrying out financial plan
Reviewing and revising the progress
Personal Financial Plan
Cash inflow
Cash outflow
Surplus/Deficit
Financial goals
Short-term
Long-term
Evaluating the feasibility of the financial plan
SMART Concept
S ⇨ Specific
M ⇨ Measurable
A ⇨ Attainable
R ⇨ Realistic
T ⇨ Time-bound

Self Reflection

Complete the following with appropriate answers based on the statements provided.

The financial management process consists of:

(a) _____

(b) _____

(c) _____

(d) _____

(e) _____

Financial goals based on SMART concept

(a) _____

(b) _____

(c) _____

(d) _____

(e) _____

Features of a short-term personal financial plan

(a) _____

(b) _____

(c) _____

Features of a long-term personal financial plan

(a) _____

(b) _____

(c) _____

(d) _____

Mathematics Exploration

You have been invited to give a talk on 'SMART Financial Management' to teenagers in an educational institution.

Instructions:

1. Divide the class into groups.
2. The speech should include the following content:
   (a) The importance of financial management
   (b) Ways to create a financial plan
   (c) Short-term goals and long-term goals setting
   (d) Examples of personal budgets for adults
3. Your group work can be presented in multimedia forms in the computer room. The speech text can also be displayed at the Mathematics Corner of your classroom.

Answers

CHAPTER 1 Quadratic Functions and Equations in One Variable

Self Practice 1.1a

1. (a) Yes
(b) No because its power is not a whole number
(c) No because there are two variables, x and y
(d) Yes
(e) No because the highest power is three
(f) No because its power is not a whole number
(g) No because its power is not a whole number
(h) Yes
(i) Yes

2. (a) $a = 2$, $b = -5$, $c = 1$
(b) $a = 1$, $b = -2$, $c = 0$
(c) $a = 2$, $b = 0$, $c = 1$
(d) $a = -\frac{1}{2}$, $b = 4$, $c = 0$
(e) $a = -2$, $b = -1$, $c = 1$
(f) $a = 4$, $b = 0$, $c = 0$
(g) $a = 1$, $b = \frac{3}{2}$, $c = -4$
(h) $a = \frac{1}{3}$, $b = 0$, $c = -2$
(i) $a = 2$, $b = -6$, $c = 0$

Self Practice 1.1b

1. (a) ∪ (b) ∩

2. (a) $a > 0$, minimum point
(b) $a < 0$, maximum point

3. (a) Minimum point $(4, -15)$, $x = 4$
(b) Maximum point $(3, 13.5)$, $x = 3$
(c) Maximum point $(-2, 4)$, $x = -2$
(d) Minimum point $(2, -2)$, $x = 2$

Self Practice 1.1c

1. (a) 5 (b) –3 (c) 4

2. $0 < p < 4$
Function $f(x)$ has a wider curve, thus $p < 4$.
For graph in the shape of ∩, $a < 0$, thus $p > 0$

3. (a) $k = -1$
(b) $h = 5$
(c) $f(x) = x^2 - 6x - 5$

Self Practice 1.1d

1. (a) $A = x^2 + 25x + 100$ (b) $x^2 + 25x - 150 = 0$

2. $p^2 + 4p - 48 = 0$

Self Practice 1.1e

1. (a) $x = -0.35$, $x = 2$
(b) $x = -4$, $x = 5$

2. (a) $x = 3$ is a root, $x = 2$ is a root
(b) $x = 1$ is a root, $x = \frac{1}{2}$ is not a root
(c) $x = -\frac{1}{3}$ is a root, $x = -2$ is not a root
(d) $x = 2$ is not a root, $x = \frac{2}{3}$ is a root

3. (a) $x = -4$ and $x = 1$ are roots, $x = 2$ is not a root
(b) $x = 3$ and $x = 5$ are roots, $x = -3$ is not a root
(c) $x = -2$ and $x = 4$ are roots, $x = 2$ is not a root

4. (a) $x = 1$ is not a root (b) $x = -3$ is a root
(c) $x = 15$ is not a root (d) $x = 5$ is a root

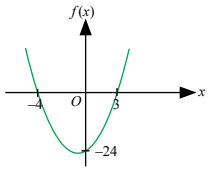
Self Practice 1.1f

1. (a) $x = 5$, $x = -2$ (b) $x = 2$, $x = 8$
(c) $x = \frac{2}{3}$, $x = 1$ (d) $x = -6$, $x = 2$
(e) $x = -3$, $x = \frac{3}{2}$ (f) $x = -\frac{5}{4}$, $x = 2$
(g) $x = -\frac{7}{3}$, $x = 2$ (h) $x = 0$, $x = 5$
(i) $x = -2$, $x = 2$

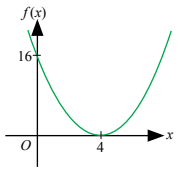
2. (a) $m^2 + 2m - 3 = 0$; $m = -3$, $m = 1$
(b) $2p^2 - 11p + 5 = 0$; $p = \frac{1}{2}$, $p = 5$
(c) $y^2 + 2y - 24 = 0$; $y = 4$, $y = -6$
(d) $a^2 - 6a + 5 = 0$; $a = 5$, $a = 1$
(e) $k^2 + 2k - 8 = 0$; $k = 2$, $k = -4$
(f) $2h^2 - 7h + 6 = 0$; $h = 2$, $h = \frac{3}{2}$
(g) $h^2 - 3h - 10 = 0$; $h = -2$, $h = 5$
(h) $4x^2 - 7x + 3 = 0$; $x = \frac{3}{4}$, $x = 1$
(i) $r^2 - 6r + 9 = 0$; $r = 3$

Self Practice 1.1g

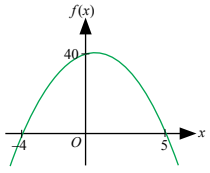
1. (a)



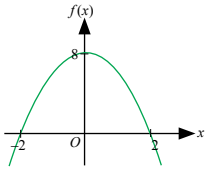
(b)



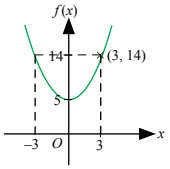
(c)



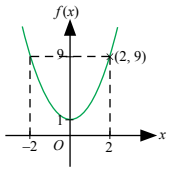
(d)



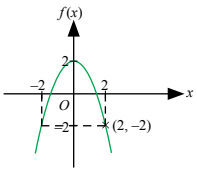
2. (a)



(b)



(c)



Self Practice 1.1h

1. (a) $A = 5x^2 + 20x$ (b) RM8 000
2. Yes

Comprehensive Practice

1. (a) Yes (b) Yes (c) No
(d) No (e) Yes (f) Yes
2. (a) $x = 2$ (b) $x = 3$
3. (a) $x = -\frac{1}{2}, x = \frac{1}{2}$
(b) $x = -9, x = 9$
(c) $y = 0, y = 4$
(d) $x = -1, x = -2$
(e) $x = -2, x = \frac{5}{2}$
(f) $x = 6, x = -2$
(g) $m = 1, m = -4$
(h) $p = 4, p = \frac{5}{2}$
(i) $k = 7, k = -2$
(j) $h = 2, h = -2$
(k) $x = 5, x = \frac{3}{2}$
4. $p = 7$
5. $m = 6, m = 4$
6. $(3, -4)$
7. $(4, 23)$
8. (a) $A(0, -5)$ (b) $x = 3$
(c) $B(6, -5)$ (d) $(3, 4)$
9. (a) $c = 6$ (b) $m = -2$
(c) $a = 2$ (d) $n = -2$
10. (a) (i) $h = 1$ (ii) $k = 5$ (iii) $a = 3$
(b) $x = 3$
(c) $P(3, -12)$
11. (a) $A = x^2 - 3x - 4$
(b) length = 8 cm, width = 3 cm
12. 20 cm
13. (a) $A = x^2 + 27x + 180$ (b) $x = 8$
(c) enough
14. (a) $A = x^2 - 5x - 4$ (b) $x = 7$
(c) 38 m

CHAPTER 2 Number Bases

Self Practice 2.1a

1. Accept pupil's correct answers.
2. 461, 371, 829
3.

(a)	234
(b)	234, 336
(c)	234, 336, 673
(d)	234, 336, 673, 281

4. (a) 2^4 (b) 5^2 (c) 7^1
(d) 6^1 (e) 3^2 (f) 9^1
(g) 4^2 (h) 8^3 (i) 6^2
(j) 5^0
5. (a) 4 (b) 10 (c) 3
(d) 72 (e) 54 (f) 2 058
(g) 8 (h) 448 (i) 12
(j) 4
6. (a) 15 (b) 277 (c) 53
(d) 278 (e) 193 (f) 15
(g) 38 (h) 655 (i) 191
(j) 43
7. (a) $p = 3, q = 2^2$
(b) $p = 2, q = 7$
(c) $p = 4, q = 3$
8. 651
9. (a) $110_2, 111_2, 1101_2, 1110_2$
(b) $112_4, 132_4, 231_4, 1123_4$
(c) $124_5, 231_5, 241_5, 324_5$
10. (a) $1213_4, 89_9, 111101_2$
(b) $313_5, 73_8, 123_4$
(c) $253_6, 161_7, 222_3$
11. 315

Self Practice 2.1b

1. (a) 111101110_2 (b) 13232_4 (c) 3434_5
(d) 756_8 (e) 608_9
2. (a) 1022_3 (b) 24_5 (c) 100001001_2
(d) 251_6 (e) 251_8 (f) 10012_4
3. 1010_3
4. (a) 75_8 (b) 16_8 (c) 367_8
(d) 52_8 (e) 70_8 (f) 725_8
5. (a) 100011_2 (b) 1001010_2
(c) 101111_2 (d) 1010001111_2
(e) 110011101_2 (f) 10100011_2

Self Practice 2.1c

1. (a) 101_2 (b) 111001_2 (c) 1101_3
(d) 1220_3 (e) 23_4 (f) 3110_4
(g) 1103_5 (h) 4002_5 (i) 513_6
(j) 213_6 (k) 452_7 (l) 1113_7
(m) 2020_8 (n) 735_8 (o) 211_9
(p) 6553_9
2. (a) 1101_2 (b) 110_2 (c) 1222_3
(d) 121_3 (e) 10_4 (f) 302_4
(g) 323_5 (h) 1141_5 (i) 3413_6
(j) 1103_6 (k) 5453_7 (l) 6313_7
(m) 746_8 (n) 4201_8 (o) 645_9
(p) 1443_9

Self Practice 2.1d

1. $x = 55_7$
2. (a) 168 (b) 134_5
3. Puan Aminah
4. 1600 m^2

Comprehensive Practice

1. (a) 240_5, 241_5, 242_5
(b) 110_2, 111_2, 1000_2
(c) 31_7, 32_7, 33_7
2. 32
3. (a) 716_8
(b) 11110111_2
4. (a) 11110001_2 (b) 1431_5
(c) 463_7 (d) 361_8
5. (a) 10101_2 (b) 442_7 (c) 56_9
6. (a) True (b) True (c) False
7. 269
8. 39
9. $y = 105$
10. (a) 65_8, 110110_2
(b) 176_8, 1003_5
11. 132_5
12. 55_8
13. 42_7

CHAPTER 3 Logical Reasoning

Self Practice 3.1a

1. (a) Not a statement. Because the truth value cannot be determined.
(b) A statement. Because it is true.
(c) Not a statement. Because the truth value cannot be determined.
(d) A statement. Because it is true.
(e) Not a statement. Because the truth value cannot be determined.
2. (a) $40 > 23 + 9$
(b) $\{3\} \subset \{3, 6, 9\}$
(c) $\frac{1}{4} \times \frac{10}{3} = \frac{5}{6}$
(d) $x^2 + 3 \leqslant (x + 3)^2$
(e) $\sqrt[3]{27} + 9 = 12$
3. (a) False (b) False (c) False
(d) True (e) True

Self Practice 3.1b

1. 819 is not a multiple of 9. False
2. A kite does not have two axes of symmetry. True
3. A cone does not have one curved surface. False
4. Two parallel lines do not have the same gradient. False
5. Not all quadratic equations have two equal roots. True

Self Practice 3.1c

1. (a) 2 or 3 is a prime factor of the number 6.
(b) A cone has one vertex and one plane.
(c) A rhombus and a trapezium are parallelograms.
2. (a) True (b) False (c) False (d) False
(e) True (f) True (g) False (h) True

Self Practice 3.1d

1. (a) If $x = 3$, then $x^4 = 81$.
(b) If $ax^3 + bx^2 + cx + d = 0$ is a cubic equation, then $a \neq 0$.
(c) If $n - 5 > 2n$, then $n < -5$.
(d) If $\frac{m}{n} > 1$, then $m^2 > n^2$.
2. (a) Antecedent: x is an even number.
Consequent: x^2 is an even number.
(b) Antecedent: set $K = \phi$.
Consequent: $n(K) = 0$.
(c) Antecedent: x is a whole number.
Consequent: $2x$ is an even number.
(d) Antecedent: A straight line AB is a tangent to a circle P.
Consequent: A straight line AB touches the circle P at one point only.
3. (a) k is a perfect square if and only if $\sqrt{k}$ is a whole number.
(b) $P \cap Q = P$ if and only if $P \subset Q$.
(c) $pq = 1$ if and only if $p = q^{-1}$ and $q = p^{-1}$.
(d) $k^2 = 4$ if and only if $(k + 2)(k - 2) = 0$.
4. (a) If PQR is a regular polygon, then $PQ = QR = PR$.
If $PQ = QR = PR$, then PQR is a regular polygon.
(b) If $\frac{m}{n}$ is an improper fraction, then $m > n$.
If $m > n$, then $\frac{m}{n}$ is an improper fraction.
(c) If 9 is the y-intercept of a straight line $y = mx + c$, then $c = 9$.
If $c = 9$, then 9 is the y-intercept of a straight line $y = mx + c$.
(d) If $f(x) = ax^2 + bx + c$ has a maximum point, then $a < 0$.
If $a < 0$, then $f(x) = ax^2 + bx + c$ has a maximum point.

Self Practice 3.1e

1. (a) Converse: If $x > -1$, then $x + 3 > 2$.
Inverse: If $x + 3 \leqslant 2$, then $x \leqslant -1$.
Contrapositive: If $x \leqslant -1$, then $x + 3 \leqslant 2$.
(b) Converse: If $k = 3$ or $k = -4$, then $(k - 3)(k + 4) = 0$.
Inverse: If $(k - 3)(k + 4) \neq 0$, then $k \neq 3$ or $k \neq -4$.
Contrapositive: If $k \neq 3$ or $k \neq -4$, then $(k - 3)(k + 4) \neq 0$.
(c) Converse: If AB is parallel to CD, then $ABCD$ is a parallelogram.
Inverse: If $ABCD$ is not a parallelogram, then AB is not parallel to CD.
Contrapositive: If AB is not parallel to CD, then $ABCD$ is not a parallelogram.

2. (a)

Implication: If 2 and 5 are the factors of 10, then 2×5 is 10.	True
Converse: If 2×5 is 10, then 2 and 5 are the factors of 10.	True
Inverse: If 2 and 5 are not the factors of 10, then 2×5 is not 10.	True
Contrapositive: If 2×5 is not 10, then 2 and 5 are not the factors of 10.	True

(b)

Implication: If 4 is a root of $x^2 - 16 = 0$, then 4 is not a root of $(x + 4)(x - 4) = 0$.	False
Converse: If 4 is not a root of $(x + 4)(x - 4) = 0$, then 4 is a root of $x^2 - 16 = 0$.	True
Inverse: If 4 is not a root of $x^2 - 16 = 0$, then 4 is a root of $(x + 4)(x - 4) = 0$.	True
Contrapositive: If 4 is a root of $(x + 4)(x - 4) = 0$, then 4 is not a root of $x^2 - 16 = 0$.	False

(c)

Implication: If a rectangle has four axes of symmetry, then the rectangle has four sides.	True
Converse: If a rectangle has four sides, then the rectangle has four axes of symmetry.	False
Inverse: If a rectangle does not have four axes of symmetry, then the rectangle does not have four sides.	False
Contrapositive: If a rectangle does not have four sides, then the rectangle does not have four axes of symmetry.	True

(d)

Implication: If $55 + 55 = 4 \times 5$, then $666 + 666 = 6 \times 6$	True
Converse: If $666 + 666 = 6 \times 6$, then $55 + 55 = 4 \times 5$.	True
Inverse: If $55 + 55 \neq 4 \times 5$, then $666 + 666 \neq 6 \times 6$.	True
Contrapositive: If $666 + 666 \neq 6 \times 6$, then $55 + 55 \neq 4 \times 5$.	True

Self Practice 3.1f

1. (a) False. A rectangle does not have four sides of equal length.
(b) True
(c) True
(d) False. 36 is not divisible by 14.

2. (a) $100_8 - 77_8 \neq 1_8$. False because $100_8 - 77_8 = 1_8$.
(b) A cuboid does not have four uniform cross sections. True
(c) If $y = 2x$ and $y = 2x - 1$ have the same gradient, then $y = 2x$ is parallel to $y = 2x - 1$. True
(d) If a triangle ABC does not have a right angle at C, then $c^2 \neq a^2 + b^2$. True
(e) If $w \geq 5$, then $w \geq 7$. False. When $w = 6$, $6 > 5$ but $6 < 7$.

Self Practice 3.2a

1. Deductive argument
2. Inductive argument
3. Inductive argument
4. Deductive argument
5. Deductive argument
6. Inductive argument
7. Inductive argument
8. Deductive argument
9. Deductive argument
10. Inductive argument

Self Practice 3.2b

1. Valid but not sound because premise 1 and conclusion are not true.
2. Valid and sound
3. Valid and sound
4. Valid but not sound because premise 1 is not true.
5. Not valid but sound because it does not comply with a valid deductive argument.
6. Valid and sound
7. Not valid and not sound because it does not comply with a valid deductive argument. A kite also has perpendicular diagonals but it is not a rhombus.
8. Valid and sound
9. Not valid and not sound because it does not comply with a valid deductive argument.
10. Valid and sound

Self Practice 3.2c

1. (a) Preevena uses digital textbook.
(b) Kai Meng gets a cash prize of RM200.
(c) Quadrilateral $PQRS$ is not a regular polygon.
(d) ΔABC has one axis of symmetry.
(e) $m : n = 2 : 3$
(f) $m + 3 > 2m - 9$

2. (a) Straight line AB has zero gradient.
(b) All multiples of 9 are divisible by 3.
(c) Polygon P is a nonagon.
(d) If $x > 6$, then $x > 4$.
(e) The room temperature is not lower than 19°C.
(f) If $3x - 8 = 16$, then $x = 8$.

Self Practice 3.2d

1. This argument is weak and not cogent because the conclusion is probably false.
2. This argument is strong and cogent.
3. This argument is weak and not cogent because the conclusion is probably false.
4. This argument is strong and cogent.
5. This argument is strong but not cogent because premise 3 is false.
6. This argument is weak and not cogent because the conclusion is probably false.

Self Practice 3.2e

1. $(3n)^{-1}$; $n = 1, 2, 3, 4, \ldots$
2. $\frac{n}{5}$; $n = 1, 2, 3, 4, \ldots$
3. $2(n)^3 + n$; $n = 0, 1, 2, 3, \ldots$
4. $20 - 4^n$; $n = 0, 1, 2, 3, \ldots$

Self Practice 3.2f

1. RM43
2. (a) 32 500 residents (b) 14th year
3. (a) $536\,100 - 15\,000n$ (b) 431 100 babies
4. (a) $\sin 60° = \frac{y}{z}$ $\sin 40° = \frac{p}{r}$ $\sin 20° = \frac{a}{c}$
   $\cos 30° = \frac{y}{z}$ $\cos 50° = \frac{p}{r}$ $\cos 70° = \frac{a}{c}$
   (b) $\sin\theta = \cos(90° - \theta)$
   (c) 0.9848

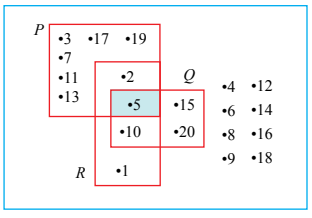
Comprehensive Practice

1. (a) A statement because it is true.
   (b) Not a statement because the truth value cannot be determined.
   (c) A statement because it is false.
   (d) Not a statement because the truth value cannot be determined.
   (e) Not a statement because the truth value cannot be determined.
   (f) A statement because it is true.
   (g) Not a statement because the truth value cannot be determined.
   (h) A statement because it is true.
   (i) A statement because it is false.
2. (a) True
   (b) False. -3 is an integer with negative value.
   (c) False. $\frac{3}{2}$ is a fraction larger than one.
   (d) False. The diagonals of a kite are not a perpendicular bisector.
3. (a) False
   (b) True
   (c) True
   (d) False
4. (a) All hexagons have 6 vertices.
   (b) Some circles have a radius of 18 cm.
   (c) Some triangles have three axes of symmetry.
5. (a) (i) Antecedent: $p < q$
   Consequent: $q - p > 0$
   (ii) Antecedent: The perimeter of rectangle A is $2(x + y)$.
   Consequent: The area of rectangle A is xy.
   (b) (i) x is a multiple of 10 if and only if x is a multiple of 5.
   (ii) 6 is a factor of 12 if and only if 6 is a factor of 24.
   (c) (i) If 20% of 30 is 6, then $0.2 \times 30 = 6$.
   If $0.2 \times 30 = 6$, then 20% of 30 is 6.
   (ii) If M is divisible by 20, then M is divisible by 2 and 10.
   If M divisible by 2 and 10, then M is divisible by 20.
6. (a) If $\alpha + \beta = 90°$, then α and β are two complementary angles. True
   (b) If $w \leq 30$, then $w \leq 20$. False because $28 < 30$ but $28 > 20$.
   (c) If $p \leq 0$, then $p^2 \leq 0$. False because $-2 < 0$ but $(-2)^2 > 0$.
   (d) The sum of exterior angles of a poligon is not 360°. False because the sum of exterior angles in each polygon is 360°.
7. (a) 2 is a factor of 8.
   (b) $x = 5$
   (c) If $\alpha = \beta$, then $\sin^2\alpha + \cos^2\beta = 1$.
   (d) 54 is a multiple of 18.
   (e) $m \leq 0$
   (f) The function $g(x)$ is a quadratic function.
8. (a) The surface area of the five similar cones is $700\,\pi$ cm^2.
   (b) The equation of the straight line PQ is $y = 3x + 5$.
9. (a) $n^2 - 5$; $n = 1, 2, 3, 4, \ldots$
   (b) $2^n + 3$; $n = 0, 1, 2, 3, \ldots$
   (c) $4n + n^2$; $n = 1, 2, 3, 4, \ldots$
   (d) $3n + 2(n - 1)^2$; $n = 1, 2, 3, 4, \ldots$
10. (a) Deductive argument
    (b) Inductive argument
11. (a) The pattern of the number of cylinders is $2n + 1$; $n = 1, 2, 3, 4, \ldots$
    (b) 104 720 cm^3
12. (a) $32(\pi + 2)$, $16(\pi + 2)$, $8(\pi + 2)$, $4(\pi + 2)$
    (c) $\frac{1}{4}(\pi + 2)$ cm

CHAPTER 4 Operations on Sets

Self Practice 4.1a

1. (a) $M = \{1, 3, 5, 7, 9\}$
   (b) $N = \{3, 6, 9\}$
   (c) $M \cap N = \{3, 9\}$
2. (a) $J \cap K = \{4, 6, 9\}$ (b) $J \cap L = \{3, 9\}$
   (c) $K \cap L = \{9\}$ (d) $J \cap K \cap L = \{9\}$
3. (a), (b)

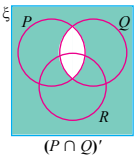


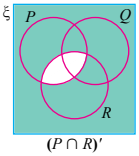
4. (a) $A \cap B = \{1\}$, $n(A \cap B) = 1$
   (b) $A \cap C = \phi$, $n(A \cap C) = 0$
   (c) $B \cap C = \phi$, $n(B \cap C) = 0$
   (d) $A \cap B \cap C = \phi$, $n(A \cap B \cap C) = 0$

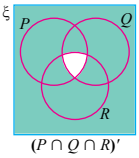
Self Practice 4.1b

1. (a) $(P \cap Q)' = \{2, 4, 6, 8, 9, 10\}$
(b) $(Q \cap R)' = \{3, 4, 5, 6, 7, 8, 9, 10\}$
(c) $(P \cap Q \cap R)' = \{2, 3, 4, 5, 6, 7, 8, 9, 10\}$

2. (a) $(G \cap H)' = \{11, 12, 14, 16, 17, 18\}$
(b) $(G \cap I)' = \{11, 12, 13, 14, 15, 16, 17, 18\}$
(c) $(H \cap I)' = \{11, 12, 13, 14, 15, 16, 17, 18\}$
(d) $(G \cap H \cap I)' = \{11, 12, 13, 14, 15, 16, 17, 18\}$

3. (a) 

$(P \cap Q)'$
(b) 

$(P \cap R)'$
(c) 

$(P \cap Q \cap R)'$

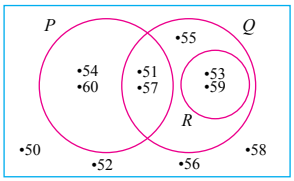
4. (a) $(M \cap L)' = \{a, b, c, d, f, g\}$
(b) $(N \cap L)' = \{a, b, c, d, g\}$
(c) $(M \cap N)' = \{a, b, d, f, g\}$
(d) $(L \cap M \cap N)' = \{a, b, c, d, f, g\}$

Self Practice 4.1c

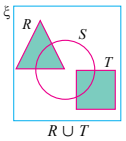
1. (a) 15 (b) 30
2. (a) 123 (b) 15 (c) 78
3. 62
4. 16
5. 8

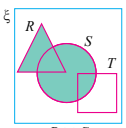
Self Practice 4.2a

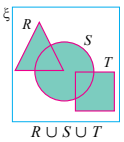
1. (a) $A \cup B = \{b, d, k, n, p, s\}$
(b) $A \cup C = \{b, d, f, g, k, l, n, p, s\}$
(c) $B \cup C = \{b, d, f, g, l, n, s\}$
(d) $A \cup B \cup C = \{b, d, f, g, k, l, n, p, s\}$

2. (a) 

(b) (i) $P \cup Q = \{51, 53, 54, 55, 57, 59, 60\}$
(ii) $P \cup R = \{51, 53, 54, 57, 59, 60\}$
(iii) $Q \cup R = \{51, 53, 55, 57, 59\}$
(iv) $P \cup Q \cup R = \{51, 53, 54, 55, 57, 59, 60\}$

3. (a) 

$R \cup T$
(b) 

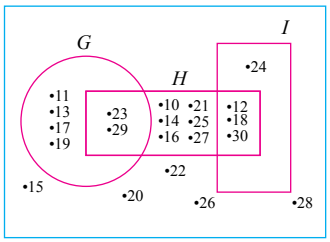
$R \cup S$
(c) 

$R \cup S \cup T$

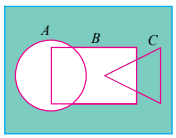
4. (a) $J \cup K = \{1, 2, 3, 5, 6, 7, 8\}$
(b) $J \cup L = \{1, 2, 3, 4, 5, 6, 9\}$
(c) $J \cup K \cup L = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

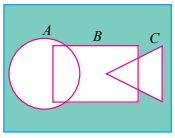
Self Practice 4.2b

1. (a) $A' = \{3, 4, 7, 8\}$
(b) $B' = \{5, 6, 7, 8\}$
(c) $(A \cup B)' = \{7, 8\}$

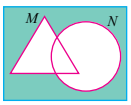
2. (a) 

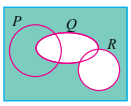
(b) (i) $(G \cup H)' = \{15, 20, 22, 24, 26, 28\}$
(ii) $(H \cup I)' = \{11, 13, 15, 17, 19, 20, 22, 26, 28\}$
(iii) $(G \cup H \cup I)' = \{15, 20, 22, 26, 28\}$

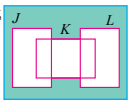
3. (a) 

$(A \cup B)'$
(b) 

$(A \cup B \cup C)'$

4. (a) 

$(M \cup N)'$
(b) 

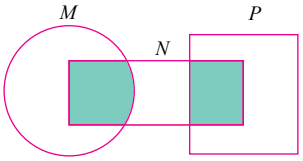
$(Q \cup R)'$
(c) 

$(J \cup K \cup L)'$

Self Practice 4.2c

1. $x = 3$
2. 8
3. 11
4. (a) 25 (b) 87 (c) 61
5. 94

Self Practice 4.3a

1. 

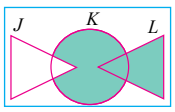
2. $(S \cup T) \cap R = \{3, 5, 7, 11, 13\}$

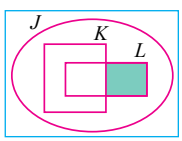
3. (a) $P \cap (Q \cup R) = \{3\}$
(b) $Q \cap (P \cup R) = \{3, 8\}$
(c) $(Q \cap R) \cup P = \{2, 3, 6, 7, 8\}$

Self Practice 4.3b

1. (a) $L' \cap (M \cup N) = \{13, 15, 19\}$
(b) $(M \cup N)' \cap L = \{12, 14, 18\}$

2. 25

3. (a) 

(b) 

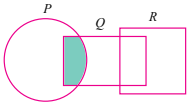
4. (a) $y = 11$ (b) 51

Self Practice 4.3c

1. 39
2. $x = 4$
3. 12
4. (a) 41
(b) 25
(c) 7

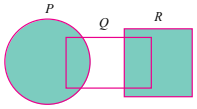
Comprehensive Practice

1. (a) $P \cap Q = \{3, 5\}$
(b) $P \cap R = \{3\}$
(c) $P \cap Q \cap R = \{3\}$
(d) $(P \cap Q \cap R)' = \{2, 5, 6\}$
2. (a) $M \cup N = \{a, b, d, i, k, u\}$
(b) $M \cup P = \{a, b, e, i, k, n, r\}$
(c) $M \cup N \cup P = \{a, b, d, e, i, k, n, r, u\}$
3. (a)



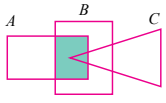
$P \cap Q$

(b)



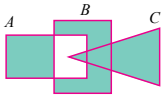
$P \cup R$

4. (a) $T' = \{1, 3, 5, 6, 8\}$
(b) $S \cup T = \{2, 4, 5, 6, 7, 8, 9\}$
(c) $S' \cap T = \{2, 4, 9\}$
(d) $(S \cup T)' = \{1, 2, 3, 4, 5, 6, 8, 9\}$
5. $A' = \{d, e, f, h, i\}$
6. (a) $Q' = \{11, 12, 13, 14, 16, 17, 18, 19, 21, 22, 23, 24, 26, 27, 28, 29\}$
(b) $P \cup R' = \{10, 11, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 29, 30\}$
(c) $(P \cup R)' \cap Q = \{10, 15, 20, 25, 30\}$
7. (a)



$A \cap (B \cup C)$

(b)



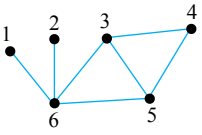
$C \cup (A \cap B)'$

8. 39
9. 31
10. 6
11. (a) 8 (b) 11 (c) 54
12. (a) 8 (b) 5 (c) 7 (d) 2
13. 50

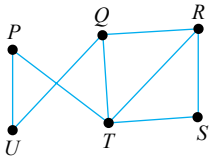
CHAPTER 5 Network in Graph Theory

Self Practice 5.1a

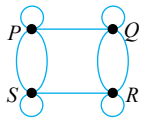
1. (a) (i) $V = \{1, 2, 3, 4, 5\}$
$n(V) = 5$
(ii) $E = \{(1, 2), (1, 5), (2, 3), (2, 4), (2, 5), (3, 4), (4, 5)\}$
$E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$
$n(E) = 7$
(iii) 14
(b) (i) $V = \{P, Q, R, S, T, U, V, W\}$
$n(V) = 8$
(ii) $E = \{(Q,P),(Q,R),(Q,W),(R,V),(S,T),(S,U),(U,V),(V,W)\}$
$n(E) = 8$
(iii) 16
(c) (i) $V = \{A, B, C, D, E, F\}$
$n(V) = 6$
(ii) $E = \{(A,B),(A,F),(B,C),(B,E),(C,D),(C,E),(D,E),(E,F)\}$
$n(E) = 8$
(iii) 16
2. (a) (i) $V = \{A, B, C, D, E\}$
$n(V) = 5$
(ii) $E = \{(A, B), (A, B), (A, E), (B, C), (B, D), (B, E), (C, C), (C, D), (D, E), (D, E)\}$
$n(E) = 10$
(iii) 20
(b) (i) $V = \{O, P, Q, R, S, T, U\}$
$n(V) = 7$
(ii) $E = \{(P,U),(P,U),(U,T),(U,T),(P,Q),(P,O),(Q,R),(Q,R),(Q,O),(R,R),(R,S),(R,S),(R,O),(S,O),(S,T),(T,O),(U,O)\}$
$n(E) = 17$
(iii) 34
3. (a)



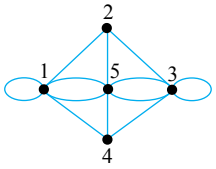
(b)



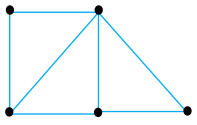
4. (a)



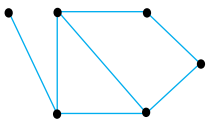
(b)



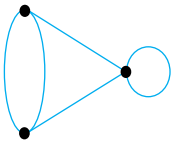
5. (a)



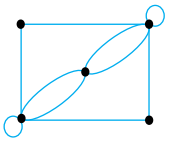
(b)



6. (a)



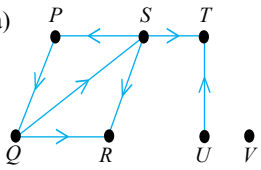
(b)



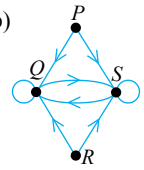
Self Practice 5.1b

1. (i) The edges in directed graphs are marked with direction.
(ii) The order of vertices in directed graphs are written according to the direction of the edges.
2. A value or information involving edge.

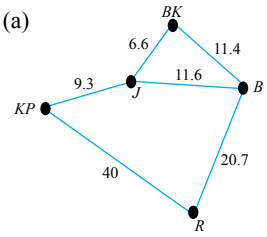
3. (a)



(b)



4. (a)



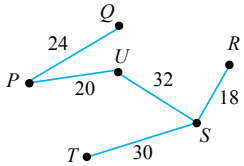
(b) 1.6 km

Self Practice 5.1c

1. Subgraph – Diagram 1, Diagram 2, Diagram 3, Diagram 4, Diagram 8, Diagram 11
Not a subgraph – Diagram 5, Diagram 6, Diagram 7, Diagram 9, Diagram 10

3. (a) Not a tree (b) Not a tree
(c) Tree (d) Not a tree

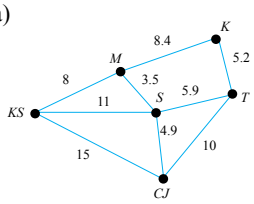
6. (a)



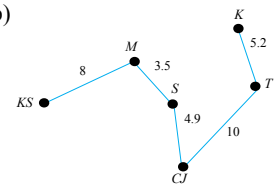
(b) Total weight = 24 + 20 + 32 + 18 + 30
= 124

Self Practice 5.1d

1. (a)

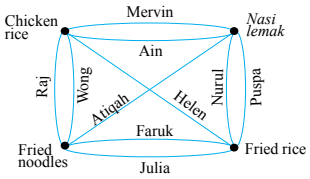


(b)



(c) 31.6 km

2. (a)



(b) Types of food. Each type of food is favoured by more than two pupils.

(c) Sum of degrees = Total number of food choices × Number of pupils

(d) Graph form

3. (b) Undirected graph. The organisation chart is a network because it shows the relationships between the individuals involved based on the chart's requirement.

Self Practice 5.1e

1. (a) Johor Bahru – Kuching (Saturday, 0605 hours) and then Kuching – Miri (Saturday, 1145 hours).

(b) Johor Bahru – Kuching (Friday, 1930 hours) and then Kuching – Miri (Friday, 2155 hours). Even though the total price of the flight tickets is RM35 higher than the cheapest package on Saturday, Encik Maswi gets to spend more time with his family.

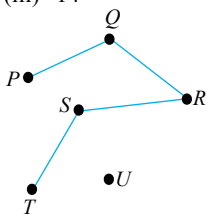
Comprehensive Practice

1. (a) (i) $V = \{P, Q, R, S, T, U\}$
(ii) $E = \{(P, Q), (P, S), (P, U), (Q, R), (Q, T), (R, S), (R, U), (S, T), (T, U)\}$
(iii) 18

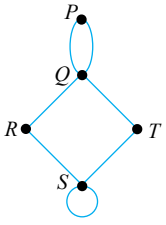
(b) (i) $V = \{P, Q, R, S, T, U\}$
(ii) $E = \{(P, P), (P, Q), (P, R), (Q, R), (R, S), (S, T), (S, T)\}$
(iii) 14

(c) (i) $V = \{P, Q, R, S, T\}$
(ii) $E = \{(P, Q), (R, Q), (S, R), (P, S), (S, P), (S, T), (T, T)\}$
(iii) 14

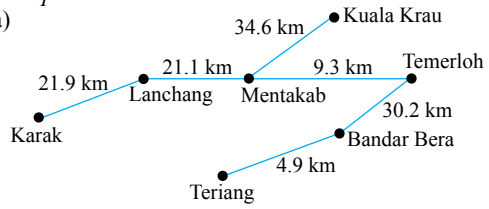
2. (a)



(b)



3. (a)



(b) Yes, because every pair of vertices is connected by one edge. Vertex = 7, Edge = 6

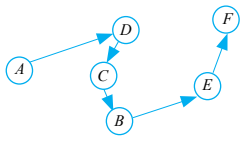
4. Route $A \rightarrow C \rightarrow D \rightarrow E$ because it is a safer route even though Lani had to cycle 300 m more.

5. (a) (i) $P \rightarrow Q \rightarrow R \rightarrow S$ (ii) $P \rightarrow S$

(b) Route $P \rightarrow Q \rightarrow S$ because I can save RM35 and the difference in time is only 9 minutes compared to route $P \rightarrow S$.

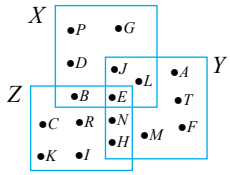
6. $11 = x_1 + x_2$, $x_4 = x_3 + 11$, $x_2 + x_3 = 20$,
$x_1 + 10 = x_5$, $x_5 + 10 = x_4$, $x_1 = 5$,
$x_2 = 6$, $x_3 = 14$, $x_4 = 25$.

7. (a)



(b) 3.08 km

8. (a)



(b) (i) $\{C, R, K, I\}$
(ii) $\{P, G, D, C, R, K, I\}$
(iii) $\{E\}$

9. (b) (i) RM1 080 (ii) 40

10. (a) third (b) 484 (c) 13 068

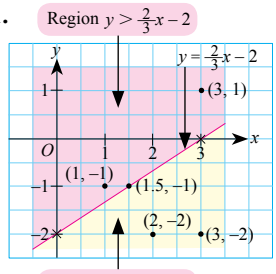
CHAPTER 6 Linear Inequalities in Two Variables

Self Practice 6.1a

1. (a) $25x + 45y \leq 250$ or $5x + 9y \leq 50$
(b) $2x + 1.5y \leq 500$ or $4x + 3y \leq 1\,000$
(c) $0.3x + 0.4y \leq 50$ or $3x + 4y \leq 500$
(d) $1.5x + 3.5y \geqslant 120$ or $3x + 7y \geqslant 240$

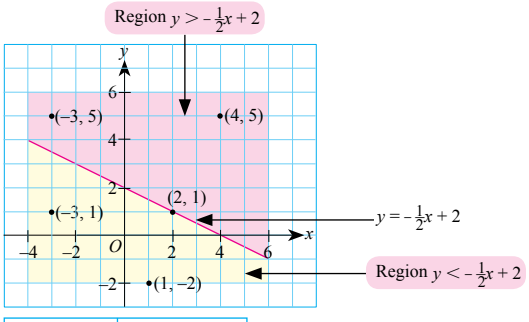
Self Practice 6.1b

1.



$y = \frac{2}{3}x - 2$	(1.5, −1)
$y > \frac{2}{3}x - 2$	(3, 1), (1, −1)
$y < \frac{2}{3}x - 2$	(2, −2), (3, −2)

2.



$y = -\frac{1}{2}x + 2$	(2, 1)
$y > -\frac{1}{2}x + 2$	(−3, 5), (4, 5)
$y < -\frac{1}{2}x + 2$	(−3, 1), (1, −2)

3.

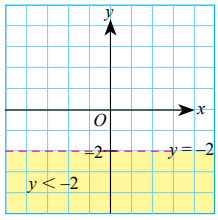
$y = 4x - 5$	(3, 7)
$y > 4x - 5$	(2, 4), (−2, 0)
$y < 4x - 5$	(0, −6), (4, 5)

4.

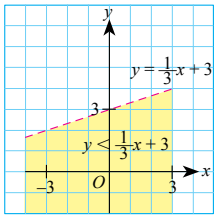
$y = -3x + 4$	(1, 1)
$y > -3x + 4$	(−1, 8), (−0.5, 7)
$y < -3x + 4$	(−2, 3), (0, 1)

Self Practice 6.1c

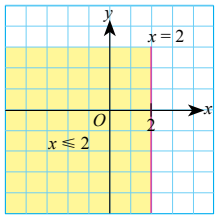
1. (a)



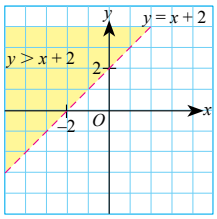
(b)



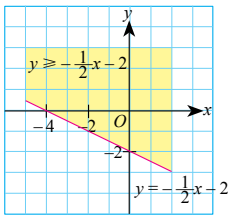
(c)



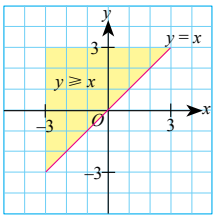
(d)



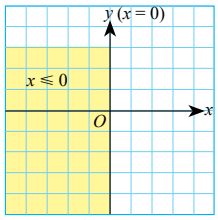
(e)



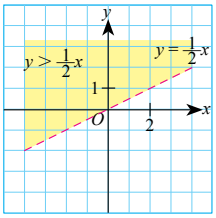
(f)



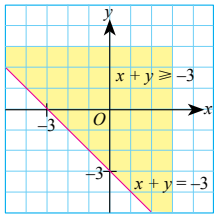
2. (a)



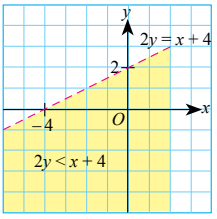
(b)



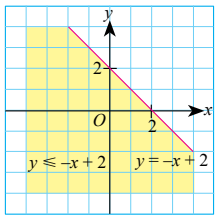
(c)



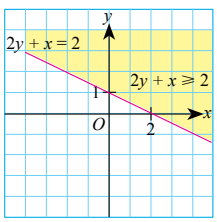
(d)

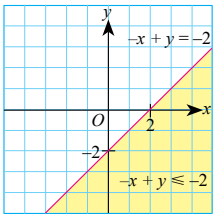


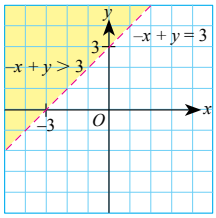
(e)



(f)



(g) 

(h) 

Self Practice 6.2a

1. (a) $x + y \leqslant 50$ (b) $x \geqslant 2y$ or $2y \leqslant x$
(c) $8x + 12y \leqslant 850$

2. (a) $x + y \leqslant 500$ (b) $x \leqslant 3y$ or $3y \geqslant x$
(c) $y \geqslant 200$

3. x = green chilli , y = *cili padi*
(a) $x + y \leqslant 250$
(b) $x \geqslant 3y$ or $3y \leqslant x$
(c) $x \geqslant 100$

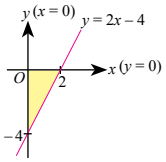
Self Practice 6.2b

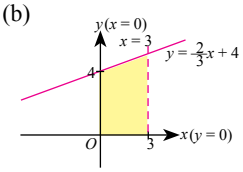
1. (a) *D* (b) *A* (c) *C* (d) *B*

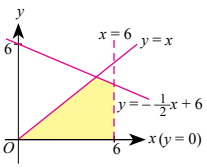
2. (a) *E* (b) *C* (c) *A* (d) *D*

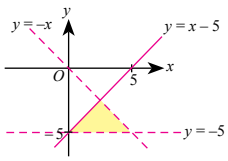
Self Practice 6.2c

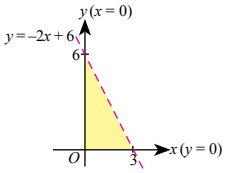
1. (a) *C* (b) *C* (c) *A*

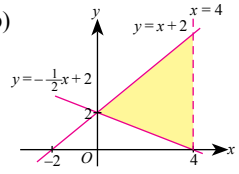
2. (a) 

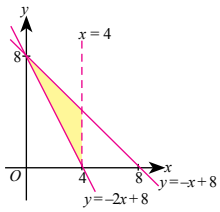
(b) 

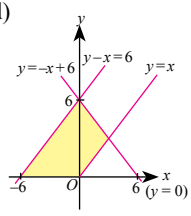
(c) 

(d) 

3. (a) 

(b) 

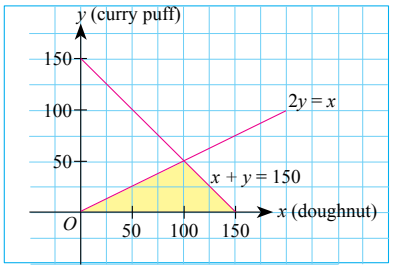
(c) 

(d) 

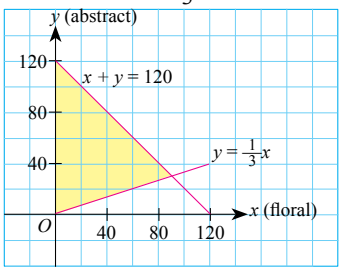
Self Practice 6.2d

1. (a) $y < x + 4$, $y \geqslant 0$ and $x \leqslant 0$
(b) $y > 2x - 4$, $y > -2x - 4$ and $y \leqslant 0$
(c) $3y \leqslant 5x$, $y > x$ and $x \leqslant 3$

2. (a) $x > -4$, $y \leqslant -\frac{1}{2}x$ and $y \geqslant 0$
(b) $y \leqslant 2x + 4$, $y \leqslant -\frac{4}{3}x + 4$ and $y \geqslant 0$
(c) $y \leqslant x - 1$, $y \geqslant \frac{2}{3}x - 2$, $x \geqslant 0$ and $y \leqslant 0$

3. (a) $x + y \leqslant 150$, $2y \leqslant x$
(b) 

(c) (i) 50
(ii) Minimum = 50; maximum = 125

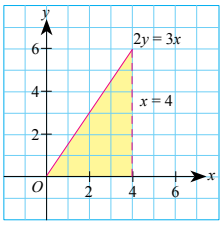
4. (a) $x + y \leqslant 120$, $y \geqslant \frac{1}{3}x$
(b) 

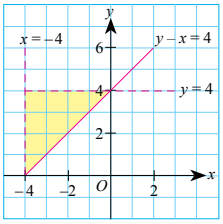
(c) 90 m

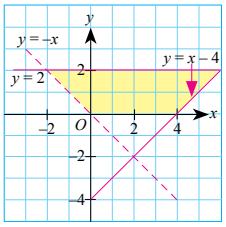
(d) No. Point (80, 60) is located outside the shaded region.

Comprehensive Practice

1. (a) $2y > x + 5$, $y - x > 8$
(b) $x \geqslant 0$, $x \geqslant -5$
(c) $y \leqslant 4 - x$, $x \leqslant 2 - y$, $y + x \leqslant 2$, $y \leqslant -\frac{1}{2}x$
(d) $y < 4$, $y < -1$
(e) $y \geqslant 0$, $y \geqslant 10$
(f) $y < 2x - 5$, $-y > 8 - 2x$, $2y < x$
(g) $y > -x - 3$, $3y + x > 4$
(h) $\frac{1}{2}y - x \geqslant 4$, $2y \geqslant x$, $-y \leqslant 4 - x$

2. (a) 

(b) 

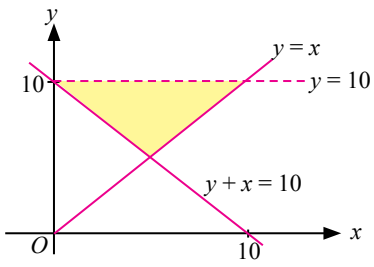
(c) 

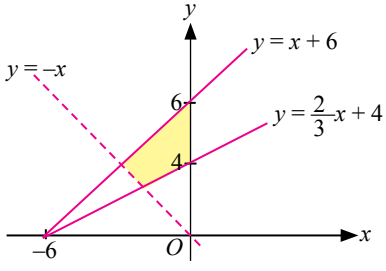
3. (a) $y \geqslant -2x$, $y > x$ and $y < 4$

(b) $y < 2x$, $y \geqslant \frac{1}{2}x$ and $y \leqslant -\frac{1}{2}x + 6$

(c) $y - x \leqslant 4$, $2y > x + 4$ and $y < 3$

(d) $y \geqslant \frac{3}{2}x + 6$, $x > -4$, $y < 5$

4. (a) 

(b) 

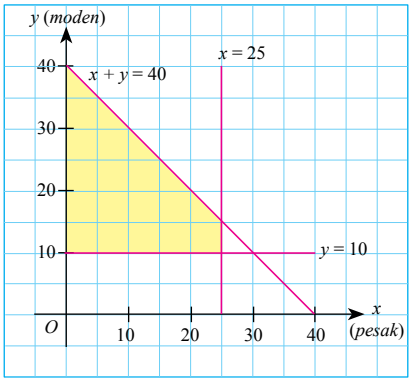
5. (a) $y \leqslant 2$, $x < 3$, $y \geqslant -x$, $y \geqslant 0$

(b) $y > -2x$, $y \geqslant 2x - 8$, $y \leqslant -\frac{1}{2}x$

6. (a) $y < -1$, $x \geqslant -5$, $y \geqslant \frac{4}{5}x - 1$

(b) $x \geqslant 2$, $y \geqslant 0$, $y < -x + 6$

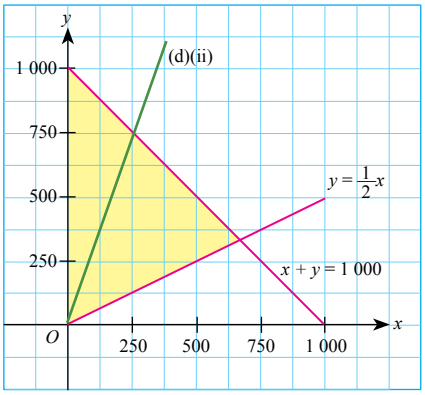
7. (a) $x + y \leqslant 40$, $y \geqslant 10$, $x \leqslant 25$

(b) 

(c) minimum = 10, maximum = 30

(d) RM2 625

8. (a) $x + y \leqslant 1\,000$, $y \geqslant \frac{1}{2}x$

(b) 

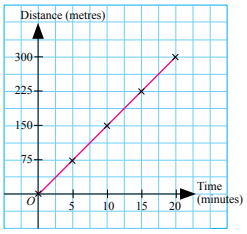
(c) minimum = 250 m, maximum = 500 m

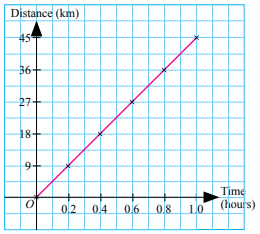
(d) (i) $y \geqslant 3x$

(ii) Refer to the graph.

CHAPTER 7 Graphs of Motion

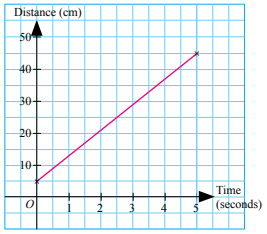
Self Practice 7.1a

1. 

2. 

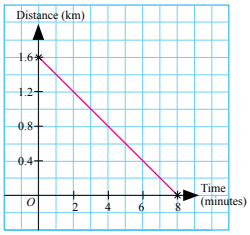
3.

Time, t (seconds)	0	5
Distance, s (cm)	5	45



4.

Time, t (minutes)	0	8
Distance, s (km)	1.6	0



Self Practice 7.1b

1. (a) 50

(b) The car is stationary.

(c) (i) 40

(ii) The car moves for a distance of 100 km with an average speed of 40 km h^{-1} in 2.5 hours.

2. (a) 2

(b) 4.8

(c) Encik Rashid runs for a distance of 4 km with an average speed of 4.8 km h^{-1} in 50 minutes.

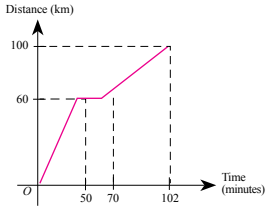
3. (a) 1424
(b) (i) The car is stationary for 66 minutes.
(ii) The car moves with an average speed of 40 km h^{-1} for a distance of 30 km in 45 minutes.

4. (a) 40
(b) The car moves with an average speed of 54 km h^{-1} for a distance of 36 km in 40 minutes.

Self Practice 7.1c

1. (a) 3
(b) Yes, Jeffrey will finish the race in 12 seconds.

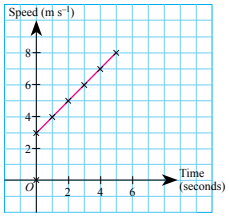
2. (a) 50 (b) 70
(c) (d) 11:12 in the morning

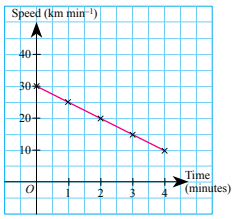


3. (a) 25 minutes
(b) (i) 27 (ii) 33 km
(c) 80
(d) 45

4. (a) 20
(b) 60
(c) The car moves with an average speed of 72 km h^{-1} for a distance of 36 km in 30 minutes.

Self Practice 7.2a

1. (a)


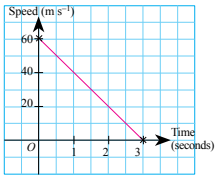
(b)


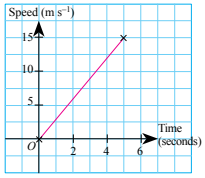
2. (a)

Time, t (seconds)	0	30
Speed, v ($m\ s^{-1}$)	60	0

(b)

Time, t (seconds)	0	5
Speed, v ($m\ s^{-1}$)	0	15





Self Practice 7.2b

1. (a) 360 (b) 0.275 (c) 2.6

2. (a) $14\frac{2}{3}$ (b) $29\frac{1}{3}$

3. (a) 16 (b) 22.5

Self Practice 7.2c

1. (a) The motorcycle is decelerating at 0.75 m s^{-2} in 20 seconds; or the speed of the motorcycle decreases from 35 m s^{-1} to 20 m s^{-1} in 20 seconds; or the motorcycle moves 550 m in 20 seconds.
(b) The motorcycle moves at a uniform speed of 20 m s^{-1} in 30 seconds; or the motorcycle travels for 600 m at a uniform speed.

2. (a) $\frac{5}{6}$ m s^{-2} (b) 260 m
(c) The particle moves at a uniform speed of 15 m s^{-1} in 7 seconds.

3. (a) $\frac{3}{8}$ m s^{-2} (b) 1 200 m
(c) Encik Merisat drives for a distance of 1.725 km in 2.5 minutes with an average speed of 41.4 km h^{-1}.

Self Practice 7.2d

1. (a) 96 (b) 18
2. (a) 1 (b) 25.5 (c) 14
3. (a) 28 (b) 15

Comprehensive Practice

1. (a) 6 minutes (b) 60 (c) 42.86
2. (a) 100 (b) 1.6 (c) 57.14
3. (a) 8 seconds (b) 17
4. (a) $-\frac{7}{6}$ (b) 6 (c) 19.68 km h^{-1}
5. (a) 12 (b) 32.4 (c) 60
6. (a) 80 (b) 0915 hours
7. (a) (i) 80
(ii) Car A moves with an average speed of 25 m s^{-1} for a distance of 2 km in 80 seconds.
(b) 1 minute

CHAPTER 8 Measures of Dispersion for Ungrouped Data

Self Practice 8.1a

1 (a) 45, 150 (b) 105
2. $p = 30$, $q = 120$
3. 2.3

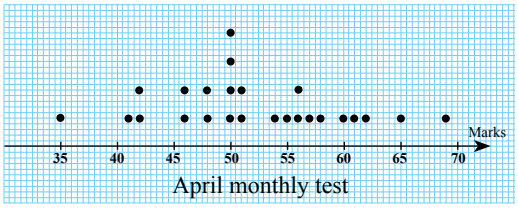
Self Practice 8.1b

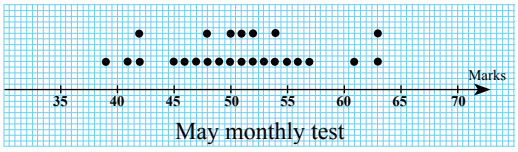
1.

Group A		Group B
9 7 6 5 4 1	4	0 0 1 2 2 6 8 9
8 6 4 4 3 2 2 0	5	2 2 4 5 6 6 7 7 8 8 9 9
9 8 8 7 6 4 3 3 2 2 1 0 0	6	0 1 3 4 4 5 6 9 9
6 6 5 3 2 0 0 0	7	0 0 2 5 6 6 8
6 4 2 2 1	8	2 3 3 4

In general, the body mass of pupils in group A is greater than the body mass of pupils in group B.

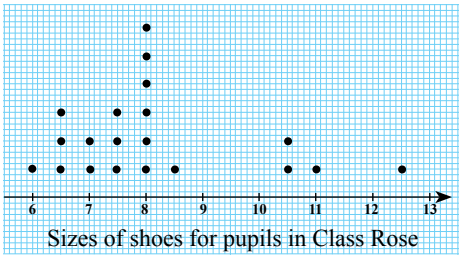
2.

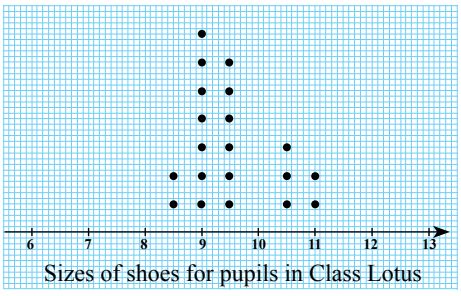




The dispersion of May monthly test is smaller.

3. (a)





(b) The size of shoes for pupils in Class Rose is dispersed wider compared to Class Lotus.
Class Rose has bigger difference in size of shoes.
Difference in size of shoes for Class Rose
$=12.5 - 6 = 6.5$
Difference in size of shoes for Class Lotus
$=11 - 8.5 = 2.5$

Self Practice 8.2a

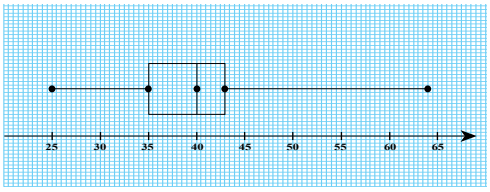
1. (a) 7, 3 (b) 12, 5.5 (c) 0.9, 0.45
2. (a) 5, 2 (b) 5, 3
3. (a) 7, 2.646 (b) 24.86, 4.986
4. 1.26, 1.122

Self Practice 8.2b

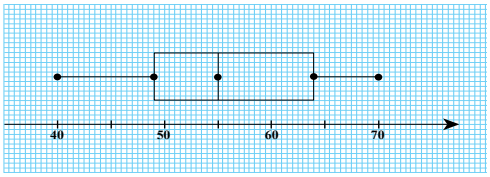
1. Range = 27
Interquartile range = 11
Interquartile range is the most appropriate measure of dispersion because there is an outlier, 2.
2. Standard deviation, pupil B
3. (a) 2100, 310, 702.2
(b) Interquartile range

Self Practice 8.2c

1. (a)



(b)



2. (a) 11 (b) 21 (c) 13 (d) 18 (e) 5 (f) 16

Self Practice 8.2d

1. 6, 3.2
2. 9, 29.16
3. 3.742, increased a lot
4. (a) 2.728
(b) (i) 5.456
(ii) 1.364
5. 100, 43.2
6. 0.9, 1.2

Self Practice 8.2e

1. $\sigma_A = 0.2506$, $\sigma_B = 0.3706$, athlete A is more consistent.
2. $\sigma_A = 12.65$, $\sigma_B = 6.321$, fertiliser B.

Self Practice 8.2f

1. (a) $h = 11$, $k = 18$
(b) 4.276
2. (a) $\sum x = 180$, $\sum x^2 = 1\ 700$
(b) 2.25

Comprehensive Practice

1. (a) 17, 13 (b) 44, 23
(c) 1.6, 0.75 (d) 20, 5.5
2. (a) 1.2, 0.4
(b) 5, 3
3. (a) variance = 5.917, standard deviation = 2.432
(b) variance = 52.8, standard deviation = 7.266
(c) variance = 0.46, standard deviation = 0.6782
(d) variance = 70.18, standard deviation = 8.377
4. variance = 130.3, standard deviation = 11.41
5. (a) range = 20, standard deviation = 10.4
(b) range = 2.5, standard deviation = 1.3
6. (a) 360
(b) 16 220
7. (a) (i) $m = 7$
(ii) 4.980
(b) 223.2

8. (a) 90.75
(b) (i) 12 (ii) 9.315

9. $\frac{56}{3}$

10. (a)

Team A	Team B
mean = 61	mean = 61
range = 22	range = 30
variance = 78.8	variance = 155.6
standard deviation = 8.877	standard deviation = 12.47

(b) No because of the existence of outlier or extreme values
(c) Team B

11. (a) mean = 18, variance = 56
(b) mean = 18.09, variance = 51.02

12. (a) 9, 2, 3.210, 1.792
(b) interquartile range

CHAPTER 9 Probability of Combined Events

Self Practice 9.1a

1. $\{(H_1, H_2), (H_1, G), (H_1, M), (H_2, H_1), (H_2, G), (H_2, M), (G, H_1), (G, H_2), (G, M), (M, H_1), (M, H_2), (M, G)\}$

2. $\{(B, B), (B, G), (G, B), (G, G)\}$

3. $\{(1, T), (2, T), (3, T), (4, T), (5, T), (6, T), (1, H), (2, H), (3, H), (4, H), (5, H), (6, H)\}$

4. {AAA, KKK, AAKA, AKAA, KAAA, KKAK, KAKK, AKKK, KAKAA, KAAKA, KKAAA, AAKKA, AKAKA, AKKAA, AAKKK, AKAKK, AKKAK, KAAKK, KAKAK, KKAAK}

Self Practice 9.2a

1. Independent events
2. Dependent events
3. Independent events
4. Dependent events
5. Independent events

Self Practice 9.2b

1. (a)

First dice	Second dice					
	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

(b) 36
(c) $\{(1, 2), (1, 3), (1, 5), (3, 2), (3, 3), (3, 5), (5, 2), (5, 3), (5, 5)\}$; $\frac{1}{4}$

2. $\{(Y_1, Y_1), (Y_1, Y_2), (Y_1, Y_3), (Y_2, Y_1), (Y_2, Y_2), (Y_2, Y_3), (Y_3, Y_1), (Y_3, Y_2), (Y_3, Y_3)\}$; $\frac{9}{64}$

3. $\{(Y_1, Y), (Y_2, Y)\}$; $\frac{1}{6}$

Self Practice 9.2c

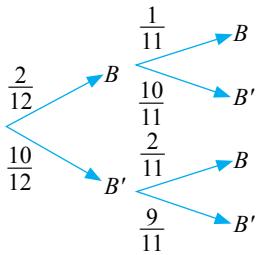
1. $\{(A, 2), (U, 2)\}$; $\frac{1}{6}$

2. $\{(1, 1), (1, 3), (3, 1), (3, 3)\}$; $\frac{1}{4}$

3. 0.2025

4. $\{(C, E), (C, I), (L, E), (L, I), (K, E), (K, I)\}$; $\frac{3}{10}$

5. B = Burnt bulb
B' = Not burnt bulb



$\frac{1}{66}$

6. (a) 0.3166 (b) 0.2108

Self Practice 9.3a

1. (a) Non-mutually exclusive events
(b) Mutually exclusive events
(c) Non-mutually exclusive events

2. (a) Non-mutually exclusive events
(b) Non-mutually exclusive events
(c) Mutually exclusive events

3. (a) Mutually exclusive events
(b) Non-mutually exclusive events
(c) Non-mutually exclusive events

Self Practice 9.3b

1. (a) $\{(4, 6), (5, 5), (5, 6), (6, 4), (6, 5), (6, 6), (1, 5), (2, 5), (3, 5), (4, 5), (5, 1), (5, 2), (5, 3), (5, 4)\}$; $\frac{7}{18}$

(b) $\{(4, 6), (5, 5), (5, 6), (6, 4), (6, 5), (6, 6), (1, 1), (2, 2), (3, 3), (4, 4)\}$; $\frac{5}{18}$

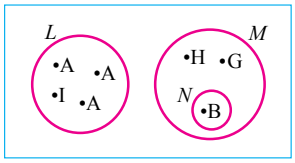
(c) $\{(1, 5), (2, 5), (3, 5), (4, 5), (5, 5), (6, 5), (5, 1), (5, 2), (5, 3), (5, 4), (5, 6), (1, 1), (2, 2), (3, 3), (4, 4), (6, 6)\}$; $\frac{4}{9}$

2. (a) $\{TT, HH\}; \frac{1}{2}$

(b) $\{TT, TH, HT\}; \frac{3}{4}$

(c) $\{HH, TH, HT, TT\}; 1$

3. (a)



(b) (i) $\{B, A, H, A, G, I, A\}; 1$

(ii) $\{A, A, A, I, B\}; \frac{5}{7}$

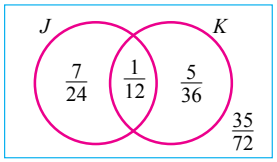
(iii) $\{B, H, G\}; \frac{3}{7}$

Self Practice 9.3c

1. $\{E, I, A, R\}, \frac{4}{7}$

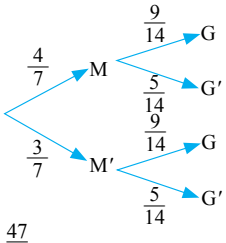
2. $\{(S, 4), (S, 5), (S, 6), (E, 6), (R, 6), (I, 6)\}, \frac{1}{2}$

3.



$, \frac{35}{72}$

4.



$\frac{47}{98}$

Self Practice 9.4a

1. 12 000

2. $\frac{13}{36}$

3. Pantai Cengal, because the probability of not raining on both days at Pantai Cengal is higher.

4. 230

Comprehensive Practice

1. $\{(B_1, B_2), (B_1, B_3), (B_1, G_1), (B_1, G_2), (B_2, B_1), (B_2, B_3), (B_2, G_2), (G_1, B_1), (G_1, B_2), (G_1, B_3), (B_2, G_1), (B_3, B_1), (B_3, B_2), (B_3, G_1), (B_3, G_2), (G_1, G_2), (G_2, G_1), (G_2, B_1), (G_2, B_2), (G_2, B_3)\}$

2. (a) $\frac{1}{15}$

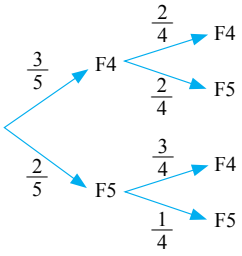
(b) $\frac{7}{30}$

3. (a) (91, R), (77, I), (77, A), (91, A)

(b) (i) $\{(77, R)\}, \frac{1}{6}$

(ii) $\{(77, R), (77, I), (77, A), (91, R)\}, \frac{2}{3}$

4.



$\frac{2}{5}$

5. (a) 0.2436

(b) 0.5128

6. (a) $\frac{199}{540}$

(b) $\frac{47}{54}$

7. (a) $\frac{2}{3}$

(b) $\frac{4}{9}$

8. (a) (i) $\frac{25}{156}$

(ii) $\frac{211}{468}$

(b) RM70

9. $\frac{43}{200}$

10. (a) $\frac{7}{33}$

(b) $\frac{5}{33}$

11. (a) (i) 0.191

(ii) 0.784

(b) Not advisable. Love your life

CHAPTER 10 Consumer Mathematics: Financial Management

Self Practice 10.1a

1. Financial management is a process that involves managing money from sources of income into savings, expenses, protection and investment.

2. The financial management process consists of setting goals, evaluating financial status, creating a financial plan, carrying out the financial plan, and reviewing and revising the progress.

3. Setting a financial goal will affect the total monthly savings in achieving the goal.
4. Short-term financial goals are less than a year and do not involve a large amount of money to be achieved. Long-term financial goals are more than five years and involve a large amount of money as compared to the short-term financial goals.
5. Puan Salmah has been practising the financial goal setting based on the SMART concept which is specific – she needs to save an amount of RM3 000 to buy a laptop, with monthly savings of RM300 and this goal is not difficult to be achieved with the total income earned as well as it is realistic with a saving of RM300 for 10 months (time-bound).

Self Practice 10.1b

1. • Inflation
   • Government policy
   • Personal health
2. (a) Mrs Thong does not spend wisely because her total monthly savings of RM250 compared to the income of RM6 000 which is less than 10%.
   (b) Mrs Thong will not be able to achieve the investment goal of RM500 000 with monthly savings of RM250.

Comprehensive Practice

1. Do not prepare for financial planning, not spending wisely, uncontrolled use of credit cards, failure to pay off the loans and car instalments.
2. Negative cash flow of an individual in a financial plan will lead to the individual's bankruptcy and he or she will not have enough savings in case of emergency.
3. A financial plan is created with the aim of estimating the initial budget to achieve goals and monthly savings that are needed to achieve short-term goals and long-term goals, analyse spending behaviour as well as setting a duration to achieve the goals.
4. When we review and revise the progress of a financial plan, it gives us some space to refine our nature of spending and it also helps in generating more income in order to achieve the goals.
5. Inflation, changes in government taxation policies, economic policies and others.
6. The way of spending can be adjusted based on financial goals. Actions can be taken such as generating more income.
7. We should start to cultivate the saving habit as early as possible to ensure that financial goals can be achieved as planned.
8. (a) Personal monthly financial plan for Encik Nabil

Income and Expenditure	RM	
Active income:		
Net salary	3 800	
Commissions	450	
Total active income		4 250
Passive income:		
House rental	600	
Total passive income		600
Total monthly income		4 850
Minus fixed monthly savings	380	
Total income after deducting savings		4 470
Minus cash outflow/expenses		
Fixed expenses:		
Housing loan instalments (1)	800	
Housing loan instalments (2)	500	
Insurance expenses	350	
Total fixed expenses		1 650
Variable expenses:		
Food expenses	900	
Utility payments	150	
Toll and petrol expenses	200	
Internet service subscription	100	
Eat at a luxury restaurant	400	
Total variable expenses		1 750
Surplus/Deficit		1 070

   (b) Encik Nabil's personal financial plan has a surplus where there is a positive cash flow when the total income is more than the total expenses. This has improved Encik Nabil's liquidity.
9. (a) Positive cash flow – enables savings and achieves goals as planned.
   (b) Negative cash flow – making it difficult for someone to achieve financial goals and more inclined to obtain loan resources such as credit cards.

Glossary

Argument (*Hujah*)
A set of statements called the premises and a conclusion.

Bankrupt (*Bankrap*)
A situation where a person is unable to settle his/her debts because his/her expenses exceed income. As a result, the court declares the person a bankrupt.

Combined events (*Peristiwa bergabung*)
An event formed from the union or intersection of two or more events.

Deceleration (*Nyahpecutan*)
A negative acceleration.

Deductive argument (*Hujah deduktif*)
A process of making a special conclusion based on general premises.

Degree (*Darjah*)
The number of edges that connect two vertices.

Digit (*Digit*)
The symbols used or combined to form numbers in the numbering system. 0, 1, 2, 3, 4, 5, 6, 8, 9 are 10 digits used in the decimal number system. For example, number 124 651 has six digits.

Directed graph (*Graf terarah*)
A graph in which a direction is assigned to the edge connecting two vertices.

Discrete (*Diskret*)
The countable values.

Displacement (*Sesaran*)
The vector distance from a fixed point is measured in a certain direction.

Distance (*Jarak*)
The length of the space between two points.

Distance-time graph (*Graf jarak-masa*)
A graph that shows the distance travelled per unit of time. The gradient of the graph shows the speed measured.

Edge (*Tepi*)
A line that connects two vertices.

Graph (*Graf*)
A series of dots that are linked to each other through a line.

Inductive argument (*Hujah induktif*)
A process of making a general conclusion based on specific cases.

Inflation (*Inflasi*)
Situation of a continuing increase in the general price level.

Intersection of sets (*Persilangan set*)
A set that contains the common elements of two or more sets. The intersection is illustrated with the symbol $\cap$.

Inverse (*Songsangan*)
An inverse of an implication "If p, then q" is "If $\sim p$, then $\sim q$".

Linear (*Linear*)
A description of something that is related to or in the form of a straight line.

Linear inequalities (*Ketaksamaan linear*)
The inequalities that involve linear expressions such as $y > mx + c$, $y < mx + c$, $y \geqslant mx + c$, $y \leqslant mx + c$ where $m \neq 0$, x and y are variables.

Linear inequality system
(*Sistem ketaksamaan linear*)
A combination of two or more linear inequalities.

Loop (*Gelung*)
An edge that is in the form of an arc that starts and ends at the same vertex.

Mutually exclusive events (*Peristiwa saling eksklusif*)
If two events, A and B, do not intersect with each other, then event A and B are said to be mutually exclusive.

Network (*Rangkaian*)
Part of a graph with vertices and edges.

Number bases (*Asas nombor*)
Numbering system of a number.

Place value (*Nilai tempat*)
The value of a digit on the basis of its position in a number. For example, the place value of digit 6 in 6 934 is thousands and the place value of digit 5 in 523 089 is hundred thousands.

Premise (*Premis*)
A statement that is assumed (considered) as something that is true for the purpose of making a conclusion later.

Probability of independent events
(*Kebarangkalian peristiwa tak bersandar*)
Two events, A and B, are said to be independent if the probability of event A does not affect the probability of event B.

Quadratic equation (*Persamaan kuadratik*)
An equation that can be written in the general form $ax^2 + bx + c = 0$ where a, b and c are constants and $a \neq 0$. This equation has one variable and the highest power of the variable is 2.

Quadratic function (*Fungsi kuadratik*)
A function in the form of $f(x) = ax^2 + bx + c$ where a, b, c are constants and $a \neq 0$. The highest power of the variable is 2 and has only one variable.

Region (*Rantau*)
An area that satisfies a system of linear inequalities.

Simple graph (*Graf mudah*)
An undirected graph without loops or multiple edges.

Speed (*Laju*)
The rate of change in distance.

Speed-time graph (*Graf laju-masa*)
A graph that shows the relationship between the speed of an object in a certain period of time. The gradient of the graph represents acceleration. The area under the graph shows the total distance travelled.

Standard deviation (*Sisihan piawai*)
A statistical measure that measures the dispersion of a set of data.

Statement (*Pernyataan*)
A sentence that the truth value can be determined.

Subgraph (*Subgraf*)
Part of a graph or the whole graph redrawn without changing the original positions of the vertices and edges.

Tree (*Pokok*)
A subgraph of a graph that has minimum connection between vertices without loops and multiple edges.

Uniform speed (*Laju seragam*)
The distance that constantly increases with time.

Union of sets (*Kesatuan set*)
A combination of all the elements of two or more sets. Its symbol is $\cup$.

Unweighted graph (*Graf tak berpemberat*)
The edges that connect two vertices of a graph is not stated with weighted values such as distance, cost, time and others.

Variable (*Pemboleh ubah*)
A quantity with a varied value and is represented by symbols such as x, y and z that can take any values from a particular set of values.

Vertex (*Bucu*)
The dot where the edge is connected to.

Weighted graph (*Graf pemberat*)
The edges that connect two vertices of a graph is stated with weighted values such as distance, cost, time and others.

References

Bondy, J.A. and Murty, U.S.R. (1982) *Graph Theory With Applications*. New York. Elsevier Science Publishing Co. Inc.

Christopher, C. (1991). *The Concise Oxford Dictionary of Mathematics*. Oxford University Press.

Glosari Matematik Pusat Rujukan Persuratan Melayu, Dewan Bahasa dan Pustaka is referred to the website http://prpmv1.dbp.gov.my

Izham Shafie. (2000). *Pengantar Statistik*. Penerbit Universiti Utara Malaysia.

James, N. (2008). *A Level Mathematics for Edexcel Statistics S1*. Oxford University Press.

Lan, F. H. and Yong, K. C. (2016). *Revision Essential Additional Mathematics SPM*. Sasbadi Sdn. Bhd.

Mok, S.S. (2011). *Logik dan Matematik Untuk Penyelesaian Masalah*. Penerbitan Multimedia Sdn. Bhd.

Murdoch, J. and Barnes, J.A. (1973). *Statistik: Masalah dan Penyelesaian*. Unit Penerbitan Akademik Universiti Teknologi Malaysia.

Nguyen-Huu-Bong. (1996). *Logik dan Penggunaannya untuk Sains Komputer*. Penerbit Universiti Sains Malaysia.

Ooi, S.H., Moy, W.G., Wong, T.S. and Jamilah Binti Osman. (2005). *Additional Mathematics Form 4*. Penerbit Nur Niaga Sdn. Bhd

Paul, Z. (1999). *The Art and Craft of Problem Solving*. John Wiley and Sons, Inc.

Ted, S. (2018). *Mathematical Reasoning: Writing and Proof*. Pearson Education, Inc.

Terlochan, S. (1986). *Buku Rujukan dan Kamus Matematik*. Kuala Lumpur, Malaysia. Tropical Press Sdn. Bhd.

Wan Fauzi Wan Mamat. (2010) *Probability*. Visual Print Sdn. Bhd.

Wong, T.S., Moy, W.G., Ooi, S.H., Khoo, C., and Yong, K.Y. (2005). SPM Focus U *Matematik Tambahan*. Penerbitan Pelangi Sdn. Bhd.

Yap, B. W. and Nooreha Husain. (1998). *Pengenalan Teori Kebarangkalian*. IBS Buku Sdn. Bhd.

Index

Acceleration *195, 200, 203*
Active income *274, 276*
Antecedent *63, 64, 68*
Argument *71, 72, 75*
Assets *274*
Attainable *273, 274, 283*
Average speed *189, 197, 203*
Axis of symmetry *8, 9, 12*

Bankruptcy *289*
Box plot *226, 227, 230*

Cash flow *274, 276, 282*
Coefficient *4*
Combined events *244, 246*
Common region *169, 174*
Complement *100, 110*
Compound statement *60, 89*
Conclusion *71, 73, 75*
Consequent *63, 64, 68*
Constant *3, 23*
Contrapositive *66, 67, 68*
Converse *66, 67, 68*
Counter-example *69, 70*
Cumulative frequency *221*

Dashed line *158, 162, 171*
Deceleration *200, 202*
Deductive *71, 73, 75*
Degree *131, 133, 136*
Dependent events *246, 247*
Digit value *36, 37, 39*
Directed graph *135, 136*
Dispersion *212, 213, 217*
Distance *184, 197*
Distance-time graph *184, 197*
Dot plot *213, 215, 217*

Edge *130, 132, 135*
Expenses *272, 273, 274*
Expression *2, 3, 16*
Extreme value *224, 228*

Factorisation *21, 23*
Financial goal *272, 273, 274, 275, 282*
Financial plan *272, 274, 277*
Financial planning *272, 273, 277, 279*
Fixed expenses *274, 278, 282*
Frequency table *212, 219, 221*

Graph *130, 135, 139*

Implication *63, 66, 68*
Independent events *246, 249*
Inductive *72, 73, 81*
Inflation *284, 285, 288*
Interquartile range *212, 219, 220*
Intersection of sets *96, 100*
Inverse *66, 67, 68*

Liabilities *274*
Linear inequality *156, 158*
Linear inequality system *167*
Long-term *272, 274, 275, 282*
Loop *132, 136*

Maximum point *7, 8, 9*
Measurable *273, 283*
Median *220, 226, 227*
Minimum point *7, 8, 9*
Multiple edges *131, 132*
Mutually exclusive events *253*

Needs *273, 280*
Negation *59, 70*
Network *130, 143*
Non-mutually exclusive events *253, 255, 259*
Number base *34, 45*

Observation *213*
Outlier *224, 229*

Passive income *274, 276, 287*
Place value *35, 36, 39*
Premise *71, 72, 81*

Quadratic *2, 3, 7, 9*
Quadratic equation *15, 16, 21*
Quadratic function *2, 5, 7, 9*

Range *212, 219, 224*
Rate of change in distance with respect to time *184, 189, 191*
Rate of change of speed with respect to time *195, 200, 201*
Realistic *273, 274, 283*
Region *158*
Root *16, 17, 18, 22*

Sample space *244, 245*
Short-term *272, 274, 282, 285*
Simple graph *131, 134, 139*
SMART *273, 279, 280, 283*
Solid line *158*
Specific *273, 277, 283*
Speed *184, 187, 189*
Speed-time graph *195, 197*
Standard deviation *212, 221, 224*
Statement *56, 58, 59, 60*
Stationary *187, 194*
Stem-and-leaf plot *214, 216*
Subgraph *139*

Time *184, 195*
Time-bound *273, 274, 283*
Tree *139, 146*
Truth value *56, 57, 59*

Undirected graph *135, 143*
Ungrouped data *219, 220*
Uniform speed *187, 200*
Union of sets *106, 110*
Unweighted graph *137, 143*

Validity *75*
Variable *2, 3, 16, 156*
Variable expenses *274, 278*
Variance *221, 228, 229*
Vertex *130, 132, 139*

Wants *273*
Weighted graph *137, 143*

SKIM PINJAMAN BUKU TEKS

Sekolah _____

Tahun	Tingkatan	Nama Penerima	Tarikh Terima

Nombor Perolehan: _____

Tarikh Penerimaan: _____

BUKU INI TIDAK BOLEH DIJUAL

RM9.30
ISBN 978-983-77-1531-8
9 789837 715318
FT084001